# Faraday Resonance in Dynamically Bar Unstable Stars

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We investigate the nonlinear behaviour of the dynamically unstable rotating star for the bar mode by both three-dimensional hydrodynamics in Newtonian gravity and our simplified mathematical model. We find that an oscillation along the rotation axis is induced throughout the growth of the unstable bar mode, and that its characteristic frequency is twice as that of the bar mode, which oscillates mainly along the equatorial plane. We also find that, by examine several azimuthal modes, mode coupling to even modes, i.e., the bar mode and higher harmonics, significantly enhances the amplitudes of odd modes, unless they are exactly zero initially. Therefore, non-axisymmetric azimuthal modes cannot be neglected at late times in the growth of the unstable bar-mode even when starting from an almost axially symmetric state.

We investigate the nonlinear effects of dynamically bar unstable stars by means of both three dimensional hydrodynamic simulations in Newtonian gravity and our simplified mathematical model.

We find interesting mode coupling in the dynamically unstable system in the nonlinear regime, and that only before the destruction of the bar. The quasi-periodic oscillation mainly along the rotational axis is induced. The characteristic frequency is twice as big as that of the dynamically unstable bar mode. This feature is quite analogous to the Faraday resonance. Although our finding is only supported by the weakly nonlinear theory of fluid mechanics, we have also found the same feature of parametric resonance even in the strongly nonlinear regime.

We also find that our mathematically simplified model provides a concrete example showing the importance of mode coupling. The amplitudes of odd modes increase without unstable odd modes being present in the axially symmetric state; instead, they are enhanced by the bar instability with $m = 2$. We also confirmed that this physical picture is consistent with the results from a three-dimensional hydrodynamics simulation. Generally, the odd modes grow only after the bar instability reaches the nonlinear regime. The timescales of the mode coupling and the growth of unstable modes may depend on the rotation law and the strength of the initial instabilities. It is very rare that the initial perturbations in the hydrodynamics simulation should consist of purely even or odd modes only. Therefore, the unstable bar mode enhances the amplitudes of the all other modes at late times, no matter whether they are even or odd.

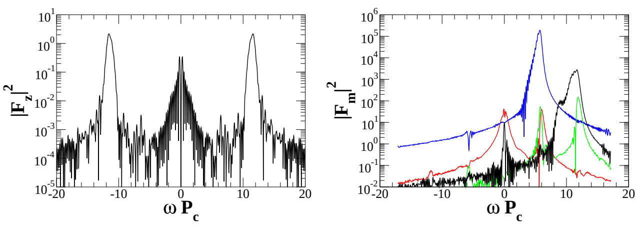


FIG. 1: Spectra $|F_m|^2$ and $|F_z|^2$ as a function of $\omega P_c$ for the differentially rotating star of model III (see Table 1 of Saijo and Kojima [2008] [1]. Red, blue, green, and black line of $|FS_m|^2$ denote the values of $m = 1$, 2, 3, and 4, respectively. Note that the spectra $|F_m|^2$ and $|F_z|^2$ are defined in Saijo and Kojima (2008) [1].

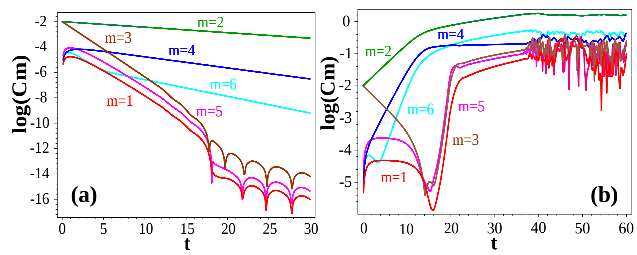


FIG. 2: The time-evolution of the amplitudes of the lowest six Fourier modes ($m = 1, \cdots, 6$). Panel (a) is the result for $\nu = 0.15$ ($m = 2$ stable), (b) for $\nu = 0.05$ ($m = 2$ unstable). The initial coefficients ($a_m$, $b_m$) of the Fourier series expansion of the flow velocity $u(t, x)$ are $a_2 = a_3 = 10^{-2}$ for both cases. The behaviour of very small amplitude, say, $\log(C_m)$ ($C_m = \sqrt{{a_m}^2 + {b_m}^2}$) approximately less than $-10$ in panel (a), mostly comes from numerical truncation errors, and is therefore unimportant.

A similar mode coupling can be seen in numerical simulations for the one-armed spiral instability and the elliptical instability of rotating stars in Newtonian gravity. The initial models and the growth mechanism are different, but the turbulent-like behaviour appears in diagnostics of the azimuthal Fourier components at late times of nonlinear growth. The behaviour is also important for the nonlinear saturation of the unstable mode. Further study is necessary to explore the origin of the similarity seen in the development of different unstable modes. It is reasonable to assume that the nonlinearity in hydrodynamics is the source of this similarity.

A more detailed discussion is presented in Saijo and Kojima (2008) [1] and Kojima and Saijo (2008) [2].

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[1] M. Saijo and Y. Kojima, Phys. Rev. D **77**, 063002 (2008).
[2] Y. Kojima and M. Saijo, Phys. Rev. D **78**, 124001 (2008).