Computation and Control of Low Energy Earth-to-Moon Transfers with Moderate Flight Time

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Object:

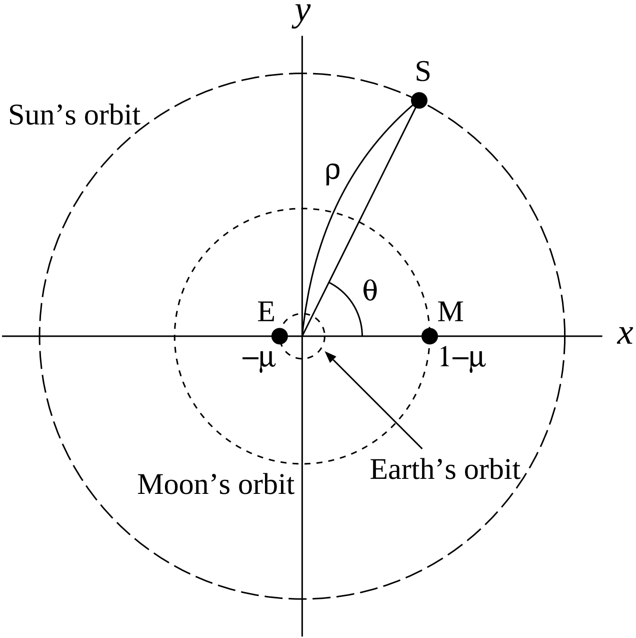
Application of dynamical systems approaches to computation of low cost Earth-to-Moon transfer trajectories of a spacecraft and their control:

- Y, *Physica D*, **197** (2004), 313–331.
- Y, *Celest. Mech. Dynam. Astron.*, **90** (2004), 197–212.
- Y, *Dyn. Syst.*, **23** (2008), 309-331.

Earth-to-Moon Transfers of a Spacecraft

- Planar circular restricted three-body problem (PCR3BP) including Earth and Moon or its perturbation due to Sun
- Traditionally, a trajectory called Hohmann transfer (based on an elliptic orbit in the two-body dynamics) is used
  $\Rightarrow$ About 4 days flight but high cost
- Several trajectories including chaotic ones
  $\Rightarrow$ New type transfers
  e.g., fuel on board 25% off & flight time $\approx$ 90 days (Belbruno and Carrico [2000])
- Present work: Low cost but moderate flight time
  e.g., fuel on board 15% off but flight time $\approx$ 43 days

Rotating Coordinate System



Equation of Motion for the Spacecraft

$$\ddot{x} - 2\dot{y} = \frac{\partial}{\partial x}\Omega_0(x, y) + \frac{\partial}{\partial x}\Omega_1(x, y, \theta),$$

$$\ddot{y} + 2\dot{x} = \frac{\partial}{\partial y}\Omega_0(x, y) + \frac{\partial}{\partial y}\Omega_1(x, y, \theta),$$

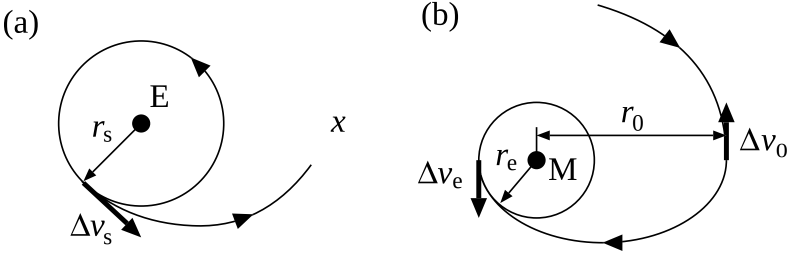
$$\dot{\theta} = \omega_S,$$

where

$$\Omega_0(x, y) = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_E} + \frac{\mu}{r_M},$$

$$\Omega_1(x, y) = \frac{m_S}{r_S} - \frac{m_S}{\rho^2}(x\cos\theta + y\sin\theta)$$

Earth-to-Moon Transfer



The velocity is

- increased by $\boldsymbol{\Delta v_s}$ on a parking orbit (PO) around the Earth;
- decreased by $\boldsymbol{\Delta v_0}$ at the closest point to the Moon;
- and further decreased by $\boldsymbol{\Delta v_e}$ on a PO around the Moon.

For $m_S = 0$
$\Delta v_M = |\Delta v_0| + |\Delta v_e|$ is the minimum at $r_0 = r_e$

Optimum Transfers

- They go from the PO around the Earth to the PO around the Moon directly ($\boldsymbol{r_0} = \boldsymbol{r_e}$)
- Their velocities at the starting and arriving points are tangent to the POs
- $\boldsymbol{\Delta v} = \boldsymbol{\Delta v_s} + \boldsymbol{\Delta v_e}$ is as small as possible

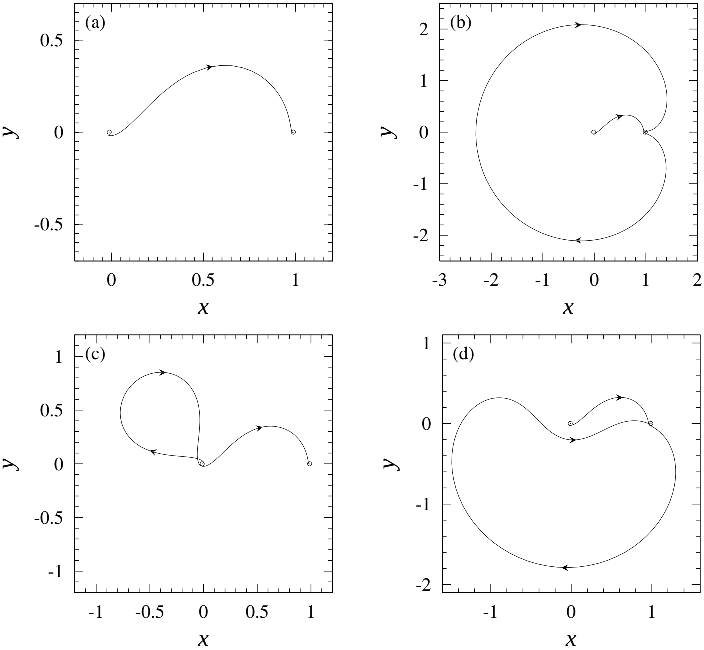
Approach

- Reduction to a nonlinear BVP
- Continuation of solutions from Hohmann type transfers (elliptic orbits) at $\boldsymbol{\mu, m_S = 0}$ by `AUTO` (Doedel et al., 1997)

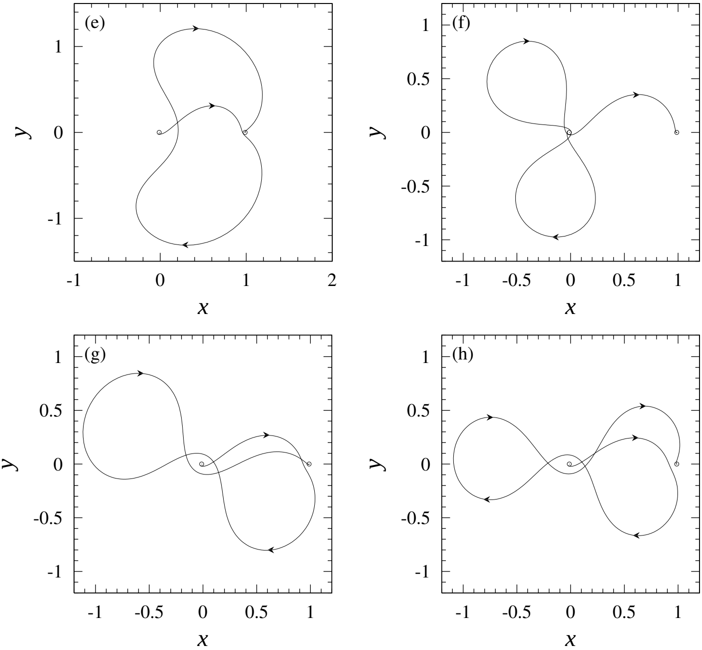
Numerical Example

- Parameter values
  $\boldsymbol{\mu = 0.01215}$, $\boldsymbol{m_S = 3.289 \times 10^5}$
  $\boldsymbol{\rho = 389.2}$, $\boldsymbol{\omega_S = -0.9253}$
  (corresponding to the real system)
  $\boldsymbol{r_s = 0.01703}$ (167 km altitude PO around the Earth)
  $\boldsymbol{r_e = 0.004781}$ (100 km altitude PO around the Moon)
- Several trajectories with the minimum cost were found

Transfer Trajectories ($m_S \neq 0$)



Transfer Trajectories ($m_S \neq 0$)

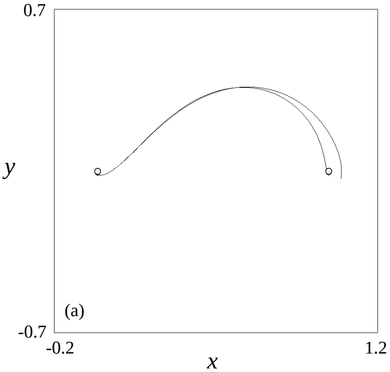


Performance of the Low Cost Transfers

For the Hohmann transfer, $\boldsymbol{\Delta v_s \approx 3.065}$ & $\boldsymbol{\Delta v_e \approx 0.828}$

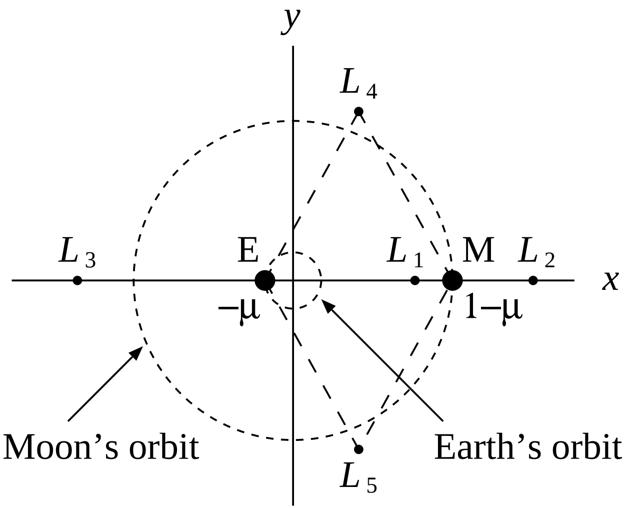
Δv	Δv_s	Δv_e	T	θ_0/π	
3.855 (3.857)	3.063 (3.063)	0.792 (0.794)	1.058 (1.052)	0.5309	(a)
3.763 (3.85)	3.062 (3.062)	0.701 (0.788)	9.993 (9.511)	0.8197	(b)
3.847 (3.853)	3.062 (3.062)	0.785 (0.79)	3.32 (3.297)	-0.7343	(c)
3.883 (3.847)	3.061 (3.061)	0.822 (0.786)	7.404 (7.269)	-0.7923	(d)
3.818 (3.843)	3.061 (3.061)	0.757 (0.782)	8.277 (8.201)	0.6011	(e)
3.848 (3.857)	3.063 (3.063)	0.784 (0.794)	5.579 (5.526)	-0.0358	(f)
3.848 (3.831)	3.058 (3.058)	0.79 (0.772)	7.105 (7.076)	-0.9918	(g)
3.808 (3.823)	3.057 (3.057)	0.751 (0.766)	7.353 (7.347)	0.4724	(h)

Stability



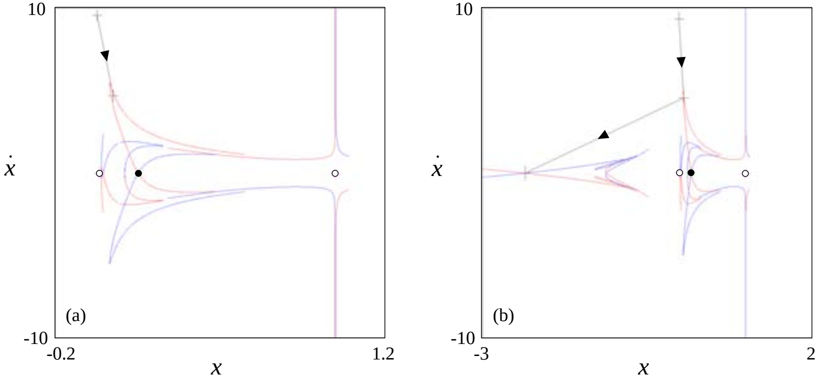
$\dot{x}(0) = 9.542599$ & 9.552599 $\qquad$ $\dot{x}(0) = 9.312612$ & 9.312632

Dynamics of the PCR3BP



Lagrange points

Transfer Trajectories and Invariant Manifolds



Poincaré section $\{(x, \dot{x}, y, \dot{y}) \mid y = 0, \dot{y} > 0\}$

力学系理論の非線形系の制御への利用

- OGY 法（Ott, Grebogi and Yorke, 1990）
  周期軌道近傍の幾何学的な構造（不変多様体）
  $\Rightarrow$ カオスアトラクターに埋め込まれた不安定周期軌道の安定化

従来の力学系理論

- 自律的あるいは時間に関して周期的なベクトル場
- $t \to \infty$ のときの解の漸近的挙動

現実的な問題

- 時間に関して非周期的なベクトル場
- 有限時間区間内での挙動

$\Downarrow$

無限あるいは有限時間区間上の非周期的なベクトル場

- 不変多様体の概念の拡張
- 流体の輸送，混合現象

双曲型軌道（有限時間区間 $[t_-, t_+]$ の場合）

$\phi(x_0, t; t_0)$: **ベクトル場の流れ（$x(t_0) = x_0$）**

定義：軌道 $\bar{\gamma}(t)$ が区間 $[t_-, t_+]$ 上で有限時間双曲型

$\Leftrightarrow$ $\mathrm{D}_x\phi(\bar{\gamma}(t_-), t_+; t_-)$ **が絶対値１の固有値をもたない**

ポアンカレ型写像 $\psi : \mathbb{R}^n \to \mathbb{R}^n$

$$\psi(x) = \phi(x + \bar{\gamma}(t_-), t_+; t_-) - \bar{\gamma}(t_+)$$

定義：有限時間不変多様体

$$\bar{W}^{\mathrm{s}}(\bar{\Gamma}) = \{(\phi(x, t; t_+), t) \,|\, x - \bar{\gamma}(t_+) \in W^{\mathrm{s}}(0),\ t \in [t_-, t_+]\},$$

$$\bar{W}^{\mathrm{u}}(\bar{\Gamma}) = \{(\phi(x, t; t_-), t) \,|\, x - \bar{\gamma}(t_-) \in W^{\mathrm{u}}(0),\ t \in [t_-, t_+]\}$$

$W^{\mathrm{s,u}}(0)$**：写像 ψ に対する平衡点 $x = 0$ の不変多様体**

安定化手法

$\{t_k \mid k \in \mathbb{Z}\}$: **時刻列** s.t. $t_k < t_{k+1}$, $\phi_k(x,\mu) = \phi(x, t_{k+1}; t_k; \mu)$

$$\begin{aligned}\phi(\gamma(t_k) + \Delta x_k, \mu + \Delta\mu_k) - \gamma(t_{k+1}) \\ \approx \mathrm{D}_x\phi(\gamma(t_k),\mu)\Delta x_k + \mathrm{D}_\mu\phi(\gamma(t_k),\mu)\Delta\mu_k\end{aligned}$$

時刻 $t = t_{k+1}$ **で** $W^{\mathrm{s}}(\Gamma) \approx E^{\mathrm{s}}(\Gamma)$ **上に存在**

$$\begin{aligned}\Delta\mu_k = -(M_{\mathrm{u}}\Lambda^{-1}(t_{k+1})\mathrm{D}_\mu\phi_k(\gamma(t_k),\mu))^{-1} \\ M_{\mathrm{u}}\Lambda^{-1}(t_{k+1})\mathrm{D}_x\phi_k(\gamma(t_k),\mu)\Delta x_k\end{aligned} \qquad (m = n_{\mathrm{u}})$$

$$E^{\mathrm{s,u}}(\tau) = \mathrm{span}(e_1^{\mathrm{s,u}}(\tau), \ldots, e_{n_{\mathrm{s,u}}}^{\mathrm{s,u}}(\tau))$$

$$\Lambda(t) = (e_1^{\mathrm{u}}(t), \ldots, e_{n_{\mathrm{u}}}^{\mathrm{u}}(t), e_1^{\mathrm{s}}(t), \ldots, e_{n_{\mathrm{s}}}^{\mathrm{s}}(t))$$

$$M_{\mathrm{u}} = \begin{pmatrix} 1 & & 0 & 0 & \cdots & 0 \\ & \ddots & & \vdots & \ddots & \vdots \\ 0 & & 1 & 0 & \cdots & 0 \end{pmatrix}$$: $n_{\mathrm{u}} \times n$ **行列**

単振り子

運動方程式 $\dot{\theta} = v, \quad \dot{v} = -\sin\theta - \delta_1 v - \delta_0 \mathrm{sgn} v + u(t)$

- **無限時間区間**

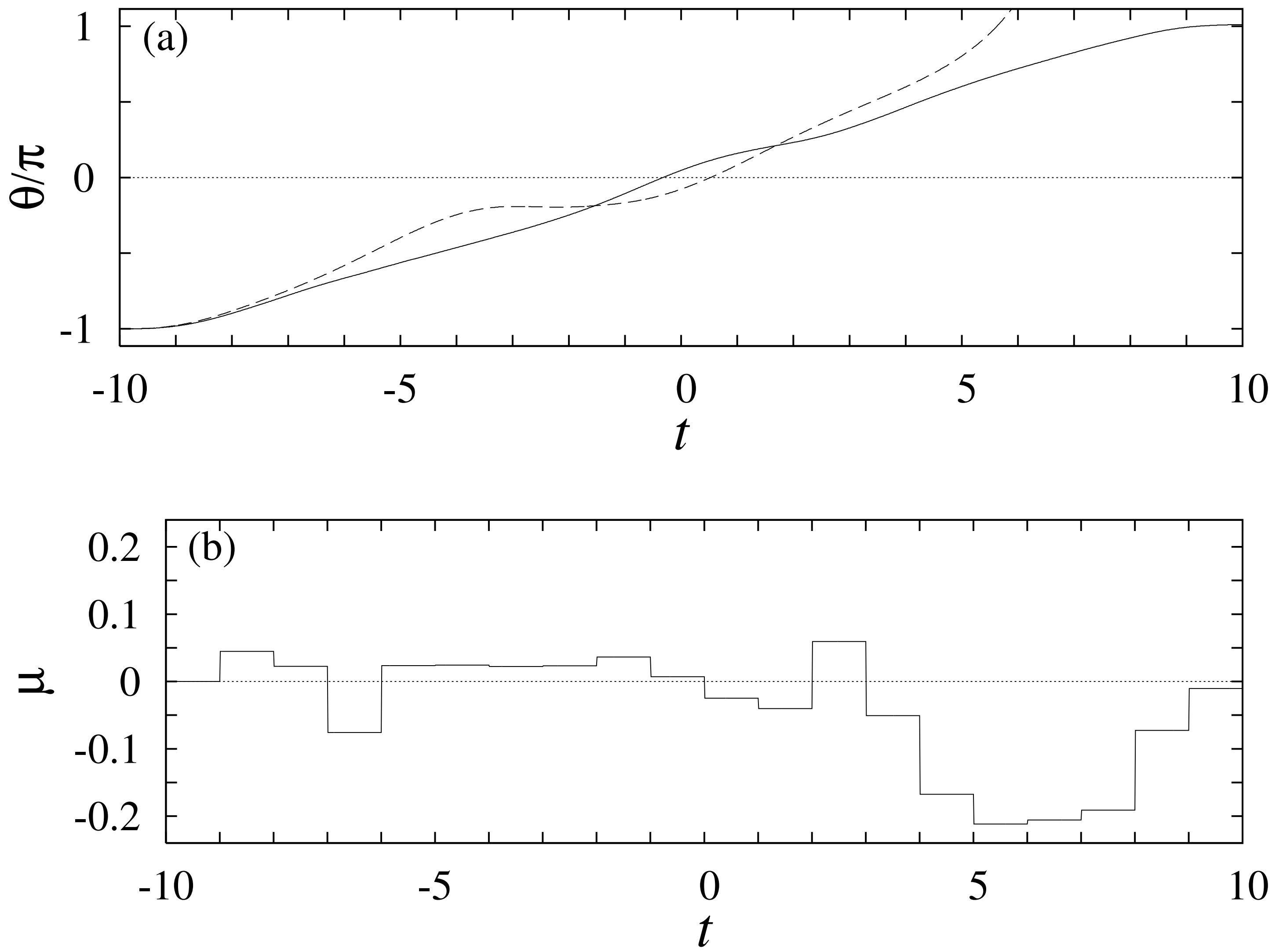
  制御力 $u(t) = 2\delta\,\mathrm{sech}\,t + \mu$

  目標軌道 $\theta = 2\arcsin(\tanh t)$ **（ホモクリニック軌道）**

- **有限時間区間**
  目標軌道

$$\theta_0(t;\theta_*) = \begin{cases} -\dfrac{\pi[(t+T)^4 - 4\Delta T(t+T)^3]}{16T_1\Delta T^3} + \theta_* & \text{for } t \in [-T, -T_2); \\ \dfrac{\pi}{T_1}t + \theta_* + \pi & \text{for } t \in [-T_2, T_2]; \\ \dfrac{\pi[(t-T)^4 + 4\Delta T(t-T)^3]}{16T_1\Delta T^3} + \theta_* + 2\pi & \text{for } t \in (T_2, T], \end{cases}$$

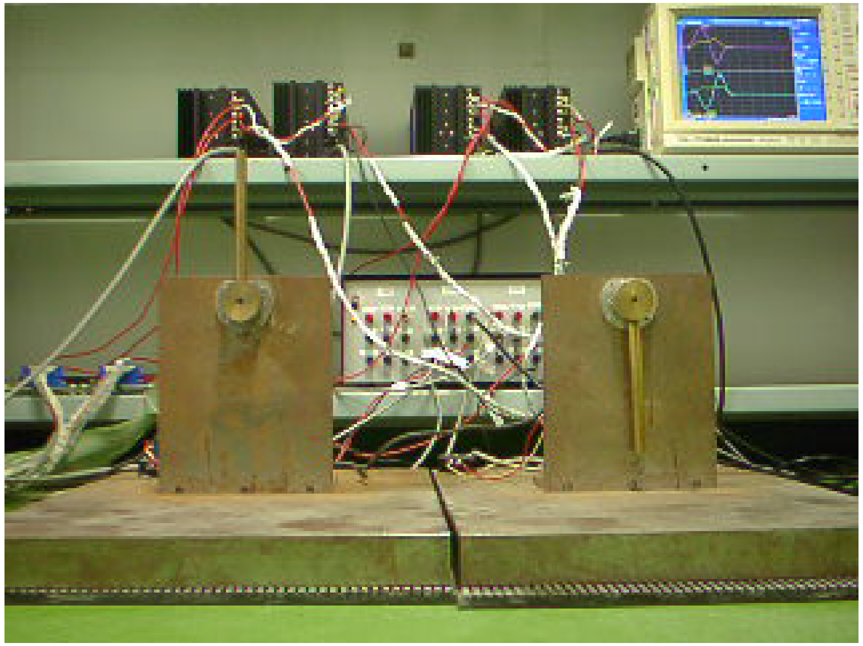
有限時間区間

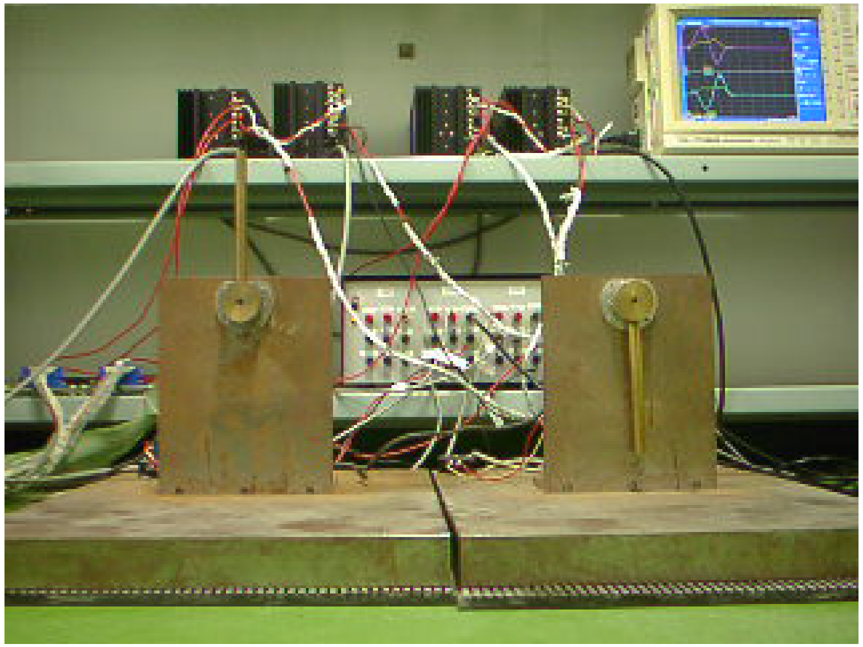


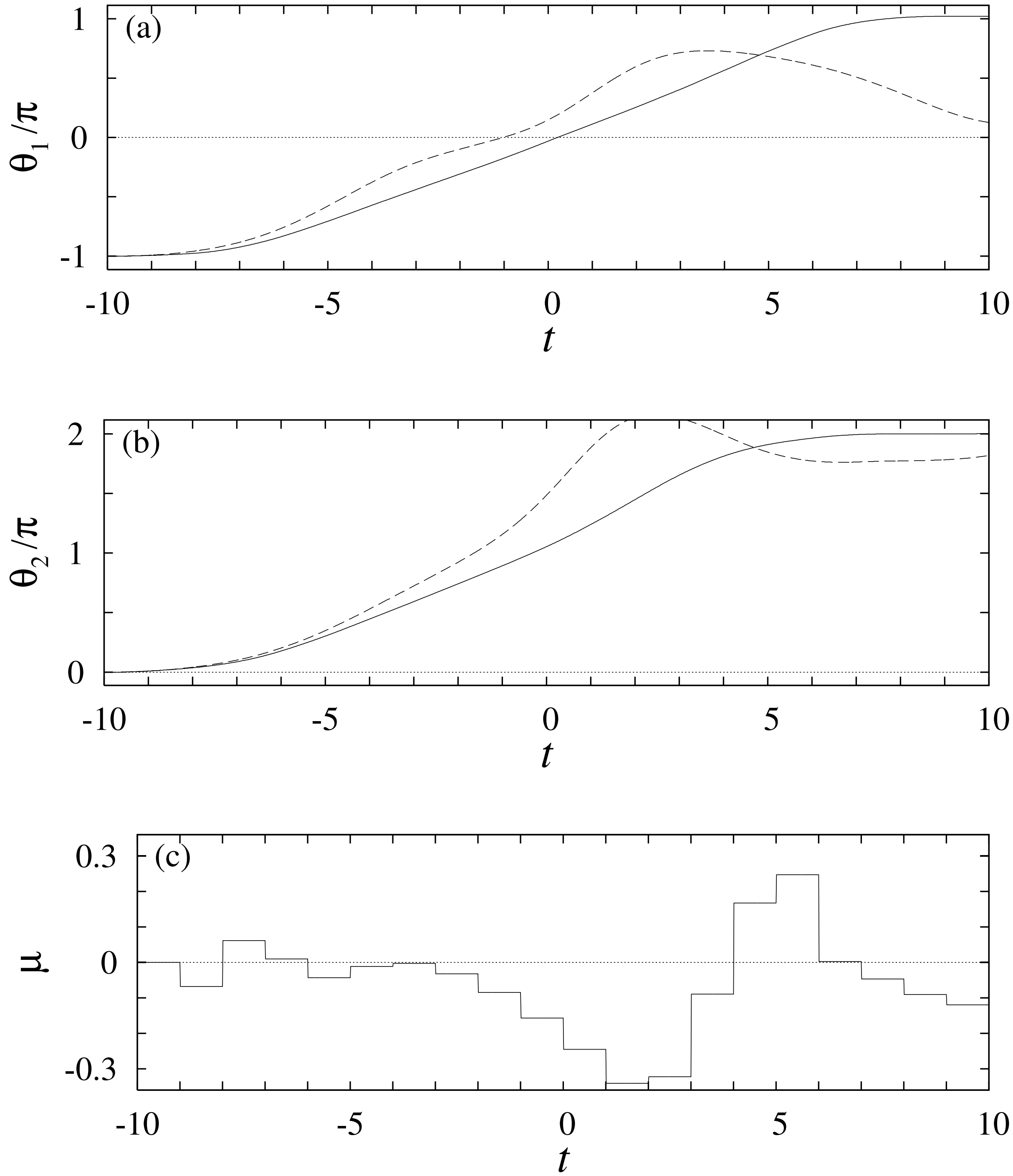
連成振り子

$$\dot{\theta}_1 = v_1, \quad \dot{v}_1 = -\sin\theta_1 - \delta_1 v_1 - \delta_0 \operatorname{sgn} v_1 - \underline{\alpha(v_1 - v_2)} + u_1(t),$$

$$\dot{\theta}_2 = v_2, \quad \dot{v}_2 = -\sin\theta_2 - \delta_1 v_2 - \delta_0 \operatorname{sgn} v_2 - \underline{\alpha(v_2 - v_1)} + u_2(t)$$






(a)
θ1/π
1
0
-1
-10
-5
0
5
10
t
(b)
θ2/π
2
1
0
-10
-5
0
5
10
t
(c)
μ
0.3
0
-0.3
-10
-5
0
5
10
t

Controlled Spacecraft

- Equation of motion for a controlled spacecraft

$$\begin{aligned}
\ddot{x} - 2\dot{y} &= \frac{\partial \Omega_0}{\partial x}(x, y) + \frac{\partial \Omega_1}{\partial x}(x, y, \theta) + \mu_1, \\
\ddot{y} + 2\dot{x} &= \frac{\partial \Omega_0}{\partial y}(x, y) + \frac{\partial \Omega_1}{\partial y}(x, y, \theta) + \mu_2, \\
\dot{\theta} &= \omega_S
\end{aligned}$$

- The Poincaré-type map ψ_{μ_1,μ_2}

$$(x(0), \dot{x}(0), y(0), \dot{y}(0)) \mapsto (x(T), \dot{x}(T), y(T), \dot{y}(T))$$

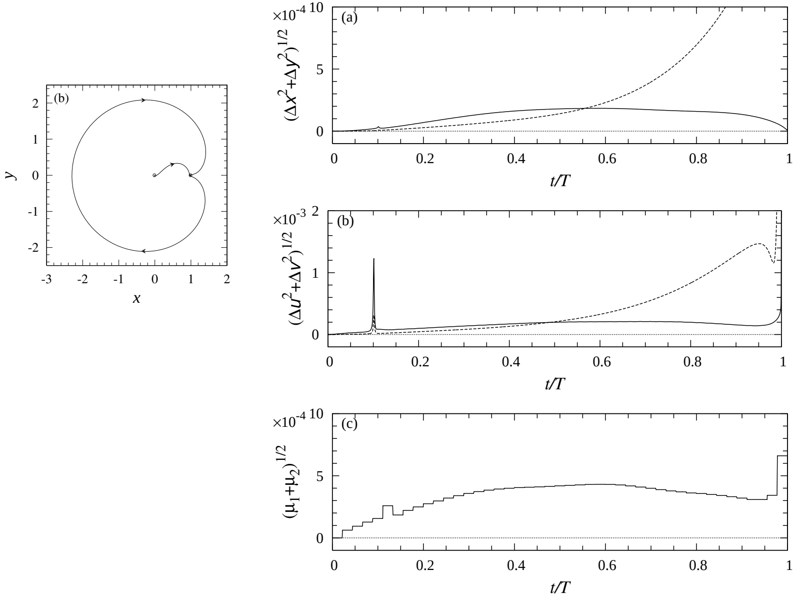
- Stable and unstable manifolds can be defined for ψ_{μ_1,μ_2} in a usual manner
- An extension of the OGY method can be applied: The trajectry is kept to remain on the stable manifold

Finite-Time Lyapunov Exponents

$$\nu_j = \frac{1}{t_+ - t_-} \log |\lambda_j|$$

n	ν_1	ν_2	ν_3	ν_4	
0	8.441	3.777	$\mathbf{-3.777}$	$\mathbf{-8.441}$	(a)
	1.454	0.537	$\mathbf{-0.537}$	$\mathbf{-1.454}$	(b)
1	3.102	1.185	$\mathbf{-1.185}$	$\mathbf{-3.102}$	(c)
	1.759	0.518	$\mathbf{-0.518}$	$\mathbf{-1.759}$	(d)
	1.383	0.692	$\mathbf{-0.692}$	$\mathbf{-1.383}$	(e)
2	1.987	0.682	$\mathbf{-0.682}$	$\mathbf{-1.987}$	(f)
	1.699	0.556	$\mathbf{-0.556}$	$\mathbf{-1.699}$	(g)
	1.497	0.648	$\mathbf{-0.648}$	$\mathbf{-1.497}$	(h)

Case (b) ($\theta(0) = \theta_0 - 0.05$)



Conclusions

- By numerical continuation of solutions in nonlinear BVPs, Earth-to-Moon transfers with low cost and moderate flight time were computed
- The Earth-to-Moon transfers are unstable due to their passage near invariant manifolds of periodic orbits around $\boldsymbol{L_1}$
- They can be stabilized by using an extension of the OGY method