Moment Approach for Astrometric Binary

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with

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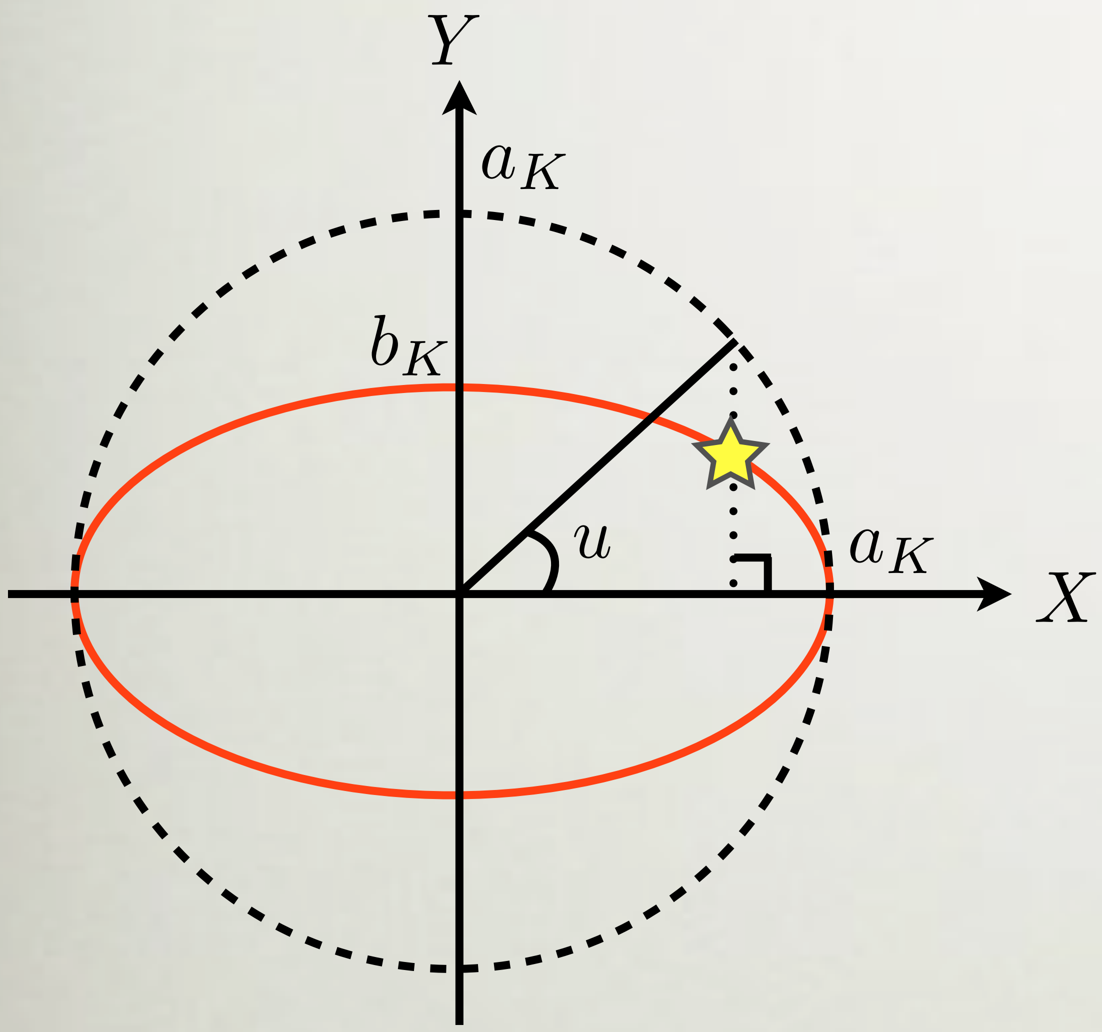
Orbit Determination

- Solar system
- Extra-solar system
  - Visual binary (Savary, etc...)
  - Astrometric binary ← This work!

Kepler Equation



$$t = t_0 + \frac{P_K}{2\pi}(u - e_K \sin u).$$

t_0 : Time of Periastron passage,

P_K: Orbital period,

e_K : Eccentricity

a_K : Semi-major axis, b_K : Semi-minor axis, u : Eccentric anomaly

Orbit Determination

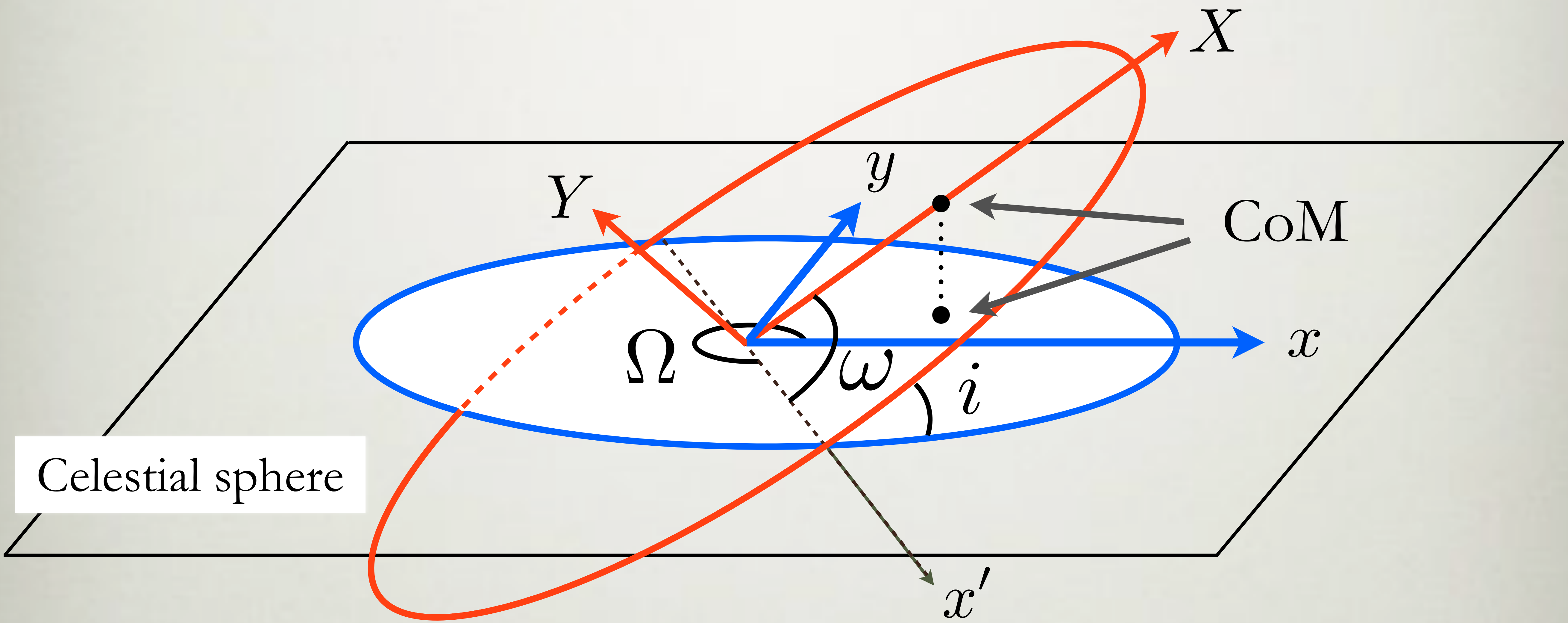
- Non-linear simultaneous equations
- Transcendental equation (Kepler equation)
- 6 parameters (t_0, e_K, a_K, i, ω, Ω)

⇒ Numerical fitting

But Time & Computer resources

Importance of analytic approach

Actual Ellipse & Apparent Ellipse

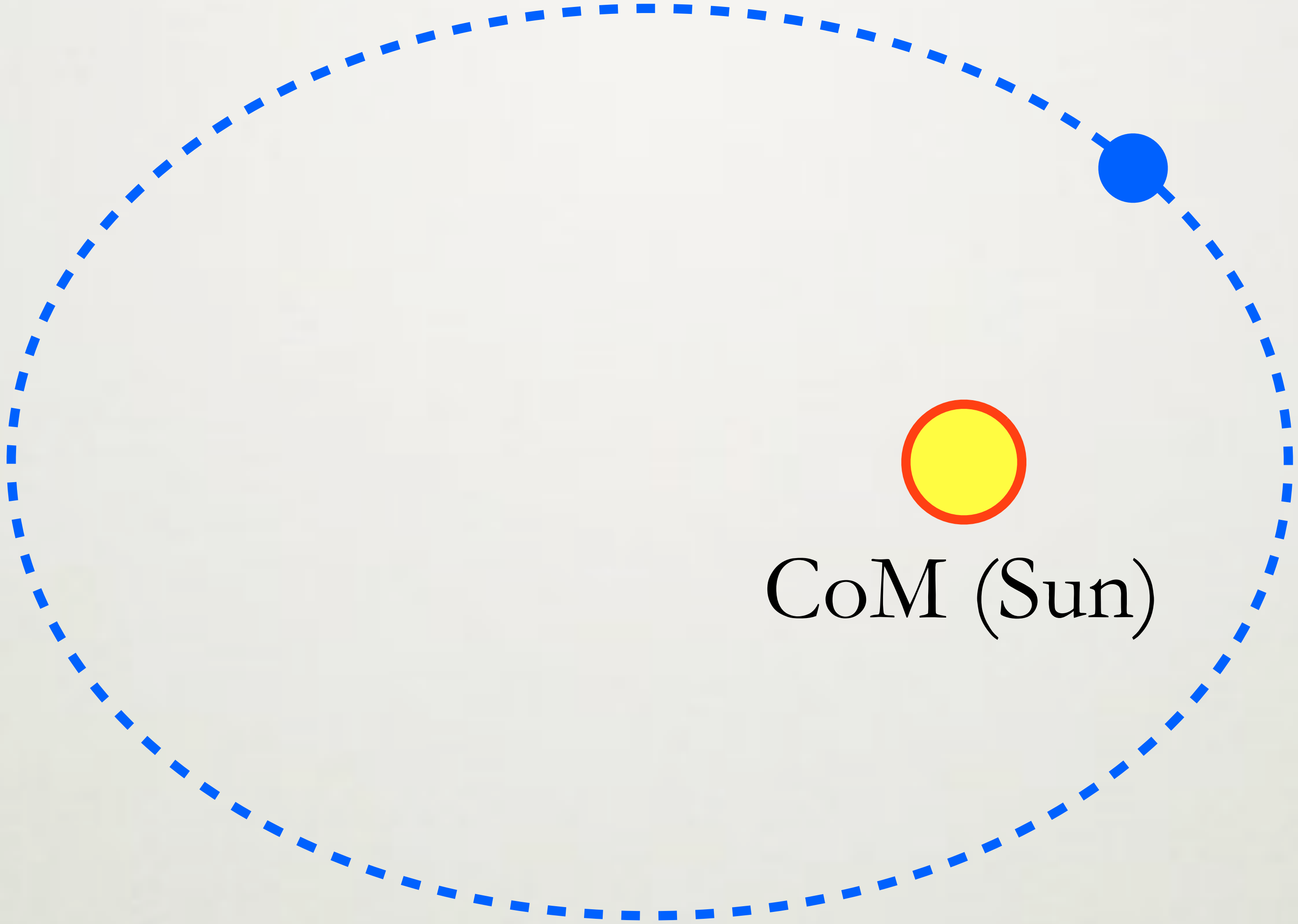


i : Inclination angle,

ω : Argument of Periastron,

Ω : Longitude of Ascending node

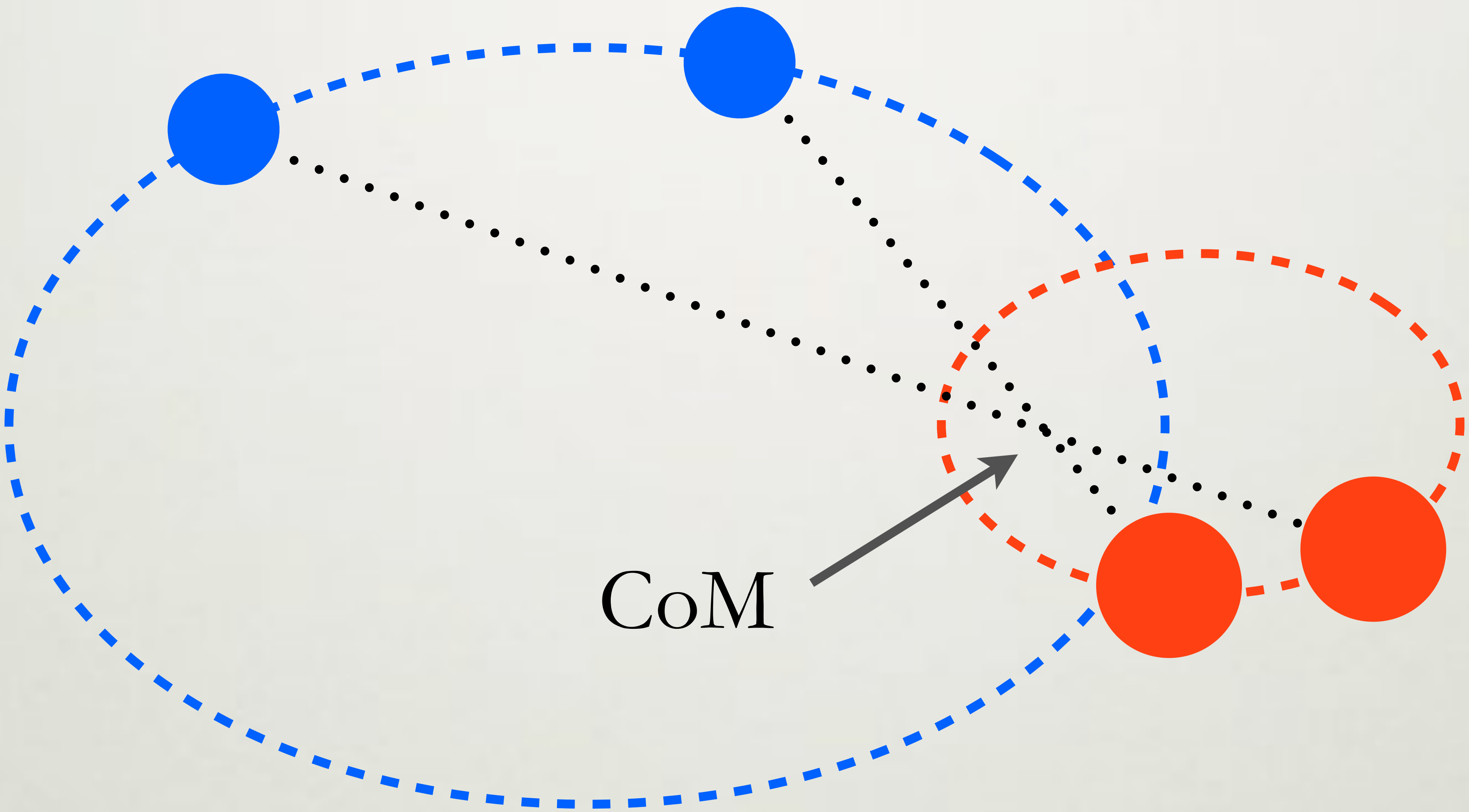
Solar System



- Center of mass (CoM) = Sun
- Orbit determination of Ceres by Gauss (1801)

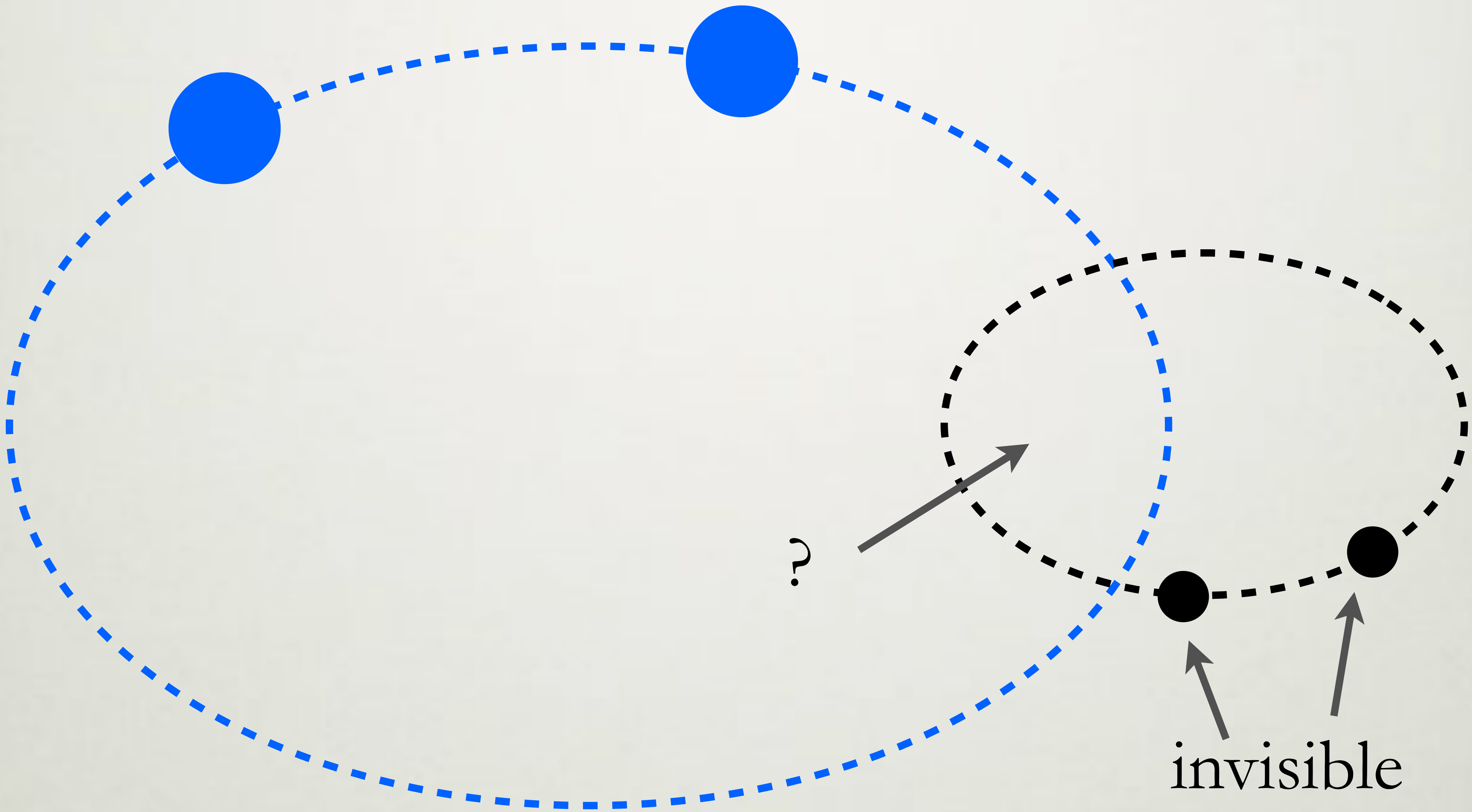
Visual Binary

Savary (1827), Encke (1832), Herschel (1833), etc.



3 Observations ⇒ Orbital parameters

Astrometric binary



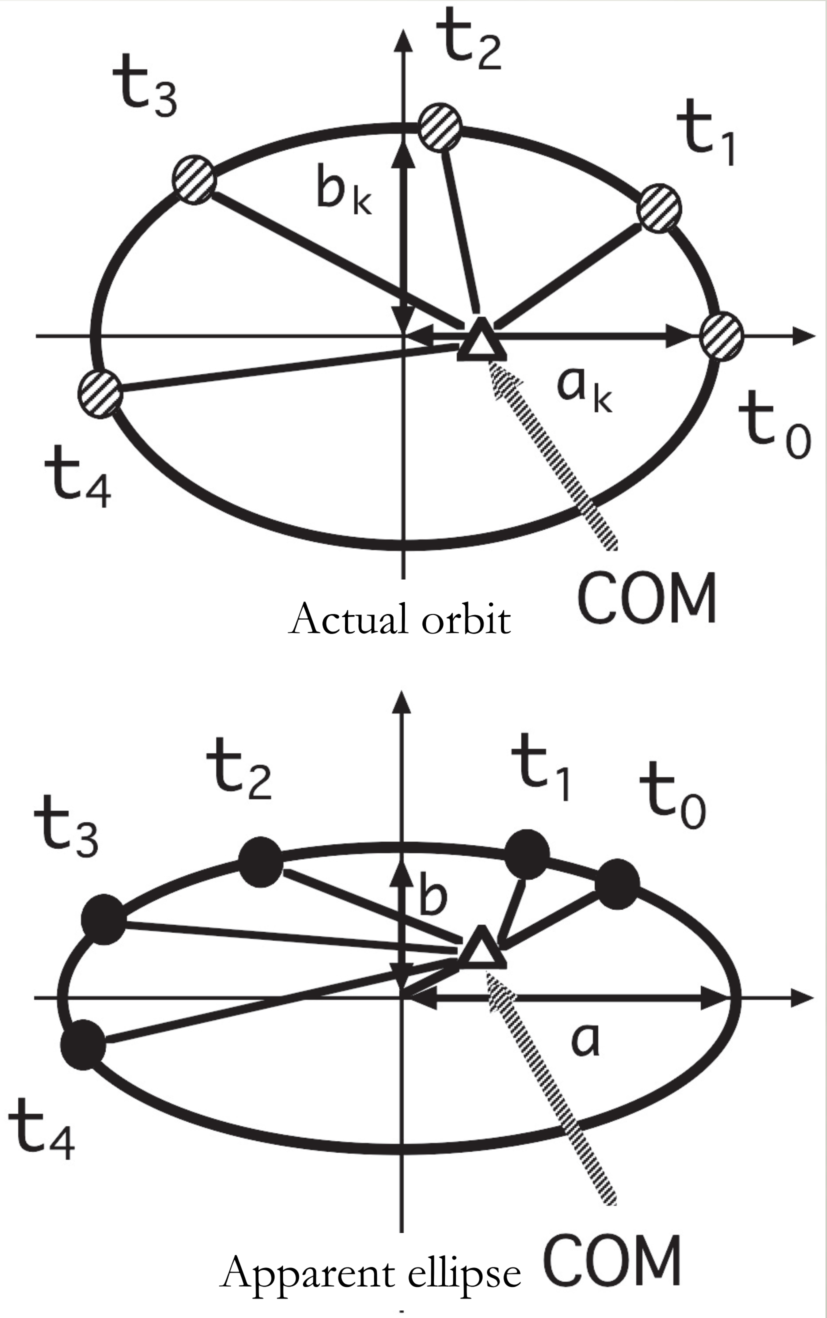
Where is CoM?

Analytic Solution by Asada et al.

- 4 observed points
- Law of constant-areal velocity

CoM's position

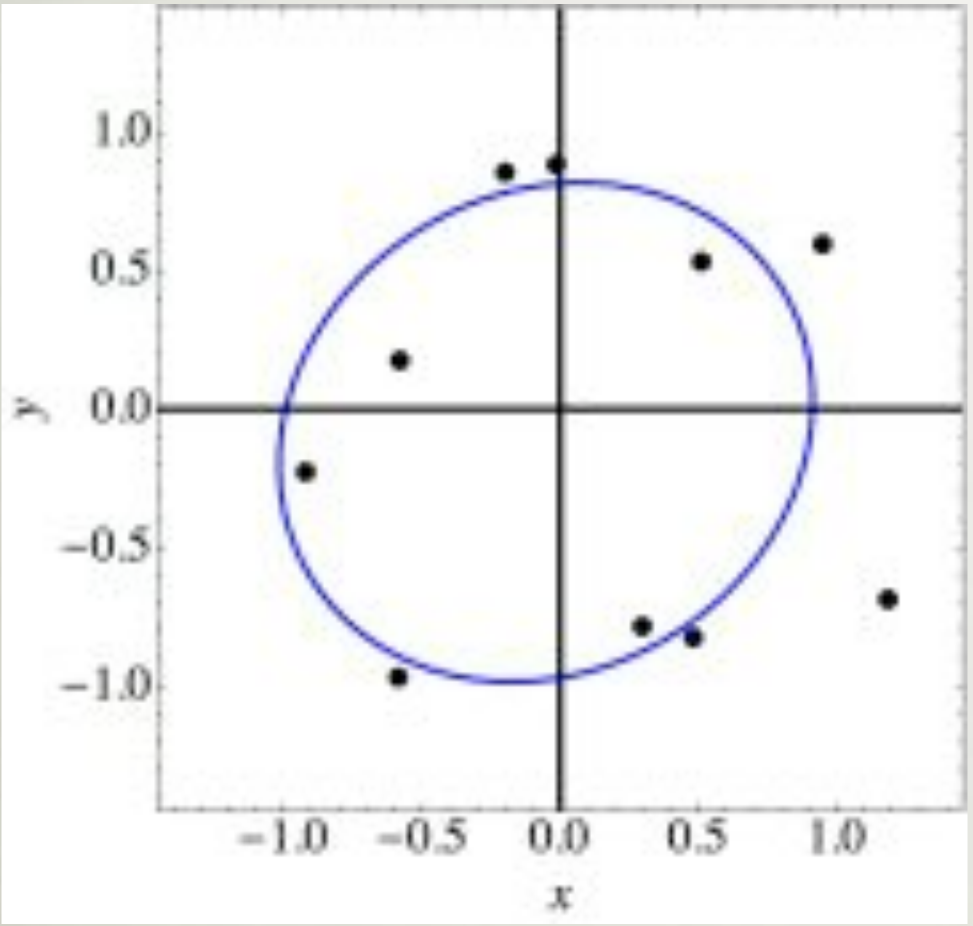
Orbital elements



[Asada et al. 2004]

Asada et al. for Close Binary

Suppose Cyg X-1 by JASMINE



	True value	Mean of estimated value
e_K	0.1	5.36259
a_K	1	$1.53289 - 9.59698 \times 10^{-18}\ i$
i	30	$77.6788 + 24.0837\ i$
ω	30	$18.8841 + 0.631039\ i$
Ω	30	$42.7195 + 1.22993 \times 10^{-13}\ i$

- too large Obs. errors
- Need Another approach!

Mass of Compact Object

from Kepler's Third Law

$$M_C = \left(\frac{4\pi^2}{G}\right)^{1/3} \left(\frac{M_{\rm tot}}{P_K}\right)^{2/3} a_{\rm K}.$$

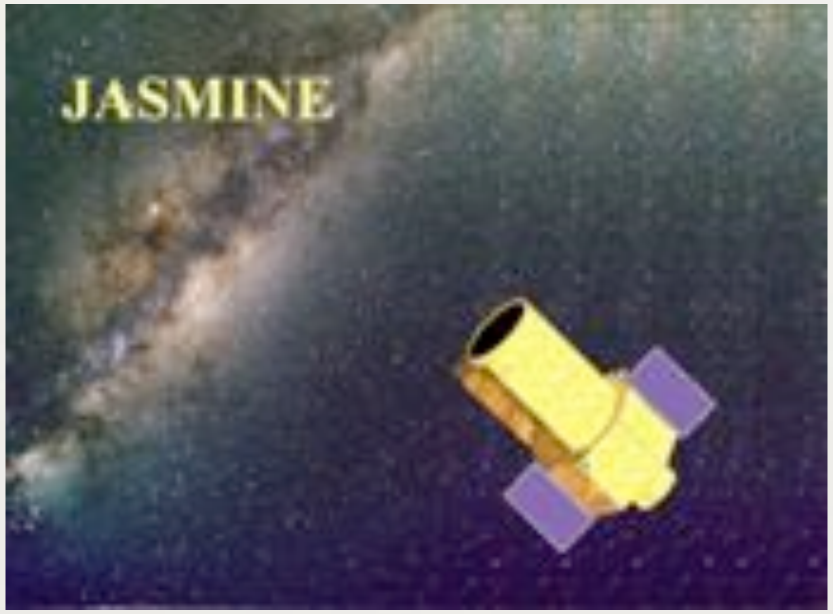
M_C: Mass of Compact object, $P_{\rm K}$: Orbital period,
M_{tot}: Total mass of System, a_K : Semi-major axis

$P_{\rm K}$, $M_{\rm tot}$: Other observations (e.g. Doppler method)

M_C is obtained from a_K

Character of Compact object (NS or BH)

JASMINE missions



[http://www.jasmine-galaxy.org/index-ja.html]

- Infrared Space Astrometry Missions
- Stars w/i 10 kpc w/ accuracy 10 μas.
- Compact binaries (e.g. Cyg X-1)
- Many observed points → Moment approach!!

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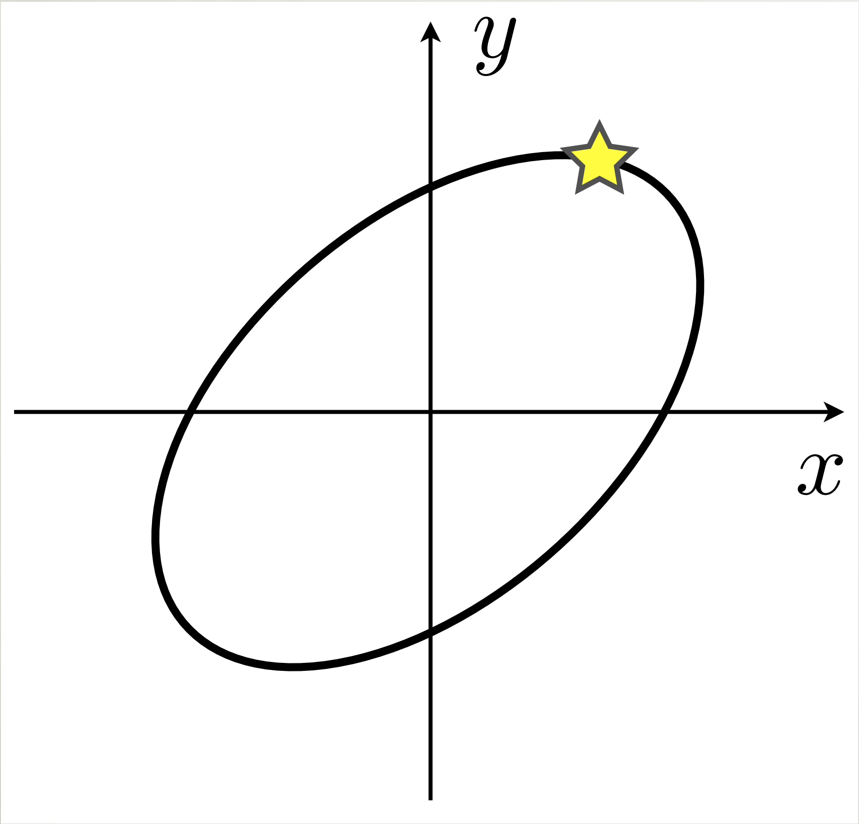
Actual Ellipse & Apparent Ellipse

Star position on Celestial sphere

$$x = x_0 + \alpha \cos u + \beta \sin u,$$
$$y = y_0 + \gamma \cos u + \delta \sin u$$



$$x_0 \equiv -a_K e_K (\cos\omega \cos\Omega - \sin\omega \sin\Omega \cos i),$$
$$y_0 \equiv -a_K e_K (\cos\omega \sin\Omega + \sin\omega \cos\Omega \cos i),$$
$$\alpha \equiv a_K (\cos\omega \cos\Omega - \sin\omega \sin\Omega \cos i),$$
$$\beta \equiv -b_K (\sin\omega \cos\Omega + \cos\omega \sin\Omega \cos i),$$
$$\gamma \equiv a_K (\cos\omega \sin\Omega + \sin\omega \cos\Omega \cos i),$$
$$\delta \equiv -b_K (\sin\omega \sin\Omega - \cos\omega \cos\Omega \cos i)$$

Statistical & Temporal Averages

Statistical average
(Observable)

$$\langle Q \rangle_{\mathrm{S}} = \frac{1}{N} \sum_{i=1}^{N} Q_i$$

N : # of Quantities

Temporal average
(Function of orbital parameters)

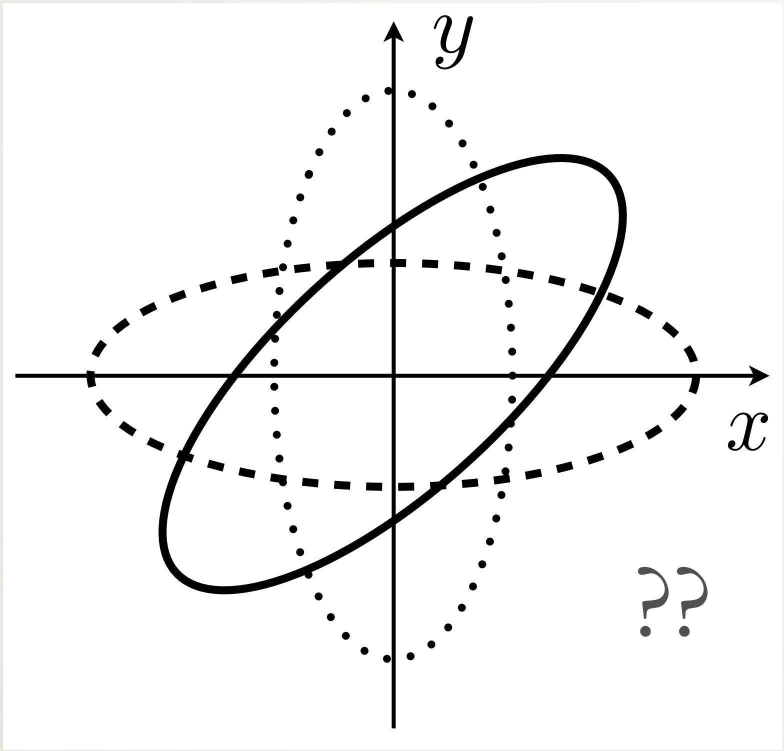
$$\langle Q \rangle_{\mathrm{T}} = \frac{1}{P_{\mathrm{K}}} \int_{t_0}^{t_0+P_{\mathrm{K}}} Q dt$$

P_{K} : Orbital period

$$\langle Q \rangle_{\mathrm{S}} = \langle Q \rangle_{\mathrm{T}}$$

for sufficiently large N

Second Moments (Dispersion)



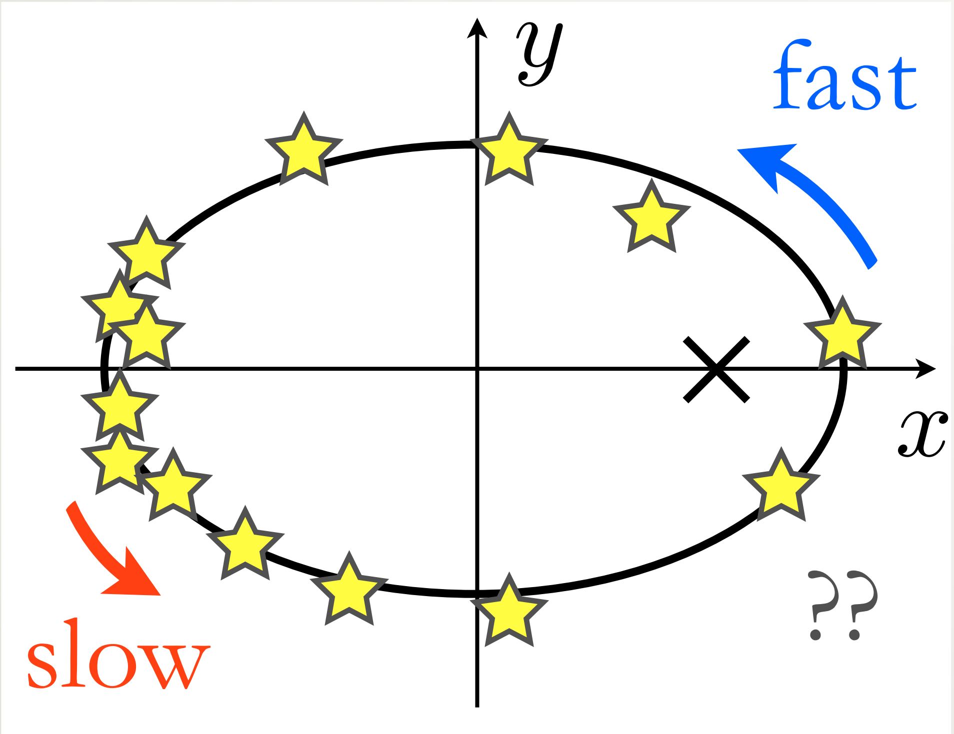
for Ellipse's shape

$$M_{xx} \equiv \langle (x - \langle x \rangle)^2 \rangle_{\mathrm{S}} = \frac{1}{2}(\alpha^2 + \beta^2) - \frac{1}{4} e_K^2 \alpha^2,$$

$$M_{yy} \equiv \langle (y - \langle y \rangle)^2 \rangle_{\mathrm{S}} = \frac{1}{2}(\gamma^2 + \delta^2) - \frac{1}{4} e_K^2 \gamma^2,$$

$$M_{xy} \equiv \langle (x - \langle x \rangle)(y - \langle y \rangle) \rangle_{\mathrm{S}} = \frac{1}{2}(\alpha\gamma + \beta\delta) - \frac{1}{4} e_K^2 \alpha\gamma,$$

Third Moments (Skewness)



for CoM's position

$$M_{xxx} \equiv \langle (x - \langle x \rangle)^3 \rangle_\mathrm{S} = \frac{3}{8} e_K \alpha(\alpha^2 + \beta^2) - \frac{1}{4} e_K^3 \alpha^3,$$

$$M_{yyy} \equiv \langle (y - \langle y \rangle)^3 \rangle_\mathrm{S} = \frac{3}{8} e_K \gamma(\gamma^2 + \delta^2) - \frac{1}{4} e_K^3 \gamma^3,$$

$$M_{xxy} \equiv \langle (x - \langle x \rangle)^2 (y - \langle y \rangle) \rangle_\mathrm{S} = \frac{1}{8} e_K (3\alpha^2\gamma + \beta^2\gamma + 2\alpha\beta\delta) - \frac{1}{4} e_K^3 \alpha^2 \gamma,$$

$$M_{xyy} \equiv \langle (x - \langle x \rangle)(y - \langle y \rangle)^2 \rangle_\mathrm{S} = \frac{1}{8} e_K (3\alpha\gamma^2 + \alpha\delta^2 + 2\beta\gamma\delta) - \frac{1}{4} e_K^3 \alpha\gamma^2.$$

Moment Approach

Second Moment: $M_{xx},\ M_{xy},\ M_{yy}$

Third Moment: $M_{xxx},\ M_{xxy},\ M_{xyy},\ M_{yyy}$

w/o Kepler eq.

- Orbital elements (except Orbital period)

+

- Orbital period by Other observations

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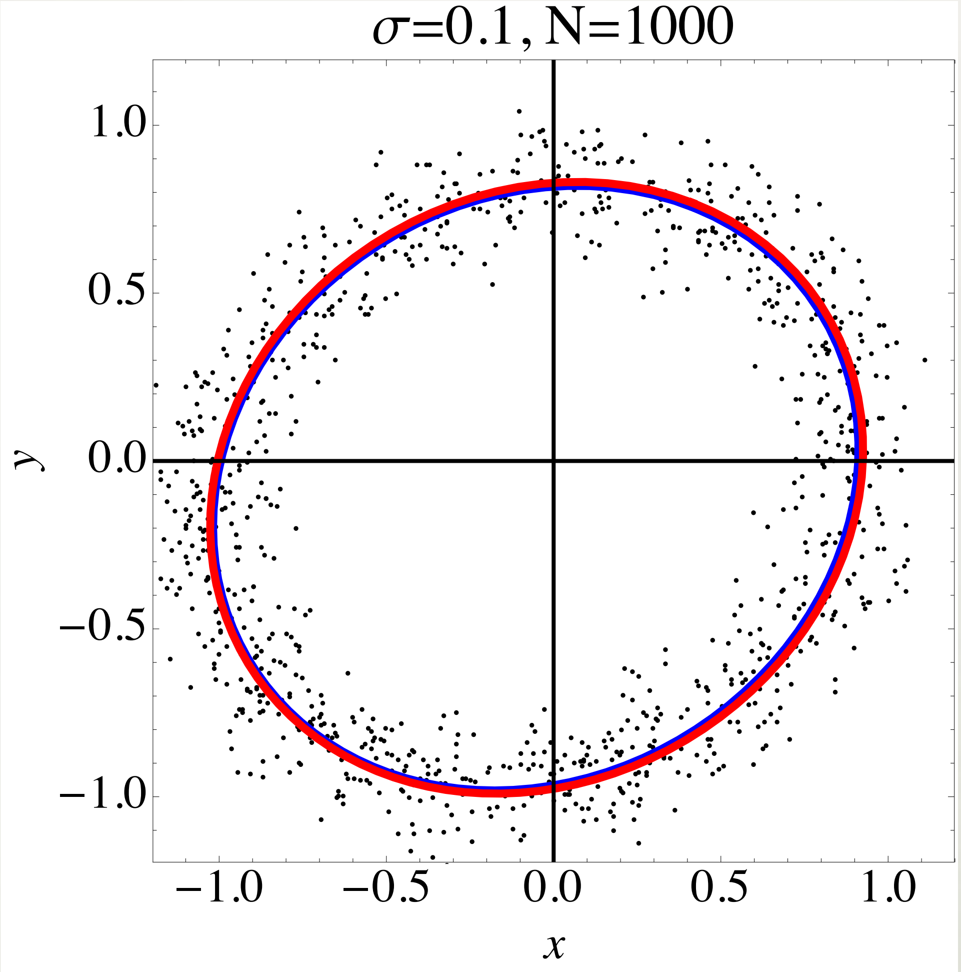
Numerical Tests

Test for Total number of Observed points $N = 1000$

Input: $e_K = 0.1$, $a_K = 1.0$, $i = \omega = \Omega = 30$ [deg.]



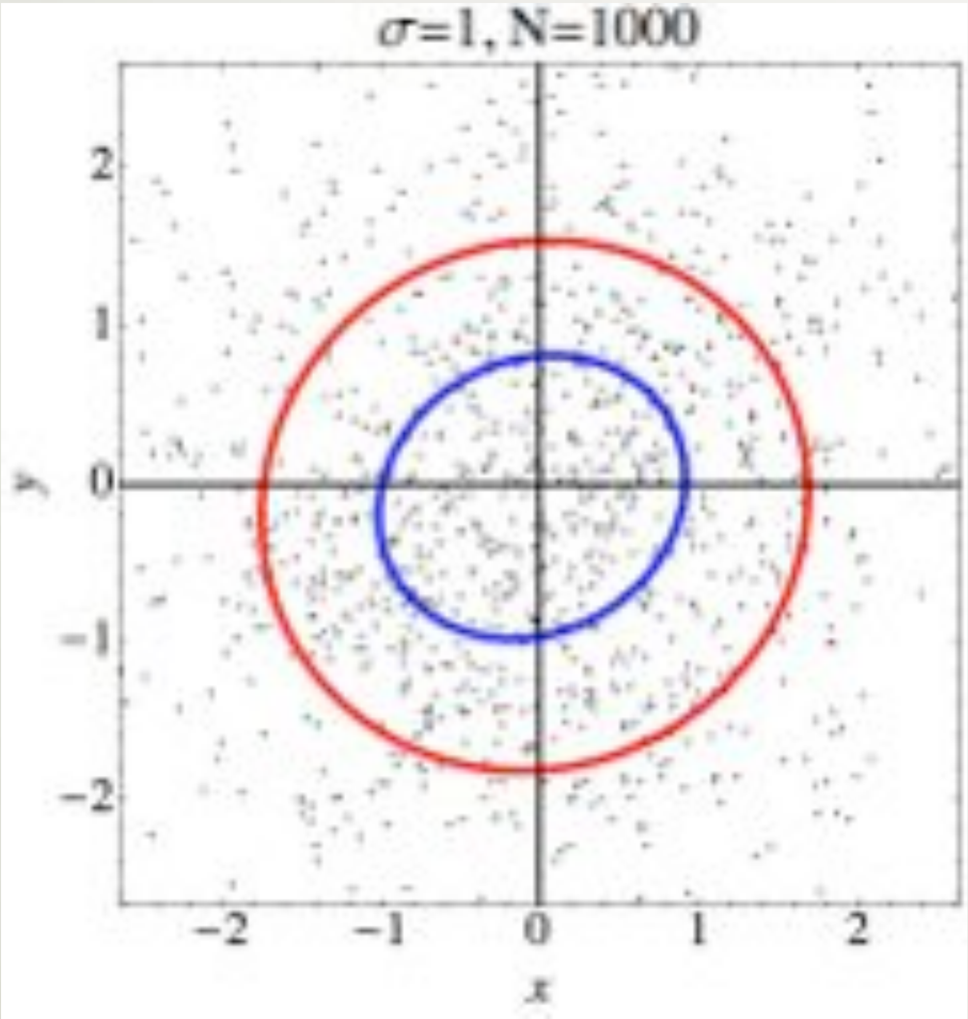
σ : Standard deviation of Obs. errors

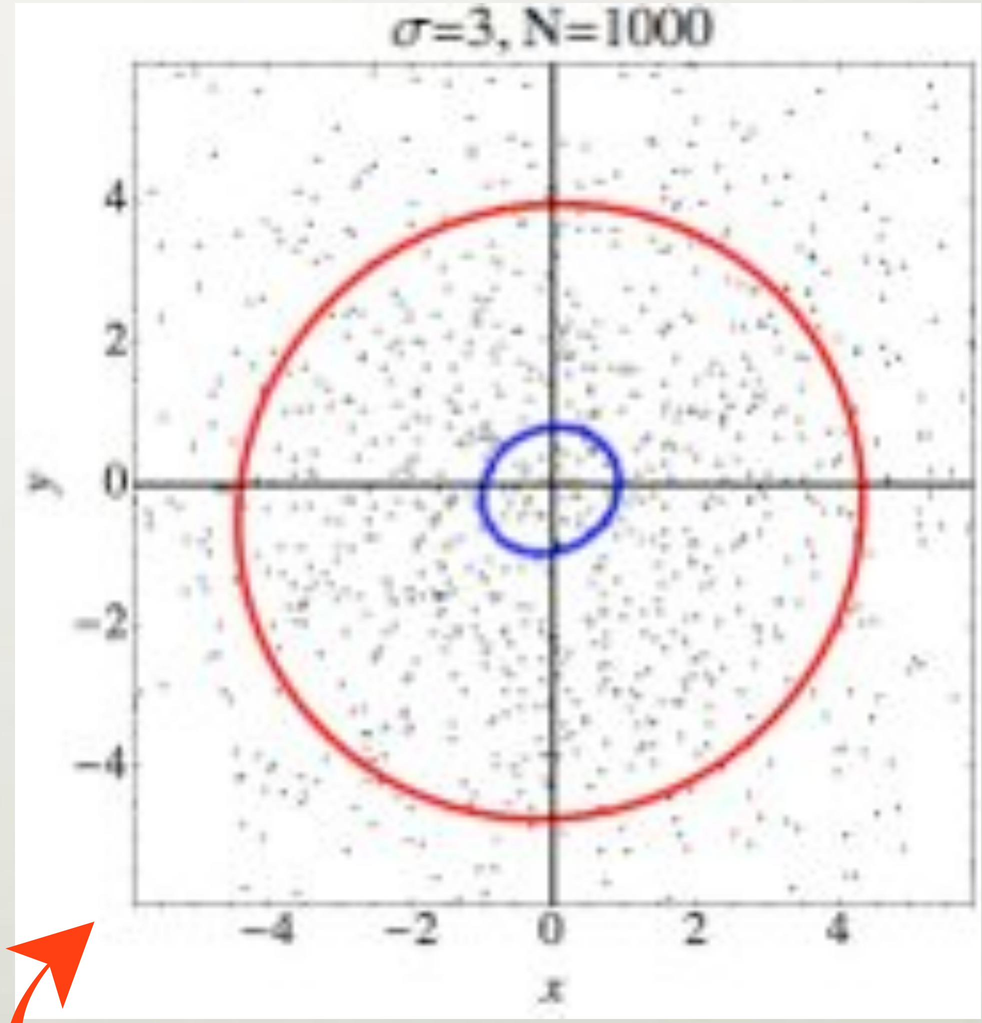
Observed points, True orbit, Estimated orbit

Numerical Tests

Test for Total number of Observed points $N = 1000$

Input: $e_K = 0.1$, $a_K = 1.0$, $i = \omega = \Omega = 30$ [deg.]





Cyg X-1 by JASMINE

Numerical Tests

Test for Total number of Observed points $N = 1000$

Input: $e_K = 0.1$, $a_K = 1.0$, $i = \omega = \Omega = 30$ [deg.]

σ : Standard deviation of Obs. errors

σ	a_K
0.1	1.01001 ± 0.00471901
1	1.74518 ± 0.0308854
3	4.4476 ± 0.0764596

Incorporate Temporal information

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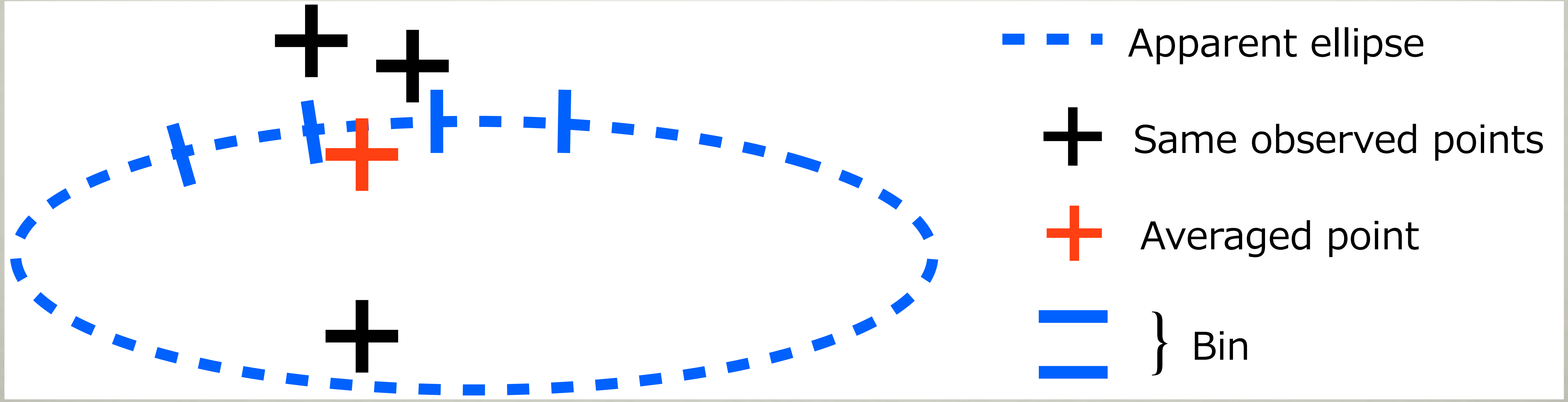
Averaged Observed Points

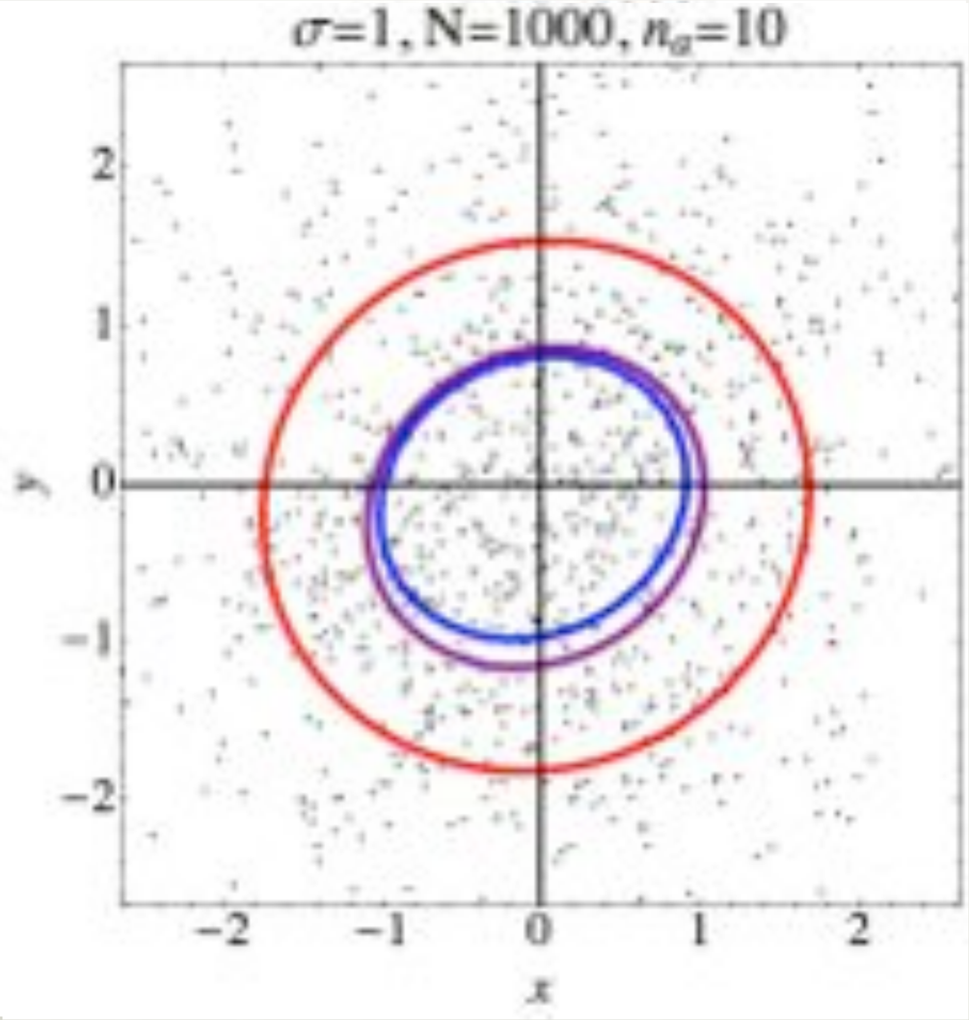
- Orbital period P_{K} is known
  $\rightarrow$ Divide Orbital period into Small bins by Δt
- Bin = Short time obs. $\rightarrow$ "Same point" obs.
- Average "Same points"

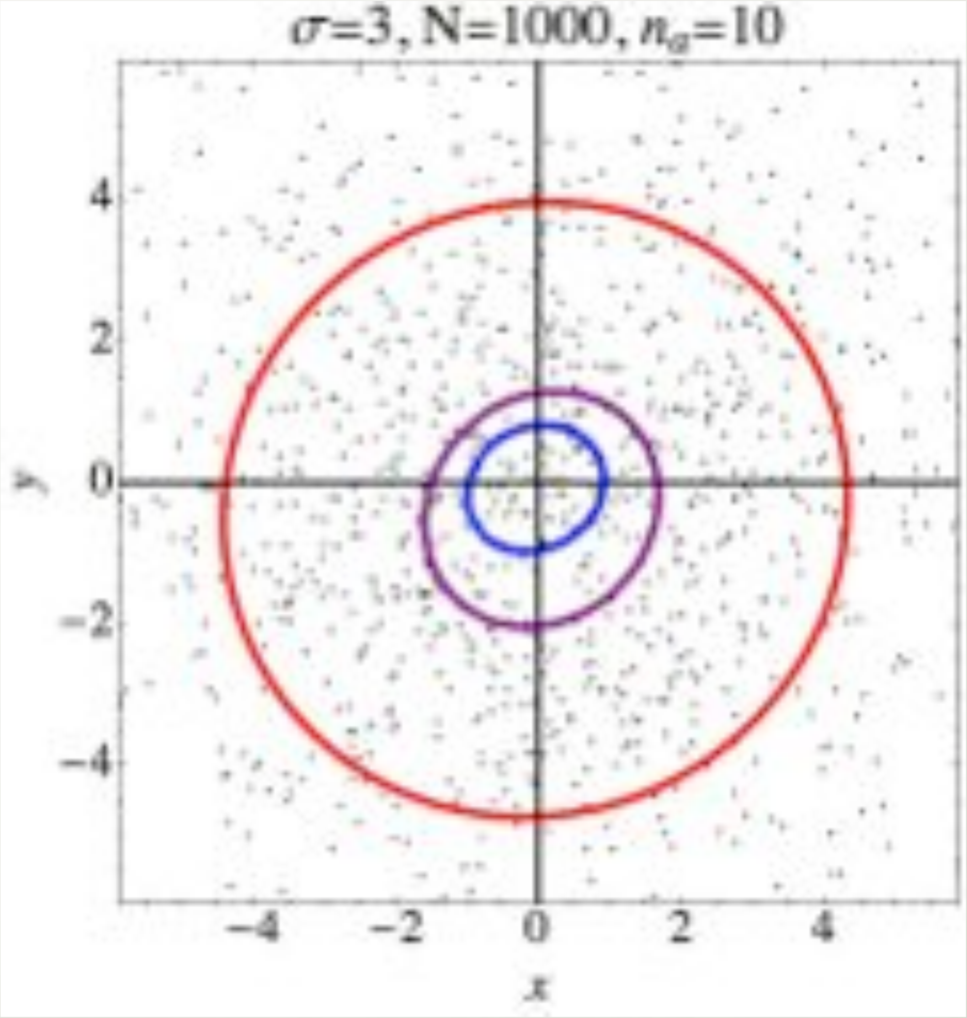


Orbit determination by Averaging

Test for 100 bins, $N = 1000$

Input: $e_K = 0.1$, $a_K = 1.0$, $i = \omega = \Omega = 30$ [deg.]





Observed points, True orbit, No-averaging, Averaging

Orbit determination by Averaging

Test for $N = 1000,\ n_a = 10$

Input: $e_K = 0.1,\ a_K = 1.0,\ i = \omega = \Omega = 30$ [deg.]

σ : Standard deviation of Obs. errors

σ	a_K
0.1	1.00099 ± 0.004836
1	1.10828 ± 0.0453001
3	1.7854 ± 0.163035

Accuracy improved!!

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Summary

- Moment approach for orbit determination
- Improved for low S/N case (w/o Kepler equation)
- Convenient as trial values for numerical fitting.
- Character of compact objects.

Thank you for your attention