Families of symmetric relative periodic orbits originating from the circular Euler solution in the isosceles three-body problem (二等辺三体問題)

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Isosceles Three-Body Problem (3BP)

Equations of motion ($m_1 = m_2 = m$, $m_3/m = \alpha$)

$$m_\ell \ddot{q}_\ell = \frac{\partial V}{\partial q_\ell}, \quad q_\ell \in \mathbb{R}^3, \quad \ell = 1, 2, 3, \quad \text{or} \quad \dot{x} = f(x),$$

where $x = (q_1, q_2, q_3, p_1, p_2, p_3)$ with $p_i = m_i \dot{q}_i$, and

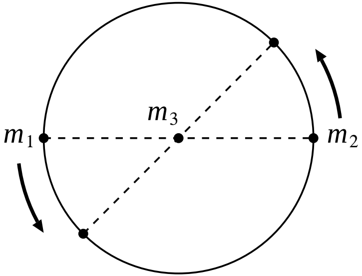
$$V(q_1, q_2, q_3) = \sum_{i<j} \frac{m_i m_j}{|q_i - q_j|}$$

G: finite dim. Lie group s.t. $f(gx) = gf(x)$, $\forall g \in G$

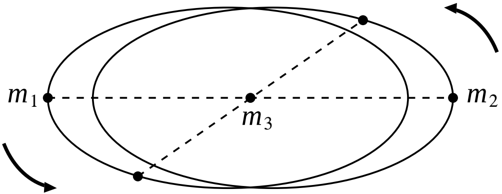
- $\bar{x}$ is said to lie on a relative periodic orbit if $\exists t > 0$ s.t. $\Phi_t(\bar{x}) \in G\bar{x}$ (Φ_t: flow)
- $T = \inf\{t > 0 \mid \Phi_t(\bar{x}) \in G\bar{x}\}$: relative period
- Relative equilibrium if $T = 0$
- $\{g\Phi_t(\bar{x}), t \in \mathbb{R}, g \in G\}$ is regarded as the same relative periodic orbit

Euler Orbits

- Circular Euler orbit ⇒ relative equilibrium



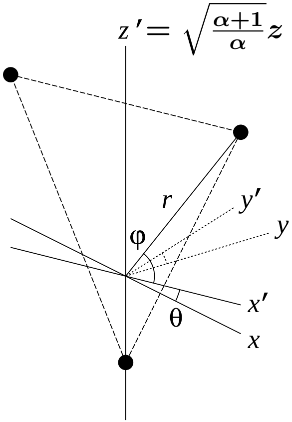
- Elliptic Euler orbit ⇒ (relative) periodic orbit



Contents

- Symmetric relative periodic orbits in isosceles 3BP using theoretical and numerical approaches
- Theoretical results
  - Another family of symmetric relative periodic orbits born from circular Euler orbit besides elliptic Euler orbits
  - Previous results (Shibayama and Y, 2009; Shibayama 2011)
    - Infinitely many families of symmetric relative periodic orbits born from heteroclinic connections between triple collisions
    - Planar periodic orbits with binary collisions
- Numerical analyses
  - Abundant families of symmetric relative periodic orbits bifurcating from 2 families born from circular Euler orbit
  - As angular momentum $\to \mathbf{0}$, many of numerically observed families $\to$ heteroclinic connections between triple collisions or planar periodic orbits with binary collisions
  - Some of them converge to "previously unknown" periodic orbits in the planar problem

Blown-up Equations of Motion



$$\dot r = vr\cos\varphi, \quad \dot\varphi = w,$$

$$\dot v = \left(U(\varphi) - \frac{1}{2}v^2 + 2rh\right)\cos\varphi,$$

$$\dot w = \frac{dU}{d\varphi}(\varphi)\cos^2\varphi - \frac{1}{2}vw\cos\varphi$$
$$- \left(2U(\varphi) - v^2 + 2rh\right)\sin\varphi\cos\varphi,$$

$$U(\varphi) = \sec\varphi + \frac{4\alpha^{3/2}}{\sqrt{\alpha + 2\sin^2\varphi}}$$

- $r = 0 \Leftrightarrow$ triple collision
- $\varphi = \pm\frac{1}{2}\pi \Leftrightarrow$ binary collision
- Conservation law

$$\frac{1}{2}\left(v^2\cos^2\varphi + w^2 + \frac{\omega^2}{r}\right) - U(\varphi)\cos^2\varphi = rh\cos^2\varphi$$

Reversibility

Rewrite (r, φ, v, w)-system as

$$\dot{\xi} = F(\xi)$$

- Reversible w.r.t. R_i, $i = 1, 2$, where

$$R_1 : (r, \varphi, v, w) \mapsto (r, \varphi, -v, -w),$$
$$R_2 : (r, \varphi, v, w) \mapsto (r, -\varphi, -v, w)$$

$\overset{\text{def}}{\Leftrightarrow} F(R_i\xi) + R_i F(\xi) = 0$

- $\xi(t) = (r(t), \varphi(t), v(t), w(t))$ is a solution

$\Rightarrow R_i\xi(-t)$ is a solution

- $\xi(t)$ is called R_i-symmetric if $R_i\xi(-t) = \xi(t)$
- $\xi(t)$: R_i-symmetric

$\Leftrightarrow \xi(t)$ intersects $\mathrm{Fix}(R_i) = \{R_i\xi = \xi\}$

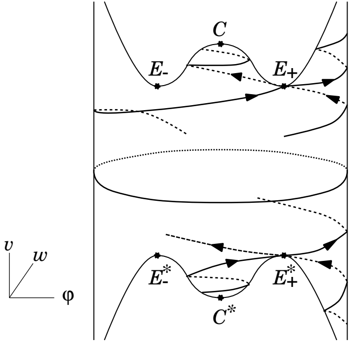
Equilibria

- $E_\pm, E^*_\pm, C_\pm$ on collision manifold

$$\mathscr{N} = \{r = 0,\ \omega = 0\}$$

where

$$\begin{aligned} E^*_\pm &= R_1 E_\pm = R_2 E_\mp, \\ W^{s,u}(E_\pm) &= R_1 W^{u,s}(E_\pm) \\ &= R_2 W^{u,s}(E_\mp) \end{aligned}$$



- $(r, \varphi, v, w) = \left(-\frac{1}{2h}(4\alpha + 1), 0, 0, 0\right)$

$\Rightarrow$ Circular Euler solution $\left(\omega = \omega_0 \equiv \dfrac{4\alpha + 1}{\sqrt{-2h}}\right)$

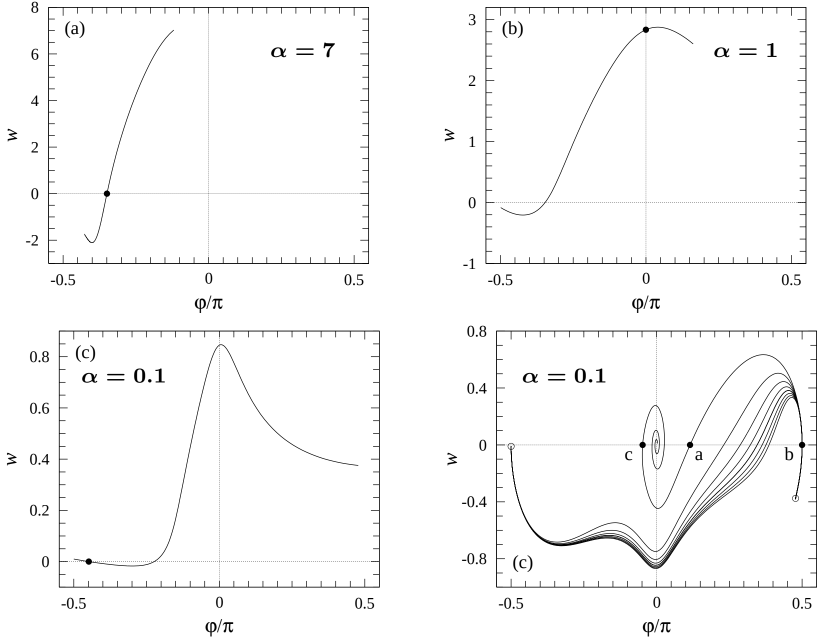
Theoretical Results

Periodic orbits of reversible system

($\Leftrightarrow$ relative periodic orbits of 3BP)

- Elliptic Euler orbits
- Another $\boldsymbol{R_1}$- and $\boldsymbol{R_2}$-symmetric family $\rightarrow$ circular Euler orbit as $\boldsymbol{\omega} \rightarrow \boldsymbol{\omega_0}$ for $\boldsymbol{\alpha} \neq \frac{1}{3}$
  $\because$) Devaney's result (1976) and proof by contradiction
- $\boldsymbol{R_1}$-symmetric (resp. $\boldsymbol{R_2}$-symmetric) families approaching to heteroclinic cycles connecting $\boldsymbol{E_\pm}$ with $\boldsymbol{E^*_\pm}$ (resp. $\boldsymbol{E^*_\mp}$) when $\boldsymbol{\alpha_1 < \alpha < \frac{55}{4}}$ (resp. $\boldsymbol{0 < \alpha < \alpha_2}$), where $\boldsymbol{\alpha_1 \leq 6.52}$ and $\boldsymbol{\alpha_2 \geq 2.25}$ (Shibayama and Y, 2009)
  Numerical observation: $\boldsymbol{\alpha_1 = 0}$ and $\boldsymbol{\alpha_2 = 2.6\ldots}$
- $\boldsymbol{R_1}$- and $\boldsymbol{R_2}$-symmetric $\boldsymbol{T}$-periodic orbit passing $\boldsymbol{\varphi = \pm\frac{1}{2}\pi}$, $\forall \boldsymbol{T, \alpha > 0}$
  ($\Leftrightarrow$ planar periodic orbits with binary collisions in 3BP)
  (Shibayama, 2011)

Numerical Computations of $W^{s,u}(E_+)$ $(v = 0)$



Numerical Approach

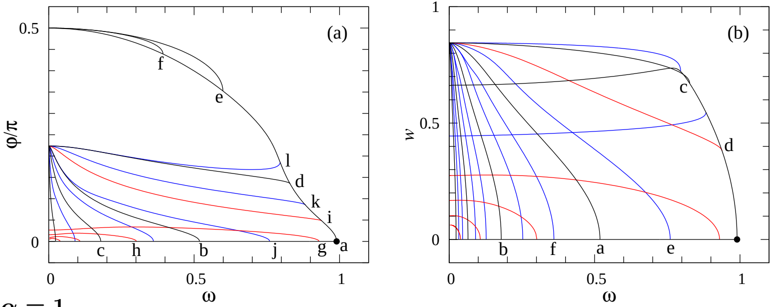
- Boundary value problem with boundary conditions

$$\xi(\pm T) \in \mathbf{Fix}(R_i), \quad i = 1, 2$$

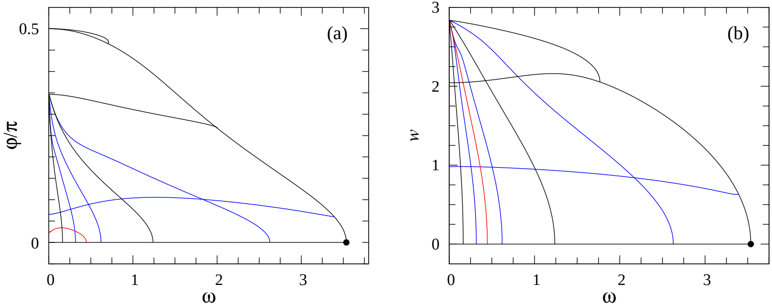
- AUTO97 (Doedel, 1997)
- Starting solution: circular Euler orbit
- Continuation of solutions with T
- $r(\pm T)$ and $\varphi(\pm T)$ or $w(\pm T)$: free parameters
- Angular momemtum ω: monitored
- $h = -1$

Bifurcation Diagrams (#1)

$\alpha = 0.1$

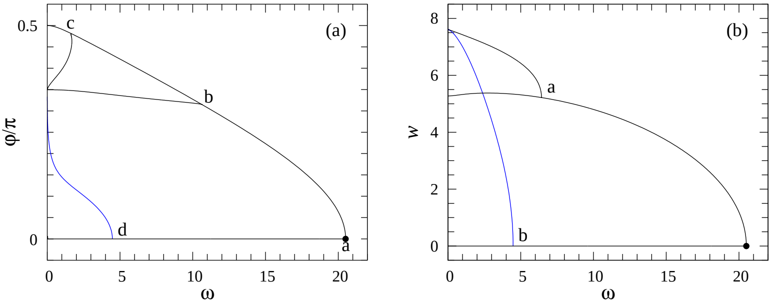


$\alpha = 1$



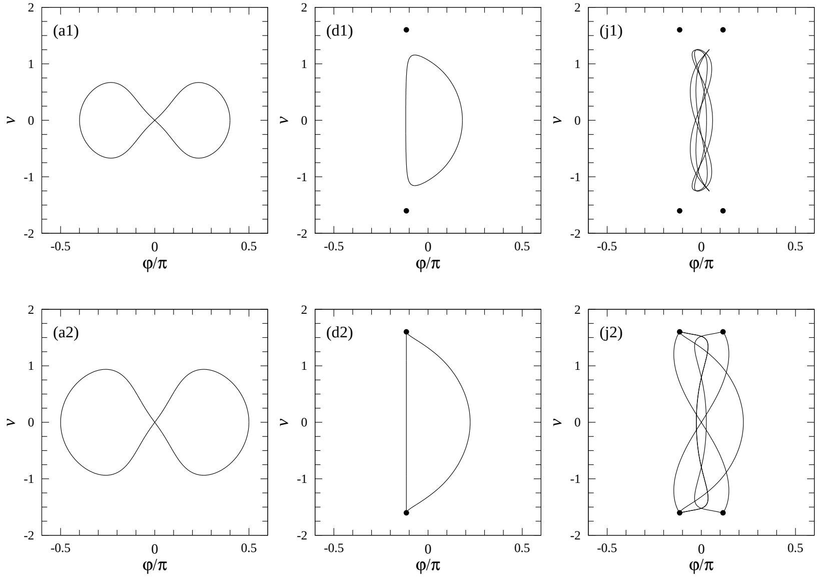
Bifurcation Diagrams (#2)

$\boldsymbol{\alpha = 7}$



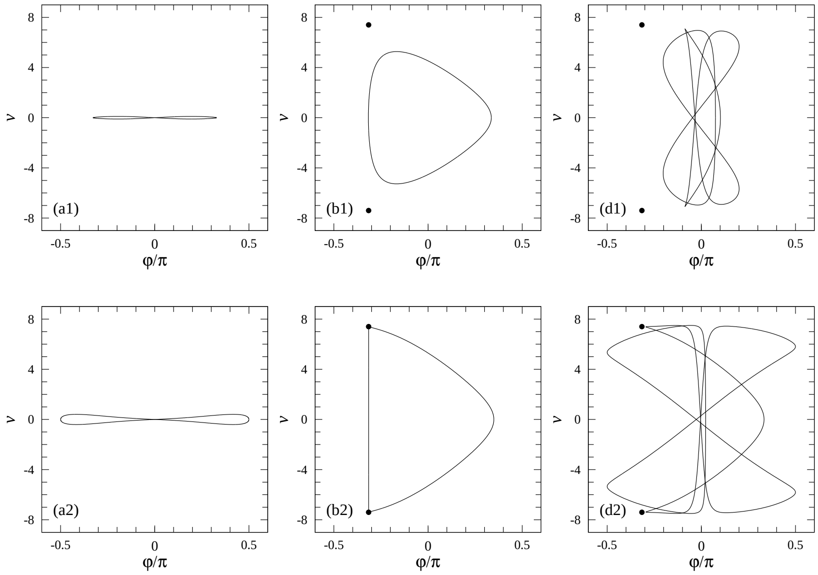
$\boldsymbol{R_1}$-Symmetric Periodic Orbits (#1)

$\boldsymbol{\alpha = 0.1}$



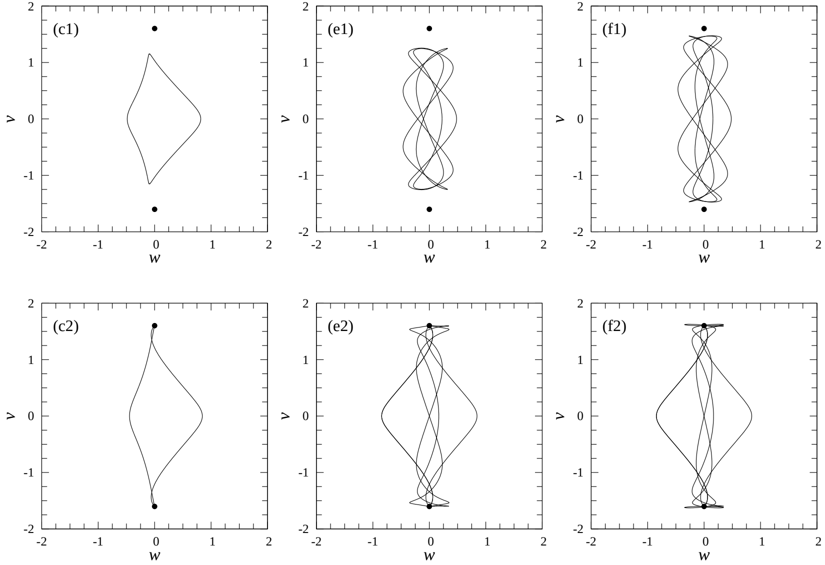
$\boldsymbol{R_1}$-Symmetric Periodic Orbits (#2)

$\boldsymbol{\alpha = 7}$



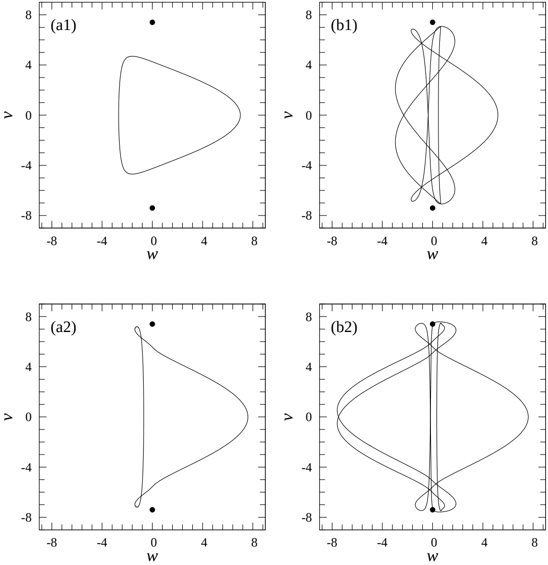
R_2-Symmetric Periodic Orbits (#1)

$\alpha = 0.1$



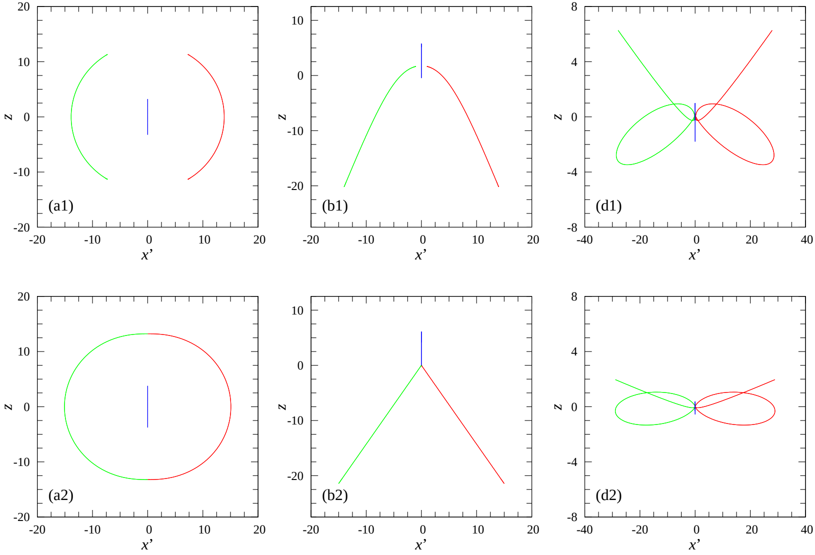
R_2-Symmetric Periodic Orbits (#2)

$\boldsymbol{\alpha = 7}$



Relative Periodic Orbits in 3BP (#1)

$\alpha = 7$



Relative Periodic Orbits in 3BP (#2)

$\alpha = 7$