Variational Methods and Periodic Solutions
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Formation of Transient Binaries in the Outer Solar System

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Abstract. Performing local three-body orbital integration, we investigate temporary capture of planetesimals by a planet from their heliocentric eccentric orbits. We find that typical orbital size and direction of revolution around the planet change depending on planetesimals' initial eccentricity and energy. We obtain the rate of temporary capture of planetesimals and find that they increase nearly monotonically with increasing eccentricity and that prograde temporary capture rates have a peak at a certain value of planetesimal orbital eccentricity.

1. Introduction

When planetesimals encounter a planet, planetesimals sometimes become captured by the planet's gravity and orbit about the planet for long time, before they escape from the vicinity of the planet. This phenomenon is called temporary capture. Temporary capture may play an important role in the origin and dynamical evolution of various kinds of small bodies in the Solar System. Recently, temporary capture of planetesimals by a planet from heliocentric orbits has been investigated in detail using three-body orbital integration (Iwasaki & Ohtsuki 2007). In the case of planetesimals initially on circular orbits, the rate of temporary capture was evaluated, and it was found that it increases with increasing semi-major axis of the planet. However, cases of large orbital eccentricities were not examined in detail. In the present work, we examine temporary capture of planetesimals initially on eccentric orbits about the Sun.

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2. Numerical Method

We suppose that a planetesimal (mass m) and a planet (mass M) are orbiting about the Sun, and that their orbital eccentricities and inclinations are sufficiently small. We use a rotating coordinate system centered on the planet, and scale time by Ω^{-1} (Ω is the planet's orbital angular frequency) and distance by the mutual Hill radius $R_{\mathrm{H}} = a_0 h = a_0 \left(\left(M + m \right) / 3 M_{\odot} \right)^{1/3}$ (a_0 is the semi-major axis of the planet). Then the non-dimensional equation of relative motion between the planet and the planetesimal can be written as (Petit & Hénon 1986; Nakazawa et al. 1989)

$$\begin{aligned} \ddot{\tilde{x}} &= 2\dot{\tilde{y}} + 3\tilde{x} - 3\tilde{x}/\tilde{r}^3, \\ \ddot{\tilde{y}} &= -2\dot{\tilde{x}} - 3\tilde{y}/\tilde{r}^3, \\ \ddot{\tilde{z}} &= -\tilde{z} - 3\tilde{z}/\tilde{r}^3, \end{aligned} \tag{1}$$

where tildes denote non-dimensional quantities and $\tilde{r} = \left(\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2 \right)^{1/2}$. When the mutual gravity represented by the last term in the right-hand side of Eq.(1) can be neglected, the solution for Eq.(1) can be written as

$$\begin{aligned} \tilde{x} &= \tilde{b} - \tilde{e} \cos \left(t - \tau \right), \\ \tilde{y} &= -\tfrac{3}{2} \tilde{b} \left(t - t_0 \right) + 2 \tilde{e} \sin \left(t - \tau \right), \\ \tilde{z} &= \tilde{i} \sin \left(t - \omega \right). \end{aligned} \tag{2}$$

Here, $\tilde{b}$ denotes the initial semi-major axis difference between the planet and a planetesimal scaled by R_{H}; the initial orbital eccentricity and inclination of the planetesimal scaled by h are denoted by $\tilde{e}$ and $\tilde{i}$, respectively; and τ and ω are the horizontal and vertical phase angles. Eq.(2) holds an energy integral, which is written in terms of the initial orbital elements $\tilde{e}, \tilde{i}$ and $\tilde{b}$ as

$$E = \frac{1}{2} \left(\tilde{e}^2 + \tilde{i}^2 \right) - \frac{3}{8} \tilde{b}^2 + \frac{9}{2}, \tag{3}$$

where $9/2$ is added so that the potential vanishes at the Lagrangian points $(\tilde{x}, \tilde{y}, \tilde{z}) = (\pm 1, 0, 0)$(Nakazawa et al. 1989).

We integrate a large number of orbits by numerically solving Eq.(1), and obtain the duration of temporary capture (T_{cap}) as a function of initial orbital elements. Iwasaki & Ohtsuki (2007) defined T_{cap} by the time interval between a planetesimal's first passage of the $\tilde{x}$-axis and its last passage of the same axis during an encounter. We also adopt their definition in the present work.

From results of orbital calculation, we obtain non-dimensional collision rates per unit surface number density of planetesimals defined as

(Nakazawa et al. 1989; Iwasaki & Ohtsuki 2007)

$$P_{\rm col} = \int p_{\rm col}(\tilde{b}, \tilde{e}, \tilde{i}, \tau, \omega) \frac{3}{2} |\tilde{b}| d\tilde{b} \frac{d\tau d\omega}{(2\pi)^2}, \tag{4}$$

where set $p_{\rm col} = 1$ for collision orbits and zero otherwise. Similarly, we evaluate rates of temporary capture with $T_{\rm cap} \geq nT_{\rm K}$ ($T_{\rm K} = 2\pi/\Omega$ is the planet's orbital period) from

$$P_{{\rm cap},n} = \int p_{{\rm cap},n}(\tilde{b}, \tilde{e}, \tilde{i}, \tau, \omega) \frac{3}{2} |\tilde{b}| d\tilde{b} \frac{d\tau d\omega}{(2\pi)^2}, \tag{5}$$

where $p_{{\rm cap},n} = 1$ for orbits with $T_{\rm cap} \geq nT_{\rm K}$ and zero otherwise. We examine the cases with $n = 1 - 10^3$ in the following.

3. Results

3.1. Temporary Capture Orbits

We examine orbital characteristics during temporary capture. Fig. 1(a) shows typical retrograde temporary capture orbit in the case of low eccentricity ($\tilde{e} \lesssim 1$). Temporary capture occurs when planetesimals are scattered by the planet into the vicinity of pseudo-periodic orbits around the planet. Since the orbital shapes remind us of a cross-section of an apple, we call this type of capture orbit Apple type or type A. However, type-A orbits become less common and eventually disappear with increasing $\tilde{e}$.

In the case of moderate eccentricities of pre-capture heliocentric orbits ($1 \lesssim \tilde{e} \lesssim 5$), another type of retrograde capture orbits become possible (Fig. 1(b)). We call this type of orbits Ribbon-type, or type R, from its shape. Type-R orbits appear at $E \simeq 1$, which is lower than the typical value of E for type-A orbits ($2 \lesssim E \lesssim 3.5$). This type of orbits appear only in the case of $1 \lesssim \tilde{e} \lesssim 5$, because with this range of eccentricities planetesimals with low energy ($0 \lesssim E \lesssim 1$) can enter the planet's Hill sphere and become temporarily captured.

In the case of still lower energy ($E \simeq 0$), long capture in the prograde direction appears (Fig. 1(c)). We find that such long prograde capture takes place at a very narrow range of $\tilde{e}$ around $\tilde{e} \simeq 3$. In this case, planetesimals enter the Hill sphere through the vicinity of the Lagrangian points, because their energy is very low. Then they bounce back many times at the equi-potential surface near the Hill sphere before escaping from it. As a result, the shape of the region swept by their trajectories during temporary capture becomes very similar to the shape of the Hill sphere. Therefore we call this type of capture orbits Hill-type, or type H.

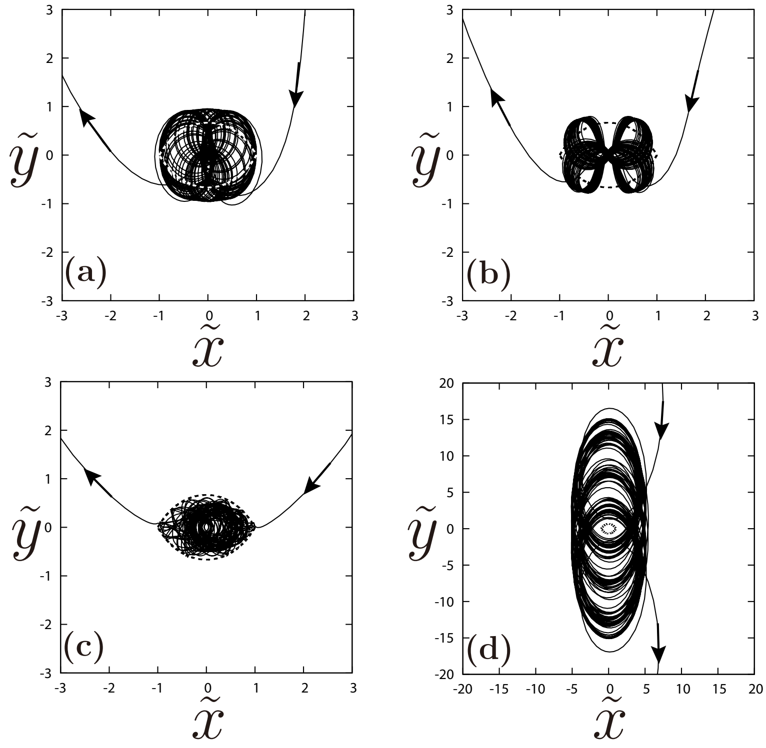

Figure 1. Examples of temporary capture from heliocentric eccentric orbits. (a) Type A (Apple-type), (b) Type R (Ribbon-type), (c) Type H (Hill sphere type), (d) Type E (oscillating Epicycle type) Direction of revolution about the planet is prograde in (c) and retrograde in other cases. (Suetsugu et al. 2011)

The above three types of temporary capture orbits are limited to a rather narrow range of eccentricity $\tilde{e}$ or energy E, and they are orbits within or in the vicinity of the Hill sphere. For large values of $\tilde{e}$ ($\tilde{e} \gtrsim 3$) and E (say, $E \gtrsim 7$), large retrograde orbits about the planet outside of its Hill sphere become common for a wide range of parameters. Fig. 1(d) shows an example of this type of orbits. This type of orbits result from oscillation in the $\tilde{y}$-direction of a quasi-epicycle orbit outside of the Hill sphere; we call it oscillating Epicycle type, or type E. Bodies on this type of orbits are also called quasi-satellite. This type of orbits results from gravitational interaction with the planet, which scatters the planetesimal to a semi-major axis very similar to that of the planet. In the case of small initial eccentricities where the Kepler shear dominates relative velocity between planetesimals and the planet, type-E orbits do not appear, and this type of orbits appear only in the random-motion-dominated velocity regime ($\tilde{e} \gtrsim 3$). For large values of eccentricity ($\tilde{e} \gtrsim 5$), all temporary capture

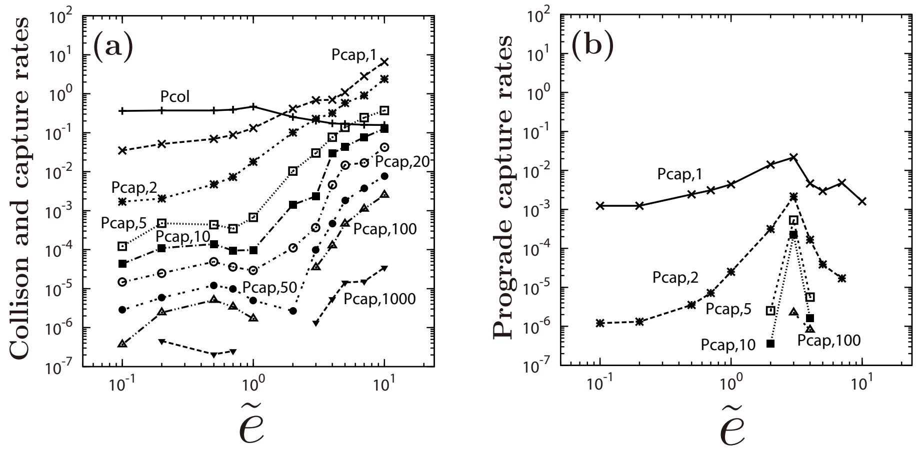

Figure 2. Collision and temporary capture rates as a function of $\tilde{e}$. (b)Rates of temporary capture in the prograde direction as a function of $\tilde{e}$. (Suetsugu et al. 2011)

orbits become this type. The size of type-E orbits increases with increasing $\tilde{e}$, and very long capture is common in this type.

3.2. Temporary Capture Rates

We obtained rates of collision and temporary capture defined by Eqs.(4) and (5) using results of three-body orbital integration. Fig. 2(a) shows the plots of these rates as a function of $\tilde{e}$ in the case of $\tilde{i} = 0$. For temporary capture with $T_{\rm cap} \leq 2T_{\rm K}$, capture rates monotonically increase with increasing eccentricity, and the capture rates with $T_{\rm cap} \lesssim 10T_{\rm K}$ are comparable to or higher than the collision rate at $\tilde{e} \simeq 10$. On the other hand, for relatively long capture with $T_{\rm cap} \geq 10T_{\rm K}$, capture rates increase with increasing $\tilde{e}$ at low ($\tilde{e} \lesssim 0.5$) and high ($\tilde{e} \gtrsim 3$) eccentricities, but take on minimum values at intermediate eccentricities ($\tilde{e} = 1 - 2$). Very long capture with $T_{\rm cap} \geq 100T_{\rm K}$ even disappears in this intermediate regime. This behavior can be explained by the transition of the types of capture orbits from type-A to type-E.

The total temporary capture rates are dominated by retrograde capture, i.e., type-A, R, and E, and the behavior of the rate of prograde capture is different from that of the retrograde capture (Fig. 2(b)). As we mentioned, relatively long prograde capture with $T_{\rm cap} \gtrsim 5T_{\rm K}$ appears in a narrow range of $\tilde{e}$ at $2 \lesssim \tilde{e} \lesssim 4$, and does not occur at low ($\tilde{e} \lesssim 1$) or high ($\tilde{e} \gtrsim 5$) eccentricities. This is because planetesimals with $E \simeq 0$ can enter the planet's Hill sphere in this case.

4. Summary

In the present work, we studied temporary capture of planetesimals by a planet from heliocentric eccentric orbits in detail, using three-body orbital integration. We found that there are four types of long capture orbits around a planet, and investigated characteristics of each type of orbits. Transition between dominant types of capture orbits occurs depending on initial heliocentric orbital eccentricity and energy of planetesimals. We also examined rates of temporary capture in detail. We found that the capture rate is dominated by that of retrograde capture, if the radial distribution of planetesimals is uniform. It increases with increasing eccentricity of planetesimals at low and high eccentricities, but in intermediate values of eccentricity ($0.5 \lesssim \tilde{e} \lesssim 2$) it decreases with increasing eccentricity, which can be explained by transition between dominant types of capture orbits. The rate of long-lived prograde capture has a rather sharp peak at $\tilde{e} \simeq 3$.

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