

研究課題名

Numerical Simulations of MHD instabilities in the Kippenhahn-Schlüter Prominence Model

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1 Introduction

Observations by the Solar Optical Telescope on the Hinode satellite have shown that on even on the smallest observable smaller scale quiescent prominences are highly dynamic phenomena. Berger et al (2008) reported dark upflows that propagated from the base of a quiescent prominence, through a height of approximately 10Mm before forming a mushroom cap profile. The dark upflows maintained an almost constant velocity of approximately 20km s^{-1} throughout their rise phase. Once the upflows had propagated through a height of 10Mm, they faded away into the background prominence.

The model that we use in this work is the Kippenhahn-Shlüter (K-S) model. The Kippenhahn-Shlüter model for the solar prominence uses the Lorentz force from a curved magnetic field to support plasma against gravity. This model has been shown to be linearly stable to ideal MHD perturbations. In this paper we present a study of how a nonlinear perturbation in the form of a low density tube placed inside the K-S model can allow the interchange of magnetic field lines causing upflows to form inside the prominence.

In section 2 we will describe the basic equations and numerical procedure. The numerical results are then presented and explained in section 3 and then summarised in section 4.

2 Numerical Method

2.1 Basic Equations

In this study, we use the ideal MHD equations. Gravity was included, but viscosity and radiative terms were neglected. The equations are expressed as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \left(\rho \mathbf{v} \mathbf{v} + p \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{4\pi} + \frac{\mathbf{B}^2}{8\pi} \right) = \rho \mathbf{g} \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad (3)$$

$$\frac{\partial}{\partial t} \left(\epsilon + \frac{\mathbf{B}^2}{8\pi} \right) + \nabla \cdot \left[(\epsilon + p) \mathbf{v} + \frac{c}{4\pi} \mathbf{E} \times \mathbf{B} \right] = \rho \mathbf{g} \cdot \mathbf{v} \quad (4)$$

$$\mathbf{E} = -\frac{1}{c} \mathbf{v} \times \mathbf{B} \quad (5)$$

where U is the internal energy per unit mass, \mathbf{I} is the unit tensor, $\mathbf{g} = (0, 0, -g)$ is the gravitational acceleration, γ is the specific heat ratio and the other symbols have their usual meaning, the medium is assumed to be an ideal gas. We take $\gamma = 1.05$ and $\beta = 0.5$.

2.2 Initial and Boundary Conditions

The initial model is as follows:

$$B_x(x) = B_{x0} = C \quad (6)$$

$$B_z(x) = B_{z\infty} \tanh \left(\frac{B_{z\infty}}{2B_{x0}} \frac{x}{\Lambda} \right) \quad (7)$$

$$p(x) = \frac{B_{z\infty}^2}{8\pi} \cosh \left(\frac{B_{z\infty}}{2B_{x0}} \frac{x}{\Lambda} \right)^{-2} \quad (8)$$

$$\rho(x) = \frac{B_{z\infty}^2}{8\pi} \cosh \left(\frac{B_{z\infty}}{2B_{x0}} \frac{x}{\Lambda} \right)^{-2} \quad (9)$$

where B_{x0} is the value of the horizontal field at $x = 0$, $B_{z\infty}$ is the value of the vertical field as $x \rightarrow \infty$ and Λ is the pressure scale height.

A nonuniform grid was used. the grid size is uniform in the y-direction (120 grid points were used with $dy = 0.0125$), and in the x-z plane we took a grid of 70×400 taking 40×320 were taken over a $1.2\Lambda \times 30\Lambda$ area to resolve the plumes with a total area of $3.5\Lambda \times 85\Lambda$.

2.3 Initial Perturbation and Boundaries

The Kippenhahn-Schlüter model has been shown to be linearly stable to ideal MHD perturbations, therefore the initial perturbation has to be either nonlinear or resistive effects need to be considered. We consider the effect of a low density (high temperature) tube placed inside the prominence. For this initial setting a small velocity perturbation, where the maximum value of the perturbation $v(y) < 0.01C_s$, using a random velocity field.

To reduce computational time, we assume a reflective symmetry boundary at $x = 0$. For the z and y boundaries a periodic boundary is assumed. The free boundary placed at $x = L_x$ was found to be very unstable, so a damping zone was place for $3 \leq x \leq L_x$.

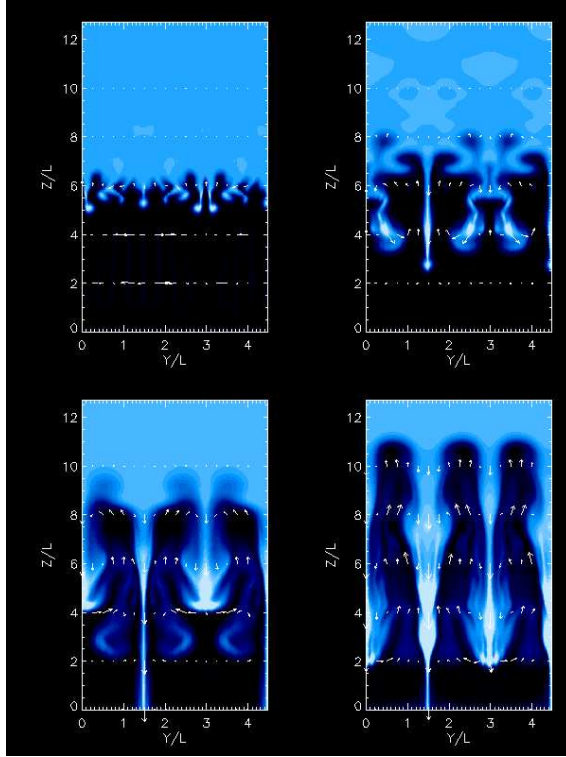


Figure 1: Temporal evolution of upflows for $t = 24.8, 37.0, 49.1$ & 61.1

3 Evolution of the Upflows

Figure (1) shows the evolution for the model. It should be noted that figure (1) shows a width that corresponds to three times L_y . The inverse cascade process and nonlinear mode coupling which creates an upflow of approximately 300km in width can be clearly seen in the figure. The upflow has a rise velocity of 3.3 km s^{-1} , and the two downflows have velocities of 3.75 km s^{-1} and 2.7 km s^{-1} respectively. The main characteristics describing the nonlinear evolution of the interchange instability in the Kippenhahn-Schlüter model found in this simulation are the constant rise velocity and nonlinear mode coupling. We discuss each of these individually below.

3.1 Constant rise velocity

The constant rise velocity is due to a force balance created at the top of the rising plumes. This happens because the magnetic field quickly relaxes so that there is an approximate force balance in the system. An increase in magnetic pressure is balanced by a decrease in gas pressure, whilst the tension balances the gravitational force.

3.2 Inverse Cascade

Our linear analysis shows that the wavelength with the largest growth rate is ~ 100 km, but our simulations show that we form flows of a larger scale. Figure (1) shows that initially the plumes produced are of the order of 50 km in width, which is smaller than can be currently observed. Though an inverse cascade process, we see that three plumes (one rising and two falling) are created from these smaller plumes. Eventually we have created an upflow that is approximately 300 km in width through the inverse cascade process.

4 Summary

The dynamics seen in the nonlinear evolution of the system can be used to shed some light on the dynamics of the rising plumes. We found that for our standard model, using a random perturbation, we obtained a rise velocity that is equivalent to $\sim 3\text{km s}^{-1}$. These velocities found were significantly lower than have been observed. However, these simulations can shed some light onto how the constant rise velocity is obtained. Initially, the bubble is out of equilibrium, but the magnetic field relaxes quickly (1 Alfvén time), this gives a force balance and allows for constant rise velocity of the plumes. Also, the process of inverse cascade allowing the formation of larger plumes from smaller flows. This is very important as it is known through theory that the magnetic Rayleigh-Taylor instability has a growth rate $\omega \propto k^{1/2}$, therefore smaller wavelengths grow faster. This would mean that the plumes would be more likely to form on much smaller scales. The inverse cascade process allows larger plumes to form, matching closer with observations.

From these results, it can be understood that the constant velocity of upflows inside prominences is maintained because the dynamical time of the upflows (15 s) is longer than the Alfvén time (~ 8 s). Therefore the magnetic field is always able to relax to a state where there is a vertical force balance, which easily explains the observed constant rise velocity. The results also give a potential process to drive the formation of large plumes, which you would not expect to form as theory suggests that smaller wavelengths are less stable. The inverse cascade process provides a way to create the observed upflow size.

After my application, I was asked to consider the question "Isn't it necessary to include heat conduction in this kind of simulation?" I believe that to study the global dynamics of a quiescent prominence, a full treatment of the non-LTE radiative transfer process could be a very interesting area, allowing for a self consistent study of the ionisation of a prominence and the effect this has on dynamics. However, the prominence model that I am currently using (the K-S model) which has a uniform temperature. Though a high temperature tube is input into the model, the timescale involved in the simulations (1000s) is short enough that I do not believe a large impact would be seen.