Helioseismic measurement of solar rotation

Takashi SEKII

sekii@solar.mtk.nao.ac.jp

NAOJ, Mitaka, Tokyo 181-8588, JAPAN

ABSTRACT

Helioseismology has been successful in investigating the otherwise invisible solar interior, and one of its greatest success is in revealing how the interior of the sun rotates. It is briefly reviewed how this is done, what we have learnt, and what they imply.

1 Introduction

The sun, the master of the solar system, rotates. We have been aware of this fact since the time of Fabricius, for around 400 years. Somewhat remarkably, it has been known for almost as long that the sun rotates *differentially* i.e. the equatorial region rotates faster than the polar regions. In both cases, it was sunspot motion that betrayed surface axisymmetric flow of the sun. How the sun rotates *internally*, on the other hand, had not been an observational subject for a long time, simply because the solar interior was — and still is, except metaphorically — invisible to us.

However, the internal rotation of the sun has always been an important issue. The solar activity cycle is thought to be caused by a dynamo mechanism, an interplay between magnetic field and flow, of the latter of which rotation is the main component. Also, although the sun is currently a slow rotator (see Section 2), its rotation is believed to have been much faster, and have been affecting the course of evolution of the sun. The so-called light-element problem, the apparent lack of light elements such as Lithium on the surface of the sun, for example, may be explained only by a mixing mechanism that involves rotation. And any theory of dynamical evolution of stars will have to be able to explain the present-day solar internal rotation. There was also the issue of measuring J_2 moment, in deciding if we need the Brans-Dicke theory of gravitation as opposed to Einstein's theory; a fast spinning core of the sun would invalidate the calculation that 'showed' the latter to be able to explain Mercury's orbit.

If the solar interior is invisible, and its rotation beyond observation and yet so important to astrophysics, why not trying a theoretical/numerical approach? This is prohibitingly difficult. The outer 30 per cent (in radius) of the sun is convective because higher opacity in surface ionization zones means photons alone cannot carry all the energy generated inside to outside. Bad news is that this convection is turbulent. Reynolds number is huge, thought to be in order of 10^{12}, which leads to, according to Kolmogorov's argument, the degree of freedom in order of 10^{27} (for Kolmogorov's argument see, e.g., [1]). If one is to take a head-on approach, he needs 10^{27} grid points to simulate the solar convection zone.

It was to this stage that helioseismology entered as a new method of probing the solar interior, to let us 'see' the interior of the sun. In the next section the early history of helioseismology is briefly presented together with some basic discussion of its diagnostic power. In Section 3 it is outlined how observation of surface velocity field can produce two-dimensional (that is, in radius and in latitude) map of solar angular velocity, and our current knowledge of the sun's rotation is summarized. Section 5 will be dedicated for discussing the future of this line of investigation.

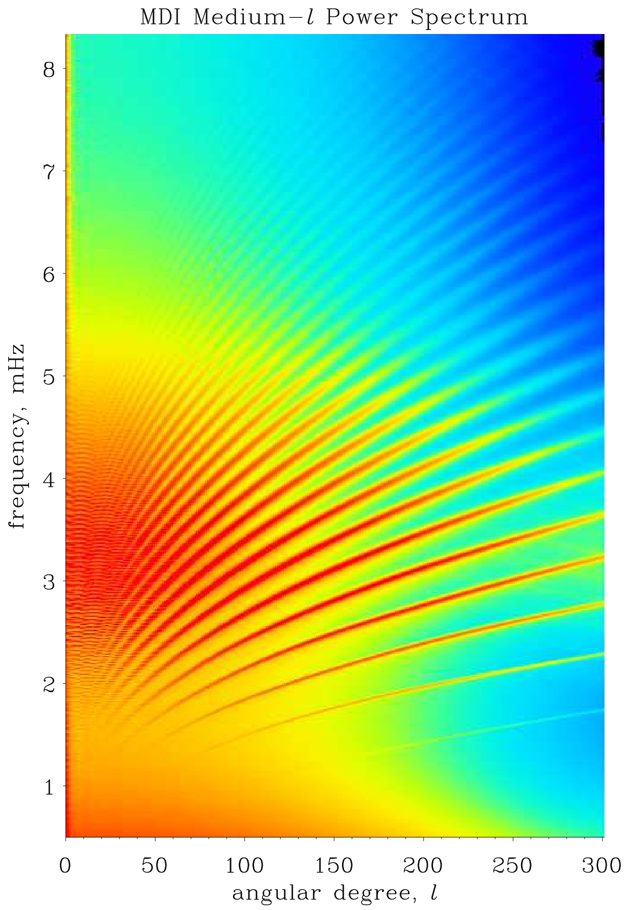

Figure 1 — Powerspectrum of 60-day Doppler velocity data by MDI, as function of spherical harmonic degree (abscissa) and frequency (ordinate). Horizontal wavenumber is $\sqrt{l(l+1)}/R_\odot$, where $R_\odot$ is the solar radius. The region of the highest power is around the period of 5 minutes (3.3mHz in frequency). The ridges correspond to the position of eigenmodes. [Image courtesy of the SOHO/SOI-MDI project of the Stanford Lockheed Institute for Space Research. SOHO is a project of international cooperation between ESA and NASA.]

2 Helioseismology

2.1 Solar acoustic eigenmodes

It all began in early 1960s, when the sun was discovered to oscillate [2]. Leighton and his collaborators were measuring Doppler velocity field of the sun, which is to say, the line-of-sight component of the solar surface velocity field. Their intention was to study supergranulation, which is another convective structure larger in size, slower in turn-over speed and harder to detect compared to granulation. During the course of their effort, they found that there was a concentration of temporal power of the velocity field around the period of 5 minutes. This was the discovery of the so-called solar 5-minute oscillation.

It took another decade or so until people came to agree that this oscillation was not a locally-driven (and damped) phenomenon but a manifestation of global acoustic eigenoscillations, also called *p modes* ('pressure modes'). One of the key was the observation by Deubner [3], who found that the concentration of power in two-dimensional Fourier space (one dimension in temporal frequency and another in horizontal wavenumber) was resolved into discrete ridges (see Figure 1 for modern data), which signify eigenoscillations[1].

Since the sun is almost perfectly spherical, we can label its normal mode by radial order n, spherical-harmonic degree l and azimuthal order m $(= 0, \pm 1, \pm 2, \ldots, \pm l)$. However, without any symmetry-breaking agent, there is no special axis, and hence degeneracy with respect to m arises i.e. eigenfrequencies depend only on n and l, which we may write ω_{nl}. By solving an eigenvalue problem, frequency spectrum $\{\omega_{nl}\}$ can be calculated. Horizontal structure of a mode is described by spherical harmonic function Y_{lm}, so the wavenumber is $\sqrt{l(l+1)}/R_\odot$ ($R_\odot$ is the radius of the sun): l is essentially the horizontal wavenumber. By plotting ω_{nl} as function l, we reproduce the ridge patterns in two-dimensional power spectrum.

Once it was established that we were observing eigenoscillations, however, it was quickly noted that these observation can be used for a diagnostic purpose. In fact, in establishing the above, people were already comparing eigenfrequencies of solar models with observed frequencies: the simplest of all the seismic approaches. Such comparison would tell you how good a model is, or whether a model is better than another. How far one can go on improving a solar model depends on the information content of eigenfrequency spectrum, and on whether one can devise a systematic approach to modify models according to the observed eigenfrequencies.

2.2 Probing the solar interior

So what determines the eigenfrequencies? Any piece of solar plasma is under influence of local gas pressure and gravity. To know how the perturbation to this piece of plasma would develop in time, we have to know how the pressure and gravity would change as results of the perturbation. To know the first is to know the so-called bulk modulus i.e. how hard a piece of fluid element would bounce back when compressed. In terms of quantities more familiar to astronomers, the bulk modulus is ρc^2, where ρ is density and c is adiabatic soundspeed. To know how the gravity would change, we need to know the distribution of material i.e. ρ as function of position in the sun.

There is also magnetic field, of course, but the effect of magnetic field is small (details of why and to what extent this is so is beyond the scope of the current review).

Then there is rotation. Rotation affects the wave propagation by advection, and by Coriolis force (to a lesser degree, in case of p-mode waves). Although in principle any change in the equilibrium structure due to rotation should also be taken into account, this is negligible since the sun is a slow rotator: the equator of the sun rotates with a period of around 25 days (sidereal), compared to the dynamical time scale of the sun of half an hour. This is even more true in considering effect of rotation on observed oscillations, since we really have to compare the rotation period with that of oscillation, which is typically 5 minutes (why the sun oscillates at such high overtones is, once again, outside the scope of this brief summary). This permits us to use linear perturbation theory to account for the effect of rotation on eigenfrequencies.

Let us forget the magnetic field. The two quantities ρ and c are what *principally* determines wave propagation in the sun and, when combined with proper boundary condition, its eigenfrequencies. Rotation affects the eigenfrequencies too, but since the effect is small, it can be treated as a perturbation. If we look at this situation from the other direction, we can conclude that by studying eigenfrequencies of the sun we can learn the distribution

[1] You may ask, why ridges, not points? Because of damping, points become blobs, and a series of such blobs constitutes a ridge. Also, horizontal wavenumber is observed as a discrete number (see Fig.1)

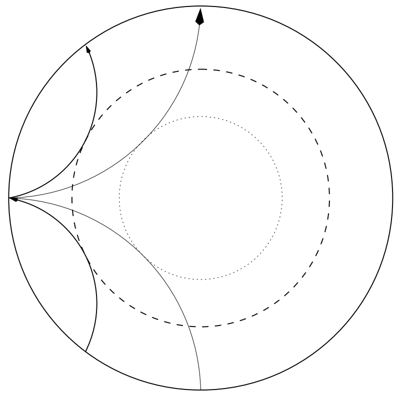

Figure 2 — Schematically drawn p-mode ray paths and their penetration depths. A wave packet travelling inward from the surface refracts back to the surface because of an inward increase in soundspeed. By comparing shallow-penetrating modes (down to dashed circle) and deeply-penetrating modes (down to dotted circle), it is possible to infer physical condition in the region that is between the two circles. In the picture are also drawn rays that travel to a surface point to be reflected inward.

of ρ and c, or the radial structure of the sun, and that by studying fine structure that is brought about by rotation, we can investigate how the solar interior rotates.

How well it can actually be done depends not only on how precisely the frequencies can be measured, but also on how many frequencies are measured. Although this may be intuitive enough, the reason is discussed in the below.

In the sun, deeper one goes and higher the temperature gets. In other words, there is an inward increase of soundspeed[2]. As a result, a packet of soundwave that starts an inward travel from a point on the surface undergoes refraction, eventually to come back to the surface to start the next trip through the solar interior (Fig. 2). Depending on the initial angle, a wavepacket penetrates to a certain depth. The region between the surface and the deepest point that the wavepacket reaches is called *propagation region*. Frequency of a mode comprising such wavepackets reflects physical condition in this propagation region. Different modes will have different penetration depths and hence different propagation regions, thereby sampling the solar interior in different fashions. If we have two modes with only slightly different penetration depths, the small difference in their frequencies will tell us about the thin layer between the two depths. With more modes added, we can probe more layers. Thus, probing the interior of the sun requires a wide variety of modes.

2.3 Rotational splitting

Let us go into more details on the effect of rotation. The presence of rotation means the spherical symmetry of the system is broken, and eigenfrequencies now depend also on azimuthal order m. We denoted eigenfrequencies by ω_{nl} before, but now we need to use ω_{nlm}. The m-independent frequency ω_{nl} is the frequency at which the mode would have been pulsating, were it not for the rotation. The difference between ω_{nlm} and ω_{nl} is calculated according to linear perturbation theory. The result is:

$$\omega_{nlm} - \omega_{nl} = m \times \langle \Omega \rangle_{nlm} \ ,$$

[2]Except near the centre, where a steep inward increase in mean molecular weight leads to an inward decrease in soundspeed.

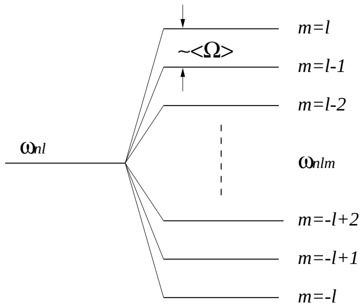

Figure 3 — Eigenfrequency ω_{nl} splits into $(2l+1)$ distinct frequencies, ω_{nlm}, due to rotation Ω. The split frequencies ω_{nlm} are spaced by averages of rotation rate when rotation is slow.

where Ω is rotational (angular) frequency which is function of radius r and colatitude θ (the component of flow that depends on azimuthal angle ϕ we do not call 'rotation' in the current context) and $\langle\Omega\rangle_{nlm}$ is a certain average of $\Omega(r,\theta)$: the single frequency ω_{nl} 'splits' into $2l+1$ frequencies (Fig. 3). In the case of the sun, the sidereal rotational period of ~ 25 days means that $\langle\Omega\rangle_{nlm}/2\pi \sim 460\,\mathrm{nHz}$. However, due to different ways by which modes sample the solar interior (Fig. 2), the average varies from mode to mode in a subtle way (hence the subscripts nlm in $\langle\Omega\rangle_{nlm}$). Accordingly,

$$\langle\Omega\rangle_{nlm} = \int dr \int d\theta K_{nlm}(r,\theta)\Omega(r,\theta) \ ,$$

where $K_{nlm}(r,\theta)$ is the sensitivity function called *splitting kernel* that can be calculated from eigenfunctions. This is the basis of helioseismic measurement of solar internal rotation.

3 Helioseismic measurement of solar rotation

3.1 Observation

One big advantage of solar physics, compared to stellar physics, is that we can observe the sun as a disc. Full-disc observations (sun-as-a-star observations) are still practiced and do have certain advantages in observing low-l modes which have a broad spatial scale. Also, although so far I have mentioned only Doppler velocity measurement in relation with helioseismology, intensity fluctuation can also be used for seismic study. Still, in outlining the observing procedure in the below, I take the resolved measurement of Doppler velocity as the canonical example.

Measuring Doppler velocity is done by observing an absorption line that is insensitive to magnetic field, for then any change in the shape and position of the line would mainly due to oscillating velocity field (and surface rotation, which can easily be subtracted). By measuring Doppler velocity over the solar disc, we eventually obtain the radial velocity field $v_{\mathrm{rad}}(\theta,\phi,t)$, which we expand in $Y_l^m(\theta,\phi)$:

$$v_{\mathrm{rad}}(\theta,\phi,t) = \sum_{lm} A_{lm}(t) Y_l^m(\theta,\phi) \ .$$

If for simplicity we assume that v_{rad} is observed all over the solar surface, then

$$A_{lm}(t) = \int d\Omega \ v_{\mathrm{rad}}(\theta,\phi,t) Y_l^{m*}(\theta,\phi) \ ,$$

where $d\Omega$ denotes surface element, gives the temporal variation of the spherical harmonic component (l, m). Then $a_{lm}(\omega)$, the Fourier component of $A_{lm}(t)$ obtained by Fourier transform, or, more typically (but not necessarily), the power spectrum $|a_{lm}(\omega)|^2$ will be examined for peaks that correspond to eigenmodes. A figure like Figure 1 will be obtained by then averaging $|a_{lm}(\omega)|^2$ over m.

A long and continuous observation is required for precise determination of eigenfrequencies through Fourier analysis. In particular, if we are to measure rotational splitting, we need at least observation time of around a month. Small gaps here and there do not hurt very much, but large gaps do, and we need to avoid them. However, even if the weather is good, the sun cannot be observed during night. To solve this problem, some people go to South Pole in summer. Another solution is to have a ground-based network, that operates by installing several telescopes around the world and then observe the sun continuously in effect, after merging data from their sites. And of course, going to space also provides a solution.

The GONG (Global Oscillation Network Group) project [4] is representative of the ground-network effort. It has been operating 6 stations since 1995 in Big Bear (California), Mauna Loa (Hawaii), Learmonth (Australia), Udaipur (India), El Teide (Canary Islands) and Cerro Tololo (Chile). The variety in site longitude cannot deal with bad weather, but still the GONG achieves a duty cycle of about 90 per cent.

The SOHO (Solar and Heliospheric Observatory) satellite [5] was launched in 1995 with a suite of solar observation instruments on-board, among which were MDI (Michelson Doppler Imager), VIRGO (Variability of Solar Irradiance and Gravity Oscillations) and GOLF (Global Oscillations at Low Frequencies). In the context of helioseismology, the main purpose of VIRGO and GOLF are to discover low-frequency, yet-to-be-found, gravity wave (not to be confused with gravitational wave) oscillations (called g modes). The MDI carries out resolved measurement of Doppler velocity (and intensity fluctuation too) for the purpose of p-mode helioseismology.

With these new-generation experiments and more sophisticated method of data analysis, these days eigenfrequencies can be measured with nominal precision of less than a tenth of a μHz. In terms of relative precision, this is of the order of 10^{-5} or even 10^{-6}, which is rarely seen in other branches of astrophysics. The number of modes observed depends on how one counts it: if one counts the (n, l) pairs, the number is around a few thousands. The number of (n, l, m) triplets would exceed a few hundreds of thousands. Equipped with these massive amount of high-precision data, we can then attempt to uncover what is beneath the photosphere.

3.2 Inversion

At the end of Section 2.1 I mentioned the importance of a systematic method of feeding information acquired by observation to our model. One such device is method of *inversion.* Suppose we have a model of rotation profile $\Omega(r, \theta)$. We can use this profile to compute ω_{nlm} and then judge if the profile matches observation or not. If not, we *may* be able to figure out how to improve the model profile. It would be much better, however, if we can work this backward, to produce an estimate of $\Omega(r, \theta)$ directly from observed set of frequencies $\{\omega_{nlm}\}$. This is what an inversion procedure aims to achieve.

There is a few minor issues concerning the determination of the internal rotation from $\{\omega_{nlm}\}$. First of all, what linear perturbation theory gives is how ω_{nlm} differs from ω_{nl}. However, ω_{nl}, being a hypothetical quantity (the frequency at which the mode would oscillate if the sun were not rotating), it cannot be measured. What can really be used, in fact, is $\omega_{nlm} - \omega_{nl,-m} = 2m \times \langle\Omega\rangle_{nlm}$. Here it is implied that $\langle\Omega\rangle_{nl,-m} = \langle\Omega\rangle_{nl,+m}$, which is guaranteed by symmetry of sensitivity function K_{nlm} (as an additional note, in inversion for soundspeed or density, m-average of ω_{nlm} substitutes for ω_{nl}). The same symmetry, by the way, implies that we are only measuring the component of $\Omega(r,\theta)$ that is north-south symmetric. Second, we do not normally use ω_{nlm} themselves but expand them in a form, for example, such as

$$\omega_{nlm} = \sum_k a_{nlk} \mathcal{P}_k(m/l)$$

with a proper polynomial $\mathcal{P}_k$, and use the expansion coefficients a_{nlk} instead as data. However, all these do not change the fact that $\Omega(r,\theta)$ is linearly related to rotational splitting data, and that we are to solve a linear inverse problem of rotational splitting to infer the internal rotation of the sun. In the below (though only in a few places), I still pretend that ω_{nlm} is the quantity we base our investigation upon.

It is not difficult to see that solving a linear inverse problem eventually comes down to invert matrices, since a linear relation has to be discretized first and the result is a matrix equation. Describing in detail how to proceed from there requires a lot of space; please see [6] and references therein. One important aspect is that if one solves the linear inverse problem via linear processes only, then the 'solution' $\hat{\Omega}(r,\theta)$ is related linearly to input data, and hence to the $\Omega(r,\theta)$, except for observational error:

$$\hat{\Omega}(r,\theta) = \int dr' \int d\theta' A(r,\theta;r',\theta')\Omega(r',\theta') + (\text{data error term}) \ .$$

It is more computational work to obtain the *averaging kernel* $A(r,\theta;r',\theta')$ than to obtain the solution itself, but it is through this function we learn spatial resolution, by examining how sharp the averaging kernel is peaked around (r,θ) or, indeed, if it is peaked around (r,θ). The latter tells us if we actually 'missed' our target in our attempt to estimate the rotation rate at (r,θ), our *target point*. These diagnostics, in addition to formal error in $\hat{\Omega}(r,\theta)$, is required to judge how reliable a solution $\hat{\Omega}(r,\theta)$ is.

3.3 Results from MDI

It has just been briefly outlined how one goes from Doppler measurement to the solar rotation. To summarize our current knowledge of the internal rotation of the sun thus obtained, I here present an example of rotation inversion of MDI data (Fig.4). In the below I summarize the features that are of importance and interest.

The solution is plotted only in the outer 50 per cent of the sun, and even there not in the very high latitudes. This is because either of a lack of precision (large error bars) or of a lack of spatial resolution, mainly in the sense that averaging kernels are not centred well around targets. Since not many modes penetrate deep into the sun, we do not have large enough variety of modes to probe the rotation in deeper layers well. With soundspeed inversion, the situation is much better and we can go down to the radius of $\sim 0.1 R_\odot$. This is because ω_{nl} (or m-average of ω_{nlm}) is more precisely measured than ω_{nlm}. As for the pole, the situation is the same; not many modes have a large amplitudes in high latitudes, hence the poor inversion.

When one look at the solution that *is* plotted, one thing that strikes eyes is that the pattern of latitudinal differential rotation observed on the surface holds through the most of the the solar convection zone. Early attempts at dynamical dynamo calculation predicted a cylindrical rotation of the convection zone [7], essentially because of the Taylor-Proudman theorem. This discrepancy has compelled numerical modelling of dynamo to be reconsidered (see below).

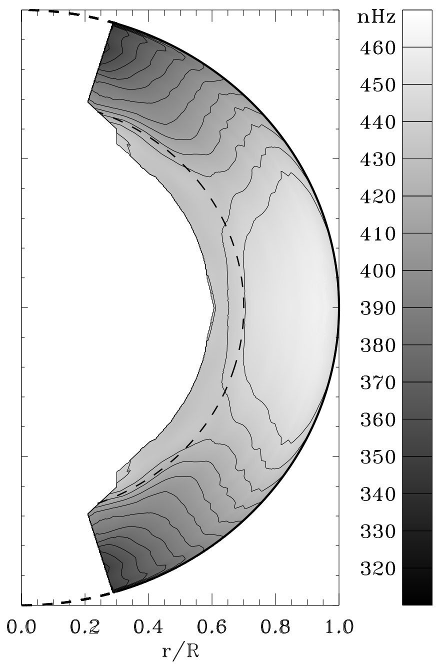

Figure 4 — Internal rotation of the sun inferred by inversion of MDI rotational-splitting data, as a contour map of rotational frequency $\Omega/2\pi$. The inner dotted curve (in fact part of a circle) indicates the base of the solar convection zone. Note that the north-south symmetry is because of the insensitivity of the method to the asymmetric component. Adapted from Schou et al.(1998)

In low- to mid-latitude, the rotation reaches its maximum around $r/R_\odot \simeq 0.95$, with a sub-photospheric shear layer (a sharp decrease in rotation rate) immediately above. It has been suggested that this layer supports a small-scale dynamo which is most likely not responsible for the 11-year cycle but playing an important role is sustaining (or magnifying) surface magnetic field [8].

If one looks deeper, one notes that differential rotation is weak, if present at all, in the radiative interior. Here, however, the spatial resolution is poor and error bars are large. It would be safer if we say we see no evidence against a near-rigid rotation, rather than we see evidence for.

In between, there is another shear layer at the base of the convection zone, named *tachocline* [9, 10]. Physics of tachocline is currently one of the hottest topic in solar and stellar astrophysical fluid dynamics. First, it can cause additional mixing in the solar interior. Indeed, a sounspeed excess found around the base of convection zone by helioseismology [11] may be explained by decreased amount of helium, as a result of such mixing offsetting an increase of helium by gravitational settling onto such layers. Second, tachocline may be playing an important role in dynamo mechanism. The standard α-ω dynamo has difficulty in explaining equatorward migration of sunspots without the cylindrical rotation and associated relatively large radial gradient of angular velocity in the solar convection zone. As an alternative, an interface dynamo mechanism at the tachocline region has been suggested [12, 13].

4 What's next?

The findings summarized in the last section could not have come via other routes, even if you count discoveries by numerical means as pure discoveries, because of the situation that was explained in Section 1. In this sense, one might say that helioseismology is even more indispensible for study of solar rotation than for study of solar (equilibrium) structure.

Still, we know very little of solar rotation in deep layers, or at polar regions, and these remain as obvious challenges for helioseismologists.

Aside from these, there is the issue of solar cycle variation. With data accumulated for a better part of solar cycle, we can now examine such variation of rotation. Though the signal still seems to be weak, it has been reported that the tachocline region is undergoing a periodic variation, with a period of 1.3 years [14].

Probably the greatest impact of helioseismic measurement of solar rotation was in the field of solar dynamo theory. The first kinematic dynamo calculation (the one that assumes a rotation rate rather than solves everything consistently as in dynamical dynamo) based on the 'observed' rotation rate was done by Dikpati & Charbonneau (1999) [15]. It was found that such a model would predict that sunspots would appear, in the beginning of a cycle, in latitudes too high compared to observation. It was suggested by Nandy & Choudhuri (2002) that meridional flow penetrating deep into the radiative zone can bring down the eruption latitude [16]. This is so-called flux-transport dynamo. Meridional flows have been measured in surface layers [17, 18, 19]. Although difficult, it will be one of the most important topic to pursue to detect the counter flow in deep layers, to provide observational evidence for such a mechanism.

How about other stars? *Asteroseimology*, seismology of stars, is at last developing fast and there has been a few reports on evidence of differential rotation in White Dwarfs [20] and a β-Cephei variable [21]. However, it will be a long time, if at all, before a detailed map of angular velocity, such as the one we have for the sun, will be available for stars.

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