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A possibility: co-evolution of the Milankovitch cycles and the earth-moon system

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Abstract

Solar insolation variation due to the gravitational perturbation among the planetary bodies in the solar system, so called Milankovitch cycle is widely believed as a major cause of the climatic change such as the glacial-interglacial cycles in Quaternary, and its typical frequencies are supposed to be constant during Quaternary. However, the periods of the Milankovitch cycles must have been largely changed in the longer time scale of billion years following to the dynamical evolution of the earth-moon system. Deccelerated rotational velocity of the earth has been making the dynamical ellipticity of the earth smaller and lengthening the major periods of both Milankovitch cycles and tidal cycles. We have studied the relation between the frequencies of the Milankovitch cycles and the rotation rate of the earth on the basis of the theoretical and computational analysis on the earth-moon system with several assumptions involved. Our conclusion is that this cyclicity which can be recorded in the sediments are mutually related well as a function of the dynamical ellipticity and the absolute age. We also performed some simple estimation about the effect of chaotic behavior of the planetary motion of the solar system for the purpose to break down the illusion of the word "chaos". We use the secular variation of the fundamental frequencies of Laskar (1990, 1991) and Nobili *et al.* (1989) and got the results which imply that the effect of chaos may be much smaller then we had expected before. This fact implies that we can establish the standard time scale for measuring the relative age, in other words, the lap time clock or the chronometer for decoding the whole history of the earth, by comparing the stripes in BIF and other sediments of Archean or Proterozoic with a set of theoretical Milankovitch cycle and tidal cycle frequencies. Here we present the preliminary reference model of the evolution of the Milankovitch cycles and tidal cycles and attempt to establish the lap time clock which will be a potential device for our project to clarify evolutionary history of our earth's environment back to −4Ga.

Introduction

Solar insolation onto the surface of the planet has been said to vary periodically and cause long period climate change. Time scale of this cycle is $10^4 \sim 10^6$ years, and it has been considered as a pacemaker of the glacial-interglacial cycles in Quaternary on the earth (Hays *et al.*, 1976). This is called the Milankovitch cycle and its mechanism has been theorized quantitatively from so early times because it consists of classical celestial mechanics about point masses and rigid bodies (Milankovitch, 1941). However in view of the longer time scale ($O(10^9)$years), secular change of the earth-moon distance and rotational velocity of the earth must have caused the great effect on the Milankovitch cycles. This effect was qualitatively shown by Walker and Zahnle (1986) up −2.5Ga, and quantitatively computed by Berger *et al.* (1992) only up to −0.5Ga. As insisted by Berger, it needs careful handling to trace the Milankovitch cycles back to the older era, because the relevant dynamics is not linear that the variation of the Milankovitch cycles have been supposed to be chaotic (Sussman & Wisdom 1992, Laskar 1990). Nevertheless we have a strong demand to identify and decode the cyclicity observed in the sediments such as banded iron formation of Archean or Proterozoic. The cyclicity recognized in the sediments was probably caused by the climate change due to Milankovitch cycles as estimated from the sedimentation rate. Therefore we have examined a possibility to clarify the evolutionary history of the Milankovitch cycles.

The Milankovitch cycle is defined as the long period variation of solar insolation on to the top of the earth's atmosphere. Figure 3 upmost right one is a typical example of calculation of the Milankovitch cycle which represents the daily average insolation at 65°N, summer solstice. The maximum amplitude is up to about 20% of the average value according to the change of axial tilt of the earth; obliquity and precession angle. This oscillation of the solar insolation is considered to trigger the glacial and interglacial cycles in Quaternary. The power spectrum of this time series data obtained by the standard Fourier transformation is shown in Figure 3 upper left. Apparently you can see four sharp peaks. The peaks of 19kyr and 23kyr are due to the oscillation of precession angle, and the peaks of 41kyr and 54Kyr is due to the oscillation of obliquity (though the peak of 54Kyr is rather weak). When a nonlinear earth system is subjected to the forcing with the spectrum as above, the output response of $\sim$ 100Ky period which was originated from the oscillation of the earth's eccentricity may be recovered as a modulated amplitude variation of the two precession components, $(23 \times 19)/(23 - 19) \approx 100$Ky. The nonlinear surface climate system of the earth might serve as a filter to extract the original orbital forcing. Milankovitch cycle consists of these four components of 19, 23, 41 and 54 Ky (we call them Mp1, Mp2, Mo1 and Mo2 respectively). Here we pay our attention to these frequencies in the Archean or Proterozoic. The length of these periods (19, 23, 41 (and 100) Ky) are thought to be nearly constant during the short time scale of Quaternary.

In view of the longer time scale (10^9years), these typical periods of the Milankovitch cycles must have been largely changed following to the dynamical evolution of the earth-moon system. It may be sure that the solar system have been evolving chaotically (Wisdom and Holman, 1991; Sussman and Wisdom, 1992) and many scientists insist that it is impossible to predict the precise orbital elements over certain time scale such as 10^8years, not to mention 10^9years. On the other hand, the process of dynamical evolution of the earth-moon system is not chaotic but absolutely secular. Hence, although the chaotic planetary perturbation exists, we noticed the effect of the planetary perturbation is non-systematic and minor in the variation of Mp1, Mp2, Mo1 and Mo2 and the evolution of the Milankovitch cycles can be traced back to the ancient times when the earth was spinning much faster. The purpose of the present paper is (1) to discuss about some special assumptions needed for the calculation, (2) to investigate in the effect of chaotic planetary motion and put the deviation error bars due to the chaotic behavior of the solar system on the evolution paths of the Milankovitch cycles by consulting the variable range of amplitudes and frequencies of the fundamental frequencies computed by Laskar (1988) and Nobili *et al.* (1989), (3) to present some results of the possible evolution diagram of the Milankovitch cycles on the basis of standard calculational result of the dynamical evolution of the earth-moon system, and (3) to provide a clue to relate the physical climate model with the actual data such as in the periodic striped bands in banded iron formation of Archean or Proterozoic (Figure 7 and 8).

Equation of motion

The annually averaged equation of motion of the rotational axis of the planet (in this case, the earth) is derived from the Euler's equation of rigid body rotation viewing from the inertial co-ordinate system (Ward, 1974; Bills, 1990)

$$\frac{d\boldsymbol{s}}{dt} = \alpha(\boldsymbol{s} \cdot \boldsymbol{n})(\boldsymbol{s} \times \boldsymbol{n}) \tag{1}$$

where α is called the precessional constant representing the magnitude of the gravitational torque obtained by the equatorial bulge of the earth (present value is 50.44 (arcsec/year)). $\boldsymbol{s} = (s_x, s_y, s_z)$ is the spin axis unit vector of the earth. $\boldsymbol{n}$ is the orbital normal unit vector of the earth and expressed by orbital inclination I and longitude of the ascending node Ω as

$$\boldsymbol{n} = (\sin I \sin\Omega, -\sin I\cos\Omega, \cos I) \tag{2}$$

We can get the time variation of the obliquity θ and the precession angle ϕ of Ward (1974) and Bills (1990) from following relationship:

$$\theta = \cos^{-1}(\boldsymbol{s}\cdot\boldsymbol{n}), \quad \phi = \sin^{-1}\left(\frac{s_y}{\sin\theta}\right) \tag{3}$$

and obtain the solar insolation variation such as in Figure 3 right side (cf. Berger & Loutre, 1978b). As for the time series of orbital elements I, Ω, and eccentricity e_s, longitude of perihelion with respect to the fixed vernal equinox ϖ, we use the solution of Laskar (1988)'s secular perturbation theory.

Dominant factors of the Milankovitch cycles

As mentioned before, precessional constant α which represents the degree of gravitational torque obtained by the equatorial bulge of the planet can be expressed as

$$\alpha = \frac{3n^2}{2\omega}\frac{C-A}{C}\left(\left(1-{e_s}^2\right)^{-\frac{3}{2}} + \frac{M_m}{M_s}\left(\frac{a_s}{a_m}\right)^3\left(1-{e_m}^2\right)^{-\frac{3}{2}}\left(1-\frac{3}{2}\sin^2 i_m\right)\right) \tag{4}$$

where A and C are the polar and the equatorial moment of inertia of the earth, n is the earth's mean motion to the sun, e_m is the eccentricity of the moon's orbit, M_m and M_s is the mass of the moon and the sun, a_m and a_s is the length of the semimajor axis of the moon's orbit and the earth's orbit, and i_m is the inclination of the moon's orbit against the orbit of the earth. The frequencies of the Milankovitch cycles are determined by the time variation of $\boldsymbol{n}$, α, and the mean obliquity $\overline{\theta} \simeq \cos^{-1}\langle\boldsymbol{s}\cdot\boldsymbol{n}\rangle$ ($\langle\rangle$ means the time average). $\boldsymbol{n}$ represents the orientation of the earth's orbital inclination which is affected by the gravitational perturbation among other planet. Since we consider that the effect of possible chaotic motion of the solar system is small, we assume that the oscillation of the earth's eccentricity e_s is not so different from the present age during these 4Ga, and use the quasi-periodic variation of e_s shown by Laskar (1988).

About the lunar orbital elements e_m and i_m, calculational results are much different among each researchers. Here we utilize two kinds of computational results of evolution of e_m and i_m: Abe *et al.* (1992) and Turcotte *et al.* (1977). em and i_m are just equal to zero in the simple model of Turcotte *et al.* (1977), and Abe *et al.* precisely calculated the change of them (Figure 4). However the absolute values of e_m and i_m are originally so small that they have only slight effect on the change of the precessional constant α (ie., $1-{e_m}^2$ and $1-\sin^2 i_m$ is almost unity). Similarly $1-{e_s}^2 \sim 1$ in a good approximation so actually we can consider that the precessional constant α is determined almost all by the relationship between three variables: dynamical ellipticity of the earth $\frac{C-A}{C}$, rotational angular velocity of the earth ω, and the earth-moon distance a_m. The changes of the earth-sun distance a_s, the masses M_s, M_m are not taken into account in this research.

About the evolution of the mean obliquity of the earth there are many hypothesis: climate friction (Rubincam, 1990), stochastic accumulation process (Dones and Tremaine, 1993), tidal evolution (Kaula, 1964). In this discussion we only take into account the tidal evolution effect calculated by Abe *et al.* (1992) because other factors are rather vague. But actually the mean obliquity at -4Ga of Abe *et al.* (1992) is about $18°$, and the difference of $\cos\overline{\theta}$ at -4Ga and the present is $\cos 18° - \cos 23.5° \approx 0.034$, which is fairly small.

In Quaternary time scale (10^6years), rotational angular velocity of the earth ω, earth-moon distance a_m, and dynamical ellipticity of the earth $\frac{C-A}{C}$ are considered to be nearly constant. However they are not constant in 10^9years time scale, because ω has been becoming smaller and a_m has been becoming larger because of the tidal friction between the earth and moon. Hence α changes with age, and so do the frequencies of the Milankovitch cycles. Here we explicitly put three assumptions for determining the evolution of α and consider about the validity of these assumptions in the next section.

Assumptions and their extent of validity

1. Conservation of the angular momentum of the earth-moon system: The earth-moon system has been losing its angular momentum along the evolution due to the tidal torque from the sun. This effect is calculated by Abe *et al.* (1992) and the results are shown in Figure 5 (c). As you can see, the angular momentum of the earth-moon system has changed no more than 1% in these 4Ga, so this assumption is quite good as the first approximation.

2. Density structure of the earth's interior has not been changed: Though the timing of core formation of the earth is still not clear, it is sure that the earth underwent the core formation stage quite earlier in its accumulation history (Newsom, 1990). Of course some events like fractionation of the crust from the mantle or mode change of mantle convection from the one layer mode to the two layer mode may have changed the density structure of the earth, but we can say that the time variation of the density structure of the earth's interior has been enough small within the practically variable range for our purpose.

3. Dynamical ellipticity is proportional to the square of the rotational angular velocity (ω^2) : Dynamical ellipticity $\frac{C-A}{C}$ of the earth is the most important quantity in our discussion. There are two concepts about the oblateness of the planet, one is the dynamical ellipticity which indicates the degree of gravitational flattening of the planet, and the other is the geometrical flattening f which is the ratio of the semimajor axis and semiminor axis of the earth. When the angular velocity ω is large, $\frac{C-A}{C}$ and f are large, and the planet is more oblate. On the contrary when ω is small, $\frac{C-A}{C}$ and f are small. The accurate determination of the dynamical ellipticity as a function of rotational angular velocity ω in the case of the actual earth is quite complicated because we should use Clairauts's theorem to calculate the equipotential surface and obtain $\frac{C-A}{C}$ of the stratified planet (Zharkov and Trubitsyn, 1978; Denis, 1986). But here we assume that the dynamical ellipticity $\frac{C-A}{C}$ is nearly equal to the geometrical flattening f and proportional to the square of rotational angular velocity (ω^2). This assumption is just correct in the case of constant density rotating body and justified well if the density structure of the earth is hydrostatic (Turcotte and Schubert, 1982; Stacy, 1992). The effects of nonhydrostatic state and the variability of the density structure within the earth are supposed to be small. But of course they are one of the important subjects of our future work.

Above we put three bold asumptions to calculate the evolutionary history of the Milankovitch cycles. Though the above assumptions are very rough, they may be essentiallyl true in the light of common sense of the geophysics of the linear system. However, we are living in a typical non-linear dynamic system — chaotic solar system. Below we investigate in a sensibility of the Milankovitch cycles to the possible chaotic motions of the planets in this solar system.

Effect of the possible chaotic motions of the planets

When the stability of our solar system is discussed, two objections often arise. First, this problem has been coming around for too long years, never getting to the final point to state clearly whether the system is stable or not; the few definite results refer to mathematical abstractions such as N-body models and do not really apply to the real solar system. Second the solar system is macroscopically stable — at least for a few 10^9 years — since it is still there, and there is not much point in giving a rigrous argument for such an intuitive proterty. By the problem of the stability of the solar system we mean to understand whether our solar system is stable for its entire lifetime or not. We are concerned only with a finite ($10^9 \sim 10^{10}$) years timespan, not with an infinite timespan which has stable solution as proved by Poincaré over a century ago. However in spite of the efforts of many scientists (Wisdom and Holman, 1991; Sussman and Wisdom, 1992) no stable (or periodic) solution is found and it even seems they give up finding the stable solution of the solar system and intend to show numerically and synthetically the instability of this solar system (Nobili *et al.*, 1989).

In Laskar (1990)(Laskar, 1990) he writes two issues about the chaotic characteres of the solar system: (1) it is impossible to compute the exact motion of the solar system over more than 100Myr and the solution over 200Myr will be just a qualitative possibility before 100Myr. (2) fundamental frequencies of the solar system g_i and s_i are not constant and slowly vary with time.

Milankovitch frequencies are determined by two factors. One is the luni-solar precession of which periods are characterized by the precessional constant α, ie., the earth's rotational angular velocity ω, earth-moon distance a_m, and the dynamical ellipticity of the earth $\frac{C-A}{C}$ without association with the motion of other planets. The other factor is called the planetary precession which represents the movement and deformation of the earth's orbital plane in the inertial coordinate system. The chaotic effect will appear in the Milankovitch cycles through the latter factor. In Berger *et al.* (1992)(Berger *et al.*, 1992) they showed the evolution of the main Milankovitch periods back to 200Myr including the variation of the fundamental seular frequencies g_i and s_i, and concludes that the impact of the changes of g_i and s_i are much less than that of the variation of the precessional constant α.

Then what about the evolutionary paths of the Milankovitch cycles over the whole 4Ga in the earth's history? If this solar system is totally chaotic we cannot manage to reach the clear conclusion about the evolution of the Milankovitch cycles, nothing to say the planetary motions. But even if so, we know there are some periodic environmental changes in Archean of Proterozoic earth's surface which indicate the existence of insolation variations at that time. It is indeed significant to get the evolutionary history of the Milankovitch cycles as long as the earth-moon system by using all data and knowledge obtained for now.

Quasi-periodic motion of the orbital plane

Orbital elements e, ϖ, I, Ω are expressed by the parameters $A_j, A'_j, \nu_j, \nu'_j, \phi_j, \phi'_j$ which represents the quasi-periodic motions of the solar system as follows: (Laskar, 1990)

$$e \exp\sqrt{-1}\varpi = \sum_j^n A_j \exp\sqrt{-1}(\nu_j t + \phi_j) \tag{5}$$

$$\sin\frac{I}{2} \exp\sqrt{-1}\Omega = \sum_j^n A'_j \exp\sqrt{-1}(\nu'_j t + \phi'_j) \tag{6}$$

Parameters $A_j, A'_j, \nu_j, \nu'_j, \phi_j, \phi'_j$ is listed in the tables of Laskar (1988). The exact mathematical definition of so called "chaos" varies widely among the researchers, but it can be summarized that "slightly small difference of the initial conditions cause the unexpectedly large difference of the output results". In this case it can be translated into the phrase that the major amplitude A_j, A'_j, frequencies ν_j, ν'_j, and the initial conditions ϕ_j, ϕ'_j is not constant during the time scale of over 10^8 years (i.e., not periodic and predictable) and we can hardly know their exact values. This brief interpretation about the term "chaotic solar system" is sufficient for us climate researchers to discuss about the climate change of surface systems of the earth. As for the parameters of orbital elements e, ϖ, I, Ω and the characteristic amplitudes A_j, A'_j, frequencies ν_j, ν'_j, and the initial phases ϕ_j, ϕ'_j, only what we have is the present values. The suffix j ($j = 1, \cdots, n$, and n in the list of Laskar (1988) is 80) has of course their physical meaning, some of them are strongly subject to changes and some of them are not.

Table 1. Left three columns: mean value of the fundamental frequencies $\overline{\nu}$ over 200Myr and numerical lower estimates Z_ν of the size of the chaotic zones (after Laskar (1990)). ν_0 are the values of the fundamental frequencies of the linear part of the secular system. Right three columns are the mean amplitudes of $\overline{A_p}$, their absolute variable ranges $\Delta\overline{A_p}$ and the relative variable ranges $\frac{\Delta\overline{A_p}}{\overline{A_p}}$ of the proper modes. All values in the right three columns are rough estimation from Figure 8 and 9 of Laskar (1991) ($g_1 \sim g_4, s_1 \sim s_4$) and Figure 4 of Nobili *et al.* . (1989) ($g_5 \sim g_8, s_6 \sim s_8$).

	ν_0 (arcsec/y)	$\overline{\nu}$ (arcsec/y)	$Z\nu$ (arcsec/y)	$\overline{A_p}$	$\Delta\overline{A_p}$	$\frac{\Delta\overline{A_p}}{\overline{A_p}}$
g_1	5.86046	5.59	0.10	1.2	0.2	0.17
g_2	7.46041	7.455	0.013	1.0	0.2	0.20
g_3	17.46509	17.30	0.17	1.0	1.0	1.0
g_4	18.11381	17.85	0.20	0.7	0.7	1.0
g_5	4.12866	4.24882	0.00002	0.005065	0.000025	0.0050
g_6	23.47280	28.203	0.0010	0.002316	0.000005	0.0022
g_7	2.98980	3.08952	0.00007	0.000595	0.00003	0.050
g_8	0.65270	0.66698	0.00003	0.000085	0.000003	0.035
s_1	-5.20087	-5.59	0.08	0.7	0.7	1.0
s_2	-6.57027	-7.00	0.23	1.0	1.0	1.0
s_3	-18.74453	-18.88	0.06	1.0	0.2	0.20
s_4	-17.63461	-17.80	0.12	1.0	0.2	0.20
s_5	0.0	0.0	0.0	0.0	0.0	0.0
s_6	-25.67345	-26.33020	0.00008	0.0000715	0.000015	0.21
s_7	-2.92885	-3.00563	0.00006	0.000083	0.000004	0.048
s_8	-0.68277	-0.69195	0.00001	0.0000342	0.0000015	0.044

As mentioned in Berger & Loutre (1987) major spectral peaks of the modern Milankovitch cycles are determined by the fundamental frequencies g_i and s_i, the precessional constant α, and time averaged obliquity $\overline{\theta}$ (Berger and Loutre, 1987). α and $\overline{\theta}$ have been changed secularly with the dynamical evolution of the earth-moon system (α has been decreased and $\overline{\theta}$ has been increased). Hence the chaotic behavior of the solar system will appear in the Milankovitch cycles through the variation of the amplitudes and the frequencies of the g_i and s_i. It is fairly difficult to precisely know the chaotic variable range of the fundamental frequencies, but in Laskar (1990) he showed the lower estimation of the possible chaotic zones of them. So we utilize them and calculate the variable range of the typical Milankovitch peaks Mp1, Mp2, Mo1 and Mo2 (Ito *et al.*, 1993) during these 4Ga.

The trouble occurs in the amplitudes A_j, A'_j. Accurate and complete calculation of the amplitudes A_j and A'_j leads to quite complicated eigenvalue problem (even in the case of a linear secular perturbation theory) so we utilized the calculational results of the amplitude of the proper modes in Nobili *et al.* (1989) and Laskar (1991). Qualitative properties of the motion of the perihelia and the nodes of the outer planets are described well by a linear secular perturbation theory (Nobili *et al.*, 1989) because of their large masses. On the other hand the analogous plots in Laskar (1991) seems to indicate that the dynamics of the inner planets is much less regular than that of the outer planets. Variable ranges of the fundamental frequencies and the amplitudes of proper modes are listed in Table 1.

Table 2. Amplitudes, periods and frequencies of the major terms of the climatic precession and obliquity on the present earth. We took major four terms for climatic precession and three terms for obliquity. Mean amplitude $\mathcal{A}_{\rm mean}$ is after tables of Berger & Loutre (1987), and $\mathcal{A}_{\rm min}, \mathcal{A}_{\rm max}$ is calculated by using $\frac{\Delta A_p}{A_p}$ in Table 1 as $\mathcal{A}_{\rm min} = \mathcal{A}_{\rm mean}\left(1 - \frac{\Delta A_p}{A_p}\right)$ and $\mathcal{A}_{\rm max} = \mathcal{A}_{\rm mean}\left(1 + \frac{\Delta A_p}{A_p}\right)$. $f_{\rm mean}$ is a simple sum of argument (ex., in the case of climatic precession No.1 term, $f_{\rm mean}(54.68861) = g_5 + k = 4.24882 + 50.4398 = 54.68861$). $f_{\rm min}, f_{\rm max}$ is calculated by using Z_ν in Table 1 as $f_{\rm min} = f_{\rm mean}(1 - Z_\nu)$ and $f_{\rm max} = f_{\rm mean}(1 + Z_\nu)$. Periods are equal to $\frac{2\pi}{f_{\rm mean}}$.

Climatic Precession (present)

	period (year)	argument	$\mathcal{A}_{\rm mean}$	$\mathcal{A}_{\rm min}$	$\mathcal{A}_{\rm max}$	$f_{\rm mean}$ (arcsec/y)	$f_{\rm min}$ (arcsec/y)	$f_{\rm max}$ (arcsec/y)
1	23697	$g_5 + k$	0.018608	0.018515	0.018701	54.68861	54.68859	54.68863
2	22385	$g_2 + k$	0.016275	0.013020	0.019530	57.89479	57.88179	57.90779
3	18977	$g_4 + k$	−0.013007	0.0	−0.026013	68.28979	68.08979	68.48979
4	19132	$g_3 + k$	0.009888	0.0	0.019777	67.73979	67.56979	67.90979

Obliquity (present)

	period (year)	argument	$\mathcal{A}_{\rm mean}$ (arcsec)	$\mathcal{A}_{\rm min}$ (arcsec)	$\mathcal{A}_{\rm max}$ (arcsec)	$f_{\rm mean}$ (arcsec/y)	$f_{\rm min}$ (arcsec/y)	$f_{\rm max}$ (arcsec/y)
1	41064	$s_3 + k$	−2462.22	−1969.78	−2954.66	31.55979	31.49979	31.61979
2	39706	$s_4 + k$	−857.32	−685.86	−1028.78	32.63979	32.51979	32.75979
3	53754	$s_6 + k$	−629.32	−497.16	−761.48	24.10959	24.10951	24.10967

Variable ranges of the Milankovitch frequencies

There are two factors which affect the amplitudes and frequencies of the Milankovitch cycles. One is called the climatic precession $e \sin \tilde{\omega}$, where $\tilde{\omega}$ is the longitude of perihelion with respect to the moving vernal equinox (Berger, 1976). Because eccentricity e is small number the maximum amplitude of the climatic precession is also small. Another factor is obliquity θ. In the case of present earth, variable range of obliquity is $\sim \pm 1^\circ$ so the order of amplitudes is $O(100(\text{arcsec/year})) \sim O(1000(\text{arcsec/year}))$. Obliquity is included in the solar insolation calculation as the term $\sin\theta \sin|\varphi|$ where φ is latitude on the earth (Milankovitch, 1941). In Table 2 we showed the possible variable ranges of the major terms

Table 3. Amplitudes, periods and frequencies of the major term of the climatic precession and obliquity on the earth of −3Ga. Contents of each columns are the same as Figure 2. Notice that the amplitudes of obliquity are lower than the present values due to the large precessional torque.

Climatic Precession (−3Ga)

	period (year)	argument	$\mathcal{A}$mean	$\mathcal{A}_{\min}$	$\mathcal{A}$max	$f$mean (arcsec/y)	$f_{\min}$ (arcsec/y)	$f$max (arcsec/y)
1	6248	g_5+k	0.018608	0.018515	0.018701	207.42544	207.42542	207.42546
2	6152	g_2+k	0.016275	0.013020	0.019530	210.63162	210.61862	210.64462
3	5863	g_4+k	−0.013007	0.0	−0.026013	221.02662	220.82662	221.22662
4	5878	g_3+k	0.009888	0.0	0.019777	220.47662	220.30662	220.64662

Obliquity (−3Ga)

	period (year)	argument	$\mathcal{A}$mean (arcsec)	$\mathcal{A}_{\min}$ (arcsec)	$\mathcal{A}$max (arcsec)	$f$mean (arcsec/y)	$f_{\min}$ (arcsec/y)	$f$max (arcsec/y)
1	7032	s_3+k	−611.26	−489.01	−733.51	184.29662	184.23662	184.35662
2	6991	s_4+k	−212.83	−170.27	−255.40	185.37662	185.25662	185.49662
3	7328	s_6+k	−156.23	−123.42	−189.04	176.84642	176.84634	176.84650

of climatic precession and obliquity. When we describe the amplitudes of the Milankovitch cycles as $\mathcal{A}$ and the contribution of luni-solar precession and the planetary perturbation as $\mathcal{A}_\alpha$ and $\mathcal{A}_p$ respectively, apparently $\mathcal{A} = \mathcal{A}_\alpha \mathcal{A}_p$, so the ratio $\frac{\Delta \mathcal{A}}{\mathcal{A}}$ is just equal to the ratio $\frac{\Delta \mathcal{A}_p}{\mathcal{A}_p}$. Here k in the column of argument equals to $\alpha \cos \overline{\theta}$ where $\overline{\theta}$ is the time average value of obliquity. At present $\overline{\theta} = 23.4°$ so $k = 50.4398$ (arcsec/year). In Table 2 $\mathcal{A}_{\rm mean}$ has the same value as in Berger (1987) and the variable ranges of the amplitudes are shown by $\mathcal{A}_{\rm min}$ and $\mathcal{A}_{\rm max}$ determined from the amplitude variation of the proper modes in Table 1. Similarly $f_{\rm mean}$ is the simple sum of the argument frequencies (eg. $g_5 + k$) and the range estimated by using Laskar (1990)'s Z_ν (Table 1) is shown by $f_{\rm min}$ and $f_{\rm max}$. Periods are equal to $2\pi/f_{\rm mean}$.

Table 2 shows the case of present age when $\alpha = 54.9600$ (arcsec/year). We can estimate the amplitudes and frequencies variation in ancient times such like Archean or Proterozoic in a similar way. Table 3 shows the ranges of major terms of climatic precession and obliquity at that time. According to the calculational results of the dynamical evolution of the earth-moon system, $\alpha = 214.8838$ (arcsec/year) at -3Ga (Abe *et al.*, 1992). There is one more point to notice here. Amplitude of obliquity oscillation is approximately proportional to the reciprocal of k (Ward, 1974). When k is large (ie., gravitational torque on the equatorial bulge of the planet is large) mean spin axis ($\overline{\boldsymbol{s}}$) tends to be fixed on the instantaneous orbital normal $\boldsymbol{n}$, and obliquity approaches constant in spite of the movement of $\boldsymbol{n}$. This is the reason why $\mathcal{A}_{\rm mean}, \mathcal{A}_{\rm min}, \mathcal{A}_{\rm max}$ of obliquity in Table 3 are much smaller than those in Table 2. Average obliquity has also been said to increase due to the lunar and solar tide. According to Abe *et al.* (1992), $\overline{\theta} \simeq 19°$ at -3Ga so $k = 214.8838 \times \cos 19° = 203.17662$ (arcsec/year).

In Figure 1 and 2 we illustrated the variable ranges of amplitude and frequency in the major spectral peaks of Table 1 and 2. Horizontal axis denotes the frequency (f) and vertical axis denotes the absolute value of the amplitude ($|\mathcal{A}|$). What we notice here is the distinguishment of each spectral peaks, that is, ranges of the frequency variation do not overlie each other. Ranges of the amplitude variation are larger the the frequency variation, but rather moderate except Mp1 which varies from 0 to 200% of the original value. However in general, we can conclude that the effect of chaotic behavior of the solar system to the amplitudes and frequencies of the Milankovitch cycles may not be so important than had expected, and we can distinguish the three major spectral peaks Mp1, Mp2 and Mo1 which we recognize in the samples from the ocean bottom sediments or polar ice cores of the Quaternary (Mo2 is originally small and not sure to be found in sedimentation records). And moreover, the difference between the Milankovitch frequencies of the present and the ancient times (such as Archean or Proterozoic, in Figure 1 and 2) is apparent even we if consider the chaotic effect.

Results and Discussions

Below we show the calculational results of the evolution of earth's rotational angular velocity ω and the earth-moon distance a_m by Abe *et al.* (1992) and Turcotte *et al.* (1977) in Figure 5. In Figure 5 (a) ω is replaced by LOD (Length of Day). The difference between two results in Figure 5 (a)(b)(c)(d)(e) is caused by the difference of the method of modeling of the tidal mechanism. Turcotte's method is a simple and analytical one, and Abe's method is completely numerical including the non-axis-symmetrical distribution of continents and oceans, and effect of the solar tide.

Then we can calculate the motion of the rotational axis of the earth by solving the equation of motion (1) and obtain ancient Milankovitch cycles. Figure 3 lower left ones are one of the results, showing the power spectrum of daily average insolation time series of summer solstice at 65°N at −30Ga. The sampling duration is four million years and integration time step is one thousand years. Of course we can directly integrate the equation of motion (1) and digitize the peak frequencies manually, but typical frequencies of the Milankovitch cycles are known to consists of linear combinations of the fundamental frequencies of planetary perturbation g_i and s_i and precessional constant α (Berger and Loutre, 1987). In this paper we concentrated on the four peak frequencies (Mp1, Mp2, Mo1, Mo2) described in Table 4 and traced their evolution back to −4Ga. The results are summarized in Figure 5 (e). The major periods of Milankovitch cycles Mp1, Mp2, Mo1 and Mo2 shift shortened as going up time. We assume the constancy of the fundamental frequencies as mentioned before. Since we are concentrating on the discussion of the frequency domain (frequencies of the Milankovitch cycles) we should take care only about the frequencies of the results, not about the phases (on the condition that the initial values are "moderate" or physically "reasonable" ones).

peak	argument	resume $\rightarrow \frac{1}{\text{period}}$
Mp1	$g_4 + \alpha$	$\frac{17.85+50.44}{3600\times360} \rightarrow \frac{1}{18977}$
Mp2	$g_5 + \alpha$	$\frac{4.249+50.44}{3600\times360} \rightarrow \frac{1}{23697}$
Mo1	$s_3 + \alpha$	$\frac{-18.88+50.44}{3600\times360} \rightarrow \frac{1}{41064}$
Mo2	$s_6 + \alpha$	$\frac{-26.33+50.44}{3600\times360} \rightarrow \frac{1}{53753}$

Table 4. Origin of the Milankovitch frequencies Mp1, Mp2, Mo1 and Mo2. α is precessional constant. Whole values above are of the present, and the unit of period is year. The factor 3600×360 denotes the conversion of unit from radian to arcsec.

There is another point here to notice. In the diagrams of Figure 3 leftsied, power of the obliquity term (Mo1 and Mo2) of older era is apparently much lower than that of the present age. This means that the amplitude of the time series data of obliquity oscillation was depressed. Obliquity oscillation is caused by the motion of instantaneous orbital normal $\boldsymbol{n}$ which gives the gravitational torque on the equatorial bulge of the planet. When the precessional constant α equals to zero, the planet never get any gravitational torque and spin axis $\boldsymbol{s}$ remains constant in the inertial space. On the other hand when $\alpha \rightarrow \infty$ (ie., magnitude of the gravitational torque reaches to infinity), time average of spin axis (precessional center) is fixed on the orbital plane and obliquity approaches constant value in spite of the movement of the orbital plane. Magnitude of gravitational torque is much larger at −3Ga than the present so Figure 3 lower left ones correspond to the latter case. This is the reason why the power of Mo1 and Mo2 becomes low in Figure 3 lower left ones. Strict mathematical formulations are given in equation (10) to (32) of Ward (1974).

The diagram with the horizontal axis of the absolute ages (Figure 5) is strongly dependent on the model of tidal dissipation mechanism between the earth and the moon. We can avoid this problem by replacing the absolute age with Length of Day (LOD) as the horizontal axis, because LOD is essentially a function only of the density structure of the earth, and the dynamical ellipticity or the rotational speed of the earth. Therefore LOD-periods plot depends only on the rotational angular velocity ω. Figure 6 is the plot using LOD by the horizontal axis. We can obtain a similar result if we take the earth-moon distance as the horizontal axis instead of LOD.

In Figure 7 we showed the Banded Iron Formation of −3.3Ga which was found in Australia. The black (actually red) and white striped bands on it are periodic and modulated, indicating the presence of environmental variation with plural periods. Hence they are believed to form either or both by the lunar tides and the Milankovitch cycles. If they were caused by Milankovitch cycles, the relative age between stripes can be known by theoretically estimating the periods of Milankovitch cycles at that time, using the results we obtained above. In Figure 8 there showed a photograph of Stromatolite.

Thus we can establish the standard measurement of relative age, ie., the lap time clock for decoding the history of the earth comparing the stripes on BIF with theoretical Milankovitch cycle frequencies. Since the Milankovitch cycles have been playing an significant role in the climate change on the earth, what we did in this discussion shows another possibility of the co-evolution of the Milankovitch cycles and the earth-moon system.

Milankovitch cycles have been evolving following to the dynamical evolution of the earth-moon system. We put three assumptions to calculate ancient Milankovitch cycles, and investigated the sensibility of the cycles to the possible chaotic motion of the solar system. Our results using these assumptions show the possibility of shorter periods of Milankovitch cycles at ancient times, and establish the preliminary reference model of co-evolution of Milankovitch cycles and tidal cycles. In further researches we will make a detailed studies on the deviation due to these assumptions, and build the more precise model of the evolution of the Milankovitch cycles.

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Figure Captions

Figure 1. Spectrum diagram of the variable ranges of climatic precession and obliquity at present (hatched area). Horizontal axis denotes the frequency (arcsec/year) and vertical axis denotes the absolute value of amplitude. Amplitude of climatic precession is dimensionless and that of obliquity is arcsec. Notice that the variable range of frequencies are small enough to distinguish each peaks. Variable ranges of amplitude is rather large especially Mp1 (200% of the mean value).

Figure 2. Spectrum diagram of climatic precession and obliquity of -3Ga ago. Horizontal and vertical scale is the same as Figure 1 but all peaks are shifted toward the high frequency region, because the gravitational torque on the equatorial bulge of the earth was much larger at this age. Notice that the amplitudes of obliquity are lower than the present values due to the large precessional torque.

Figure 3. Time series data and power spectrum diagram of the daily average solar insolation variation (65°N, summer solstice). (Right side) time series data at present, $-1Ga$, $-2Ga$, $-3Ga$. Unit of vertical axis is $\mathrm{W/m^2}$. You can see the same envelop wave caused by the oscillation of eccentrisity e. (Left side) power spectrum diagram of the right data. At the present one we notice on the peak of about 19ky ($Mp1$), 23ky ($Mp2$), and 41ky ($Mo1$). $Mp2$ is a doublet of two sharp spectral peaks owing to the modulation of several eigen frequencies of orbital parameters. But in practice these peaks are observed as a single and broad peak in the power spectrum of the time series of geological data such as δ^{18}O anomaly, because this doublet will be totally smoothed out by various processes of surface climate system on the earth (Hays *et al.*, 1976). All these peaks are shifted toward the short high frequency region as going back the age, because the precessional constant α were much larger at ancient times. Moreover the amplitude of the obliquity term (Mo1 and Mo2) are suppressed in the diagram of older era. It is also because of the strong gravitational torque from the near moon.

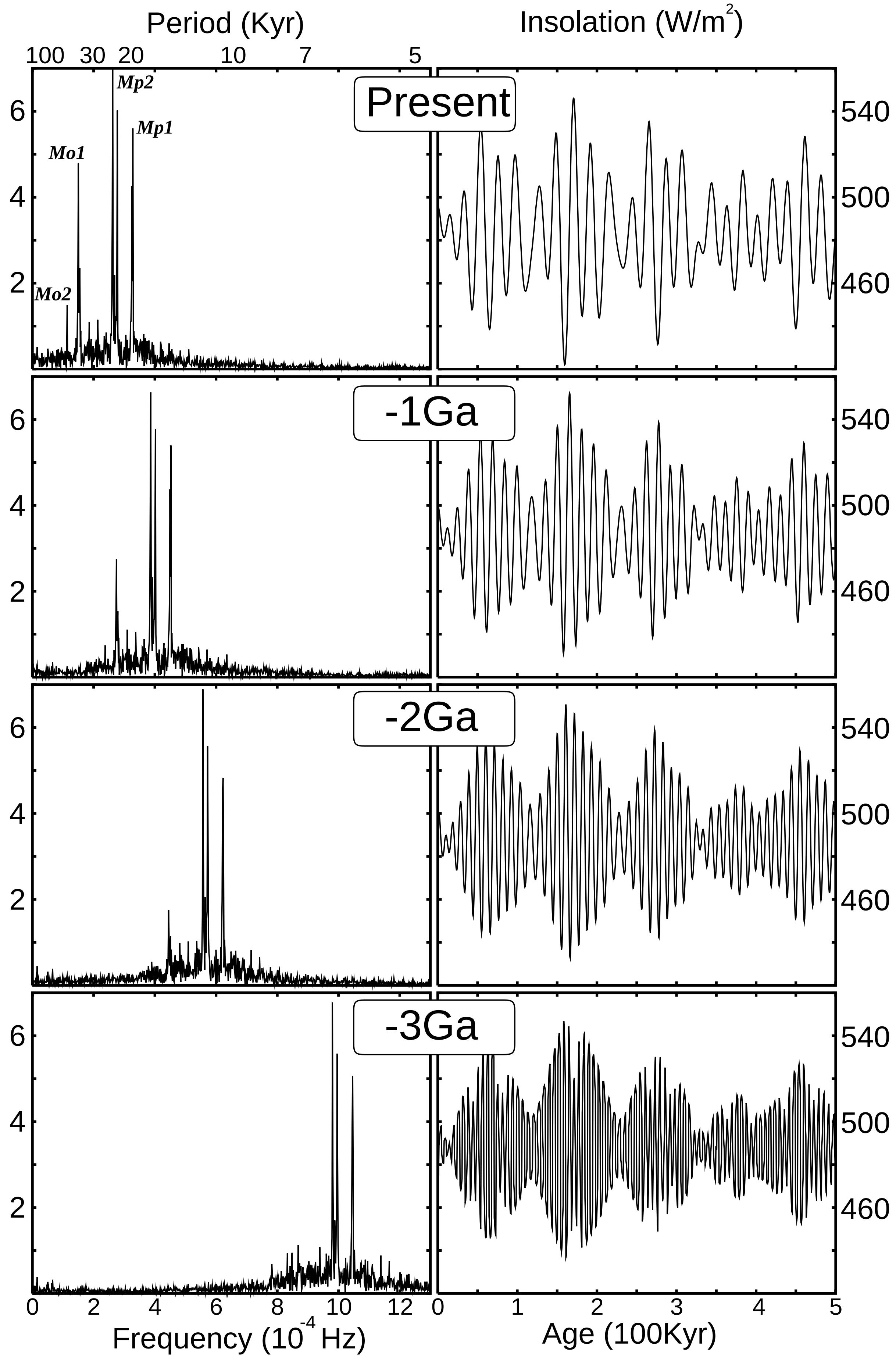
Figure 4. Orbital inclination (with respect to the earth's ecliptic) and eccentricity of the moon, mean obliquity of the earth and the total angular momentum of the earth-moon system after Abe *et al.* (1992). (a) Inclination of the moon i_m (degree) (b) eccentricity of the moon e_m (non-dimensional), (c) Mean obliquity (degree), (d) the angular momentum of the earth-moon system ($10^{34}\mathrm{Kg\cdot m^2/s^2}$). In the model of Turcotte *et al.* (1977) $e_m = i_m = 0$, and the mean obliquity is not taken into account (they only considered the secular change of the rotational angular velocity of the earth, no consideration about the precession or obliquity), and the total angular momentum is kept constant.

Figure 5. Transition diagram of the Milankovitch cycles including the dynamical evolution of the earth-moon system. (a) Length of Day (LOD, present value equals to the unity), (b) Distance between the earth and the moon (radius of the earth R_E), (c) Dynamical ellipticity of the earth (non-dimensional), (d) Precessional constant α (arcsec/year), (e) Evolution paths of the major periods of the Milankovitch cycles $Mp1$ (solid line), $Mp2$ (dashed line), $Mo1$ (dot-dash line), the unit is year. The solid lines in the column of (a)(b)(c)(d) are the result using the tidal model of Abe *et al.* (1992) and the dotted lines are using Turcotte *et al.* (1977). In (e) the four thick lines are from Abe *et al.* (1992) and the four thin lines are from Turcotte *et al.* (1977).

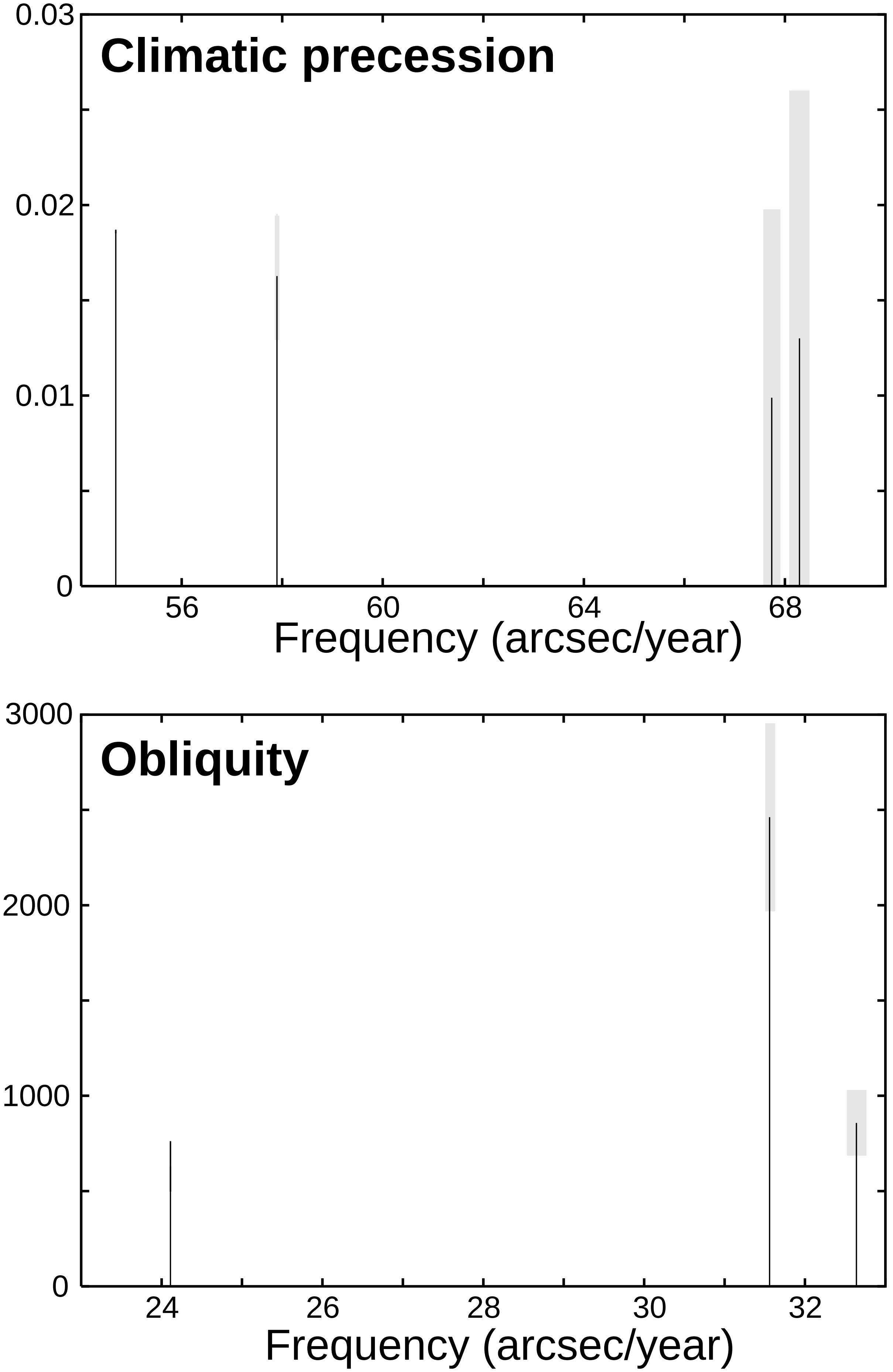
Figure 6. Same as the Figure 5, but the horizontal axis is Length of Day (LOD) instead of absolute age. In this case there is no difference between each models.

Figure 7. The banded iron formation from Creavaville, Australia. The black (actually red) bands consist of oxidized iron sediment, the white bands consist of chert. Absolute age of the whole rock is -3.3Ga.

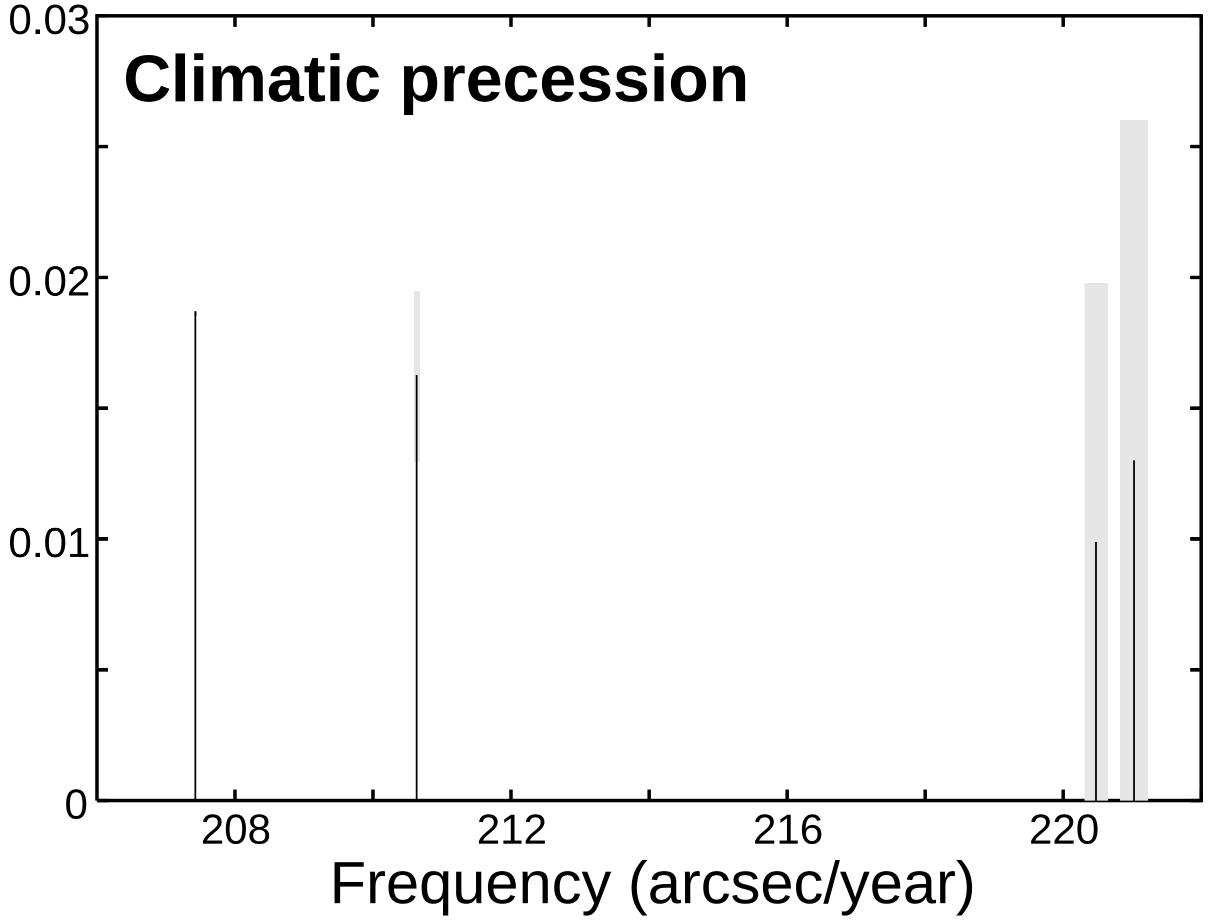
Figure 8. The stromatolite from Pilbara, Australia. Absolute age of the whole rock is -3.3Ga. These will be some significant keys to decode the history of the surface environment of the earth.



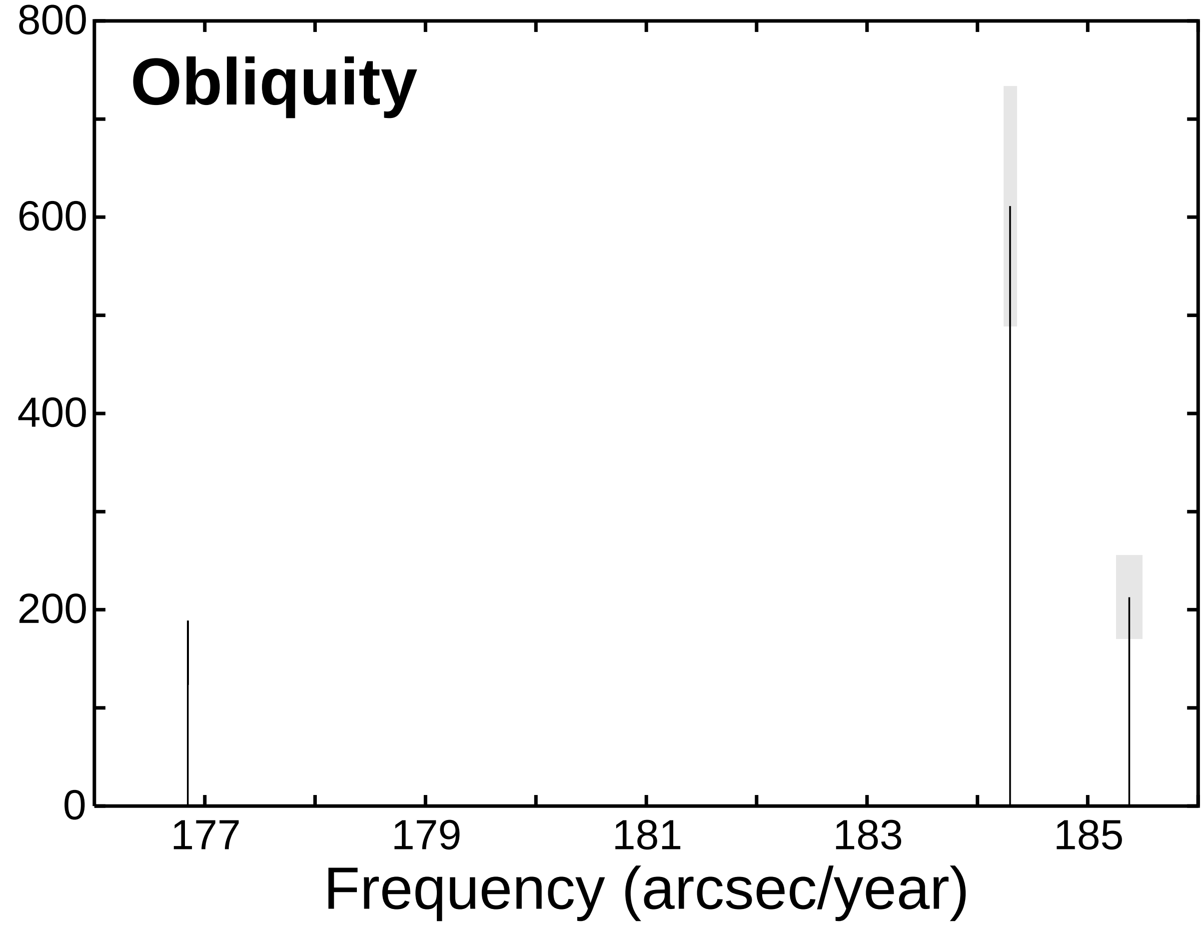
Period (Kyr)
Insolation (W/m²)
100
30
20
10
7
5
Mp2
Mp1
Mo1
Mo2
Present
-1Ga
-2Ga
-3Ga
540
500
460
6
4
2
0
8
10
12
1
3
5
Frequency (10⁻⁴ Hz)
Age (100Kyr)



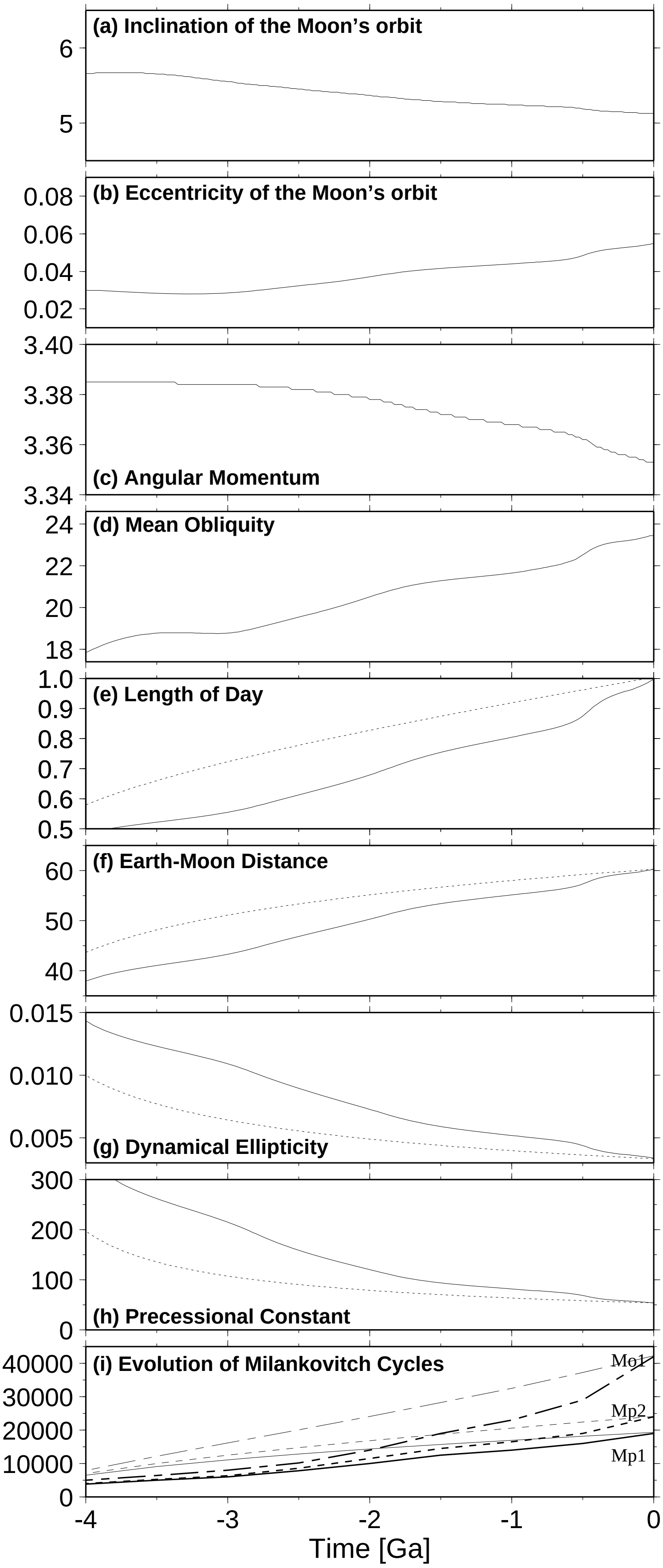
Climatic precession
0.03
0.02
0.01
0
56
60
64
68
Frequency (arcsec/year)
Obliquity
3000
2000
1000
0
24
26
28
30
32
Frequency (arcsec/year)

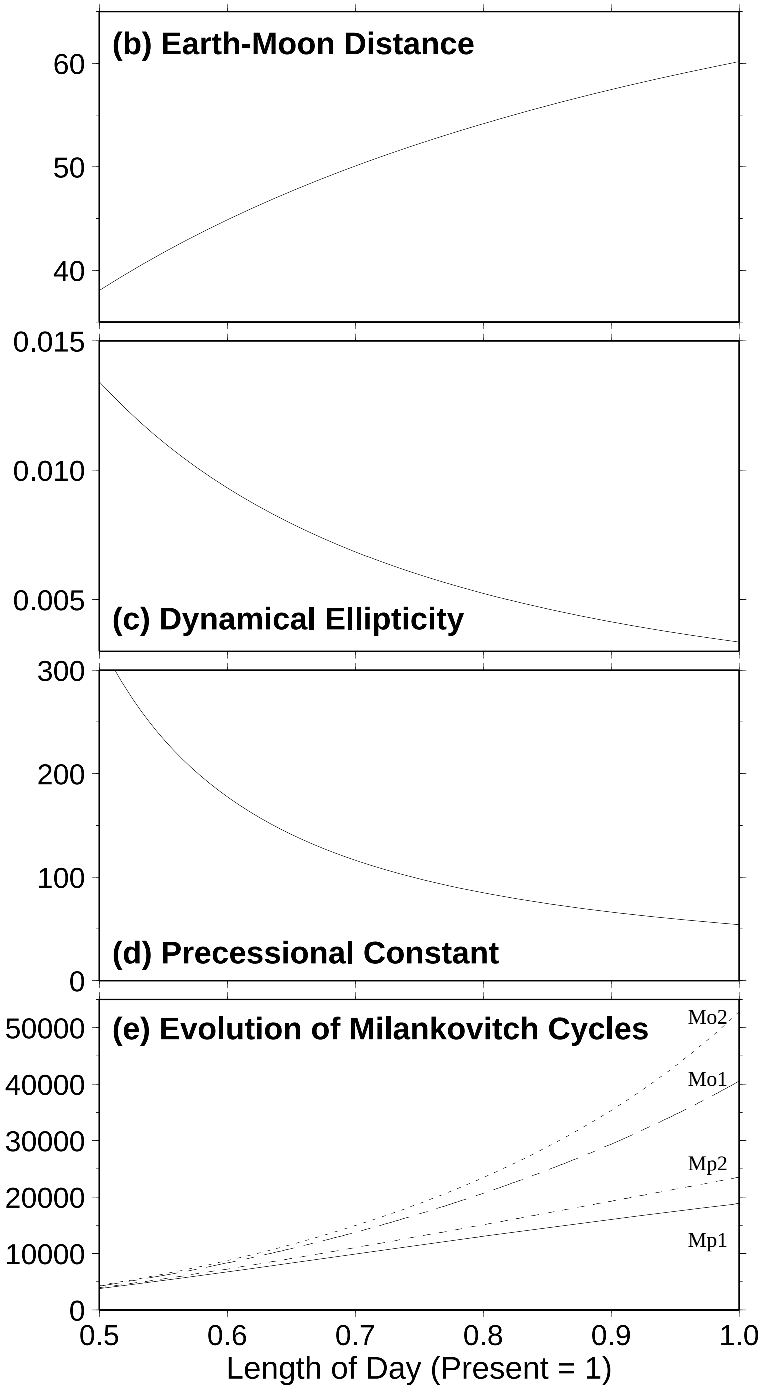


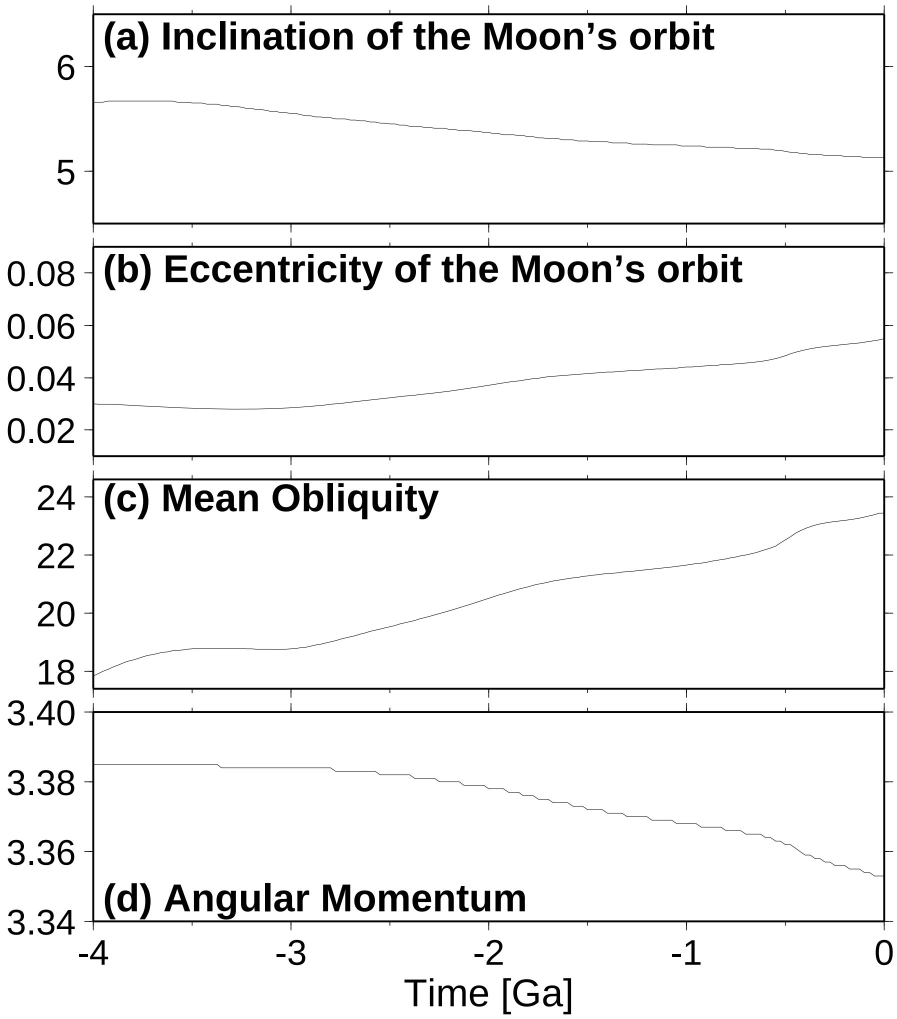
Climatic precession
0.03
0.02
0.01
0
208
212
216
220
Frequency (arcsec/year)



Obliquity
800
600
400
200
0
177
179
181
183
185
Frequency (arcsec/year)


(a) Inclination of the Moon's orbit
(b) Eccentricity of the Moon's orbit
(c) Angular Momentum
(d) Mean Obliquity
(e) Length of Day
(f) Earth-Moon Distance
(g) Dynamical Ellipticity
(h) Precessional Constant
(i) Evolution of Milankovitch Cycles
Mo1
Mp2
Mp1
Time [Ga]


(b) Earth-Moon Distance
(c) Dynamical Ellipticity
(d) Precessional Constant
(e) Evolution of Milankovitch Cycles
Mo2
Mo1
Mp2
Mp1
Length of Day (Present = 1)


(a) Inclination of the Moon's orbit
6
5
(b) Eccentricity of the Moon's orbit
0.08
0.06
0.04
0.02
(c) Mean Obliquity
24
22
20
18
3.40
3.38
3.36
3.34
(d) Angular Momentum
-4
-3
-2
-1
0
Time [Ga]