The influence of initial mass segregation on the runaway merging of stars

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ABSTRACT

We have investigated the effect of initial mass segregation on the runaway merging of stars. The evolution of multi-mass, dense star clusters was followed by means of direct N-body simulations of up to 131.072 stars. All clusters started from King models with dimensionless central potentials of $3.0 \leq W_0 \leq 9.0$. Initial mass segregation was realized by varying the minimum mass of a certain fraction of stars whose either (1) distances were closest to the cluster center or (2) total energies were lowest. The second case is more favorable to promote the runaway merging of stars by creating a high-mass core of massive, low-energy stars.

Initial mass segregation could decrease the central relaxation time and thus help the formation of a high-mass core. However, we found that initial mass segregation does not help the runaway stellar merger to happen if the overall mass density profile is kept constant. This is due to the fact that the collision rate of stars is not increased due to initial mass segregation. Our simulations show that initial mass segregation is not sufficient to allow runaway merging of stars to occur in clusters with central densities typical for star clusters in the Milky Way.

Subject headings: stellar dynamics — globular clusters: general — methods: n-body simulations

1. Introduction

The discovery of point-like, ultra-luminous X-ray (ULX) sources with luminosities larger than $L_X > 10^{40}$ ergs s^{-1} by the *Chandra* satellite (Matsumoto et al. 2001; Kaaret et.al. 2001), corresponding to a few hundred $M_\odot$ black holes (BHs) if the sources are not beamed and

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accrete at the Eddington rate, could be a first hint for the existence of so called intermediate-mass black holes (IMBH). IMBHs would bridge the gap between stellar-mass BHs which form as the end-product of normal stellar evolution and the supermassive BHs observed at the centers of galaxies. The connection between IMBHs and ULX is also supported by quasi-periodic oscillations in the X-ray spectrum found in some of the sources (Strohmayer & Mushotzky 2003; Fiorito & Titarchuk 2004).

Several additional arguments have also suggested the presence of IMBHs in globular clusters (see Baumgardt et al. (2005) for a review), such as (1) the extrapolation of the $M_{BH} - M_{bulge}$ relation found for supermassive black holes in galactic nuclei (Magorian et al. 1998) (2) the analysis of the central velocity dispersion in the globular clusters M15 (Gerssen et al. 2002) and G1 (Gebhardt et al. 2005) (3) N-body simulations of runaway merging of stars in young star clusters in M82 (Portegies Zwart et al. 2004).

How IMBHs can form is still an open question. Ebisuzaki et.al. (2001) proposed a scenario in which IMBHs form through successive merging of massive stars in dense star clusters. In a dense enough cluster, mass segregation of massive stars is faster than their stellar evolution and the massive stars sink into the center of the cluster by dynamical friction and form a dense inner core. In the inner core, massive stars undergo a runaway merging process and a very massive star forms with a mass exceeding several 100 solar masses.

A recent study of collisionally merged massive stars by Suzuki et al. (2007) showed that the merger products return to an equilibrium state on a Kelvin-Helmholtz timescale and then evolve like single homogeneous stars with corresponding mass and abundance. The final fate of the very massive stars will depend on the assumed mass loss rate, but IMBH formation is one possible outcome (Belkus et al. 2007).

Direct N-body simulations of star clusters with up to 65536 stars by Portegies Zwart & McMillan (2002) showed that runaway merging can cause the formation of a massive star with up to 0.1% of the total cluster mass before it turns into an IMBH. Formation of very massive stars, as progenitors of IMBHs through runaway collisions in young star clusters has also been studied recently by Freitag et al. (2006a,b) using Monte Carlo simulations. Utilizing a large number of particles ($10^6 - 10^8$ particles), they found that runaway collisions could lead to formation of very massive stars with masses $\geq 400\ M_{\odot}$.

Runaway merging of stars in the star cluster MGG-11 in the starburst galaxy M82, whose position is consistent with a luminous X-ray source, has been intensively examined by Portegies Zwart et al. (2004). They reported that MGG-11 can host an IMBH if its initial dimensionless central potential was high enough. A dimensionless central-potential $W_0 \geq 9.0$ was required for runaway growth through collisions to form an IMBH. Unfortunately, such a high dimensionless central potential leads to a central density $\rho_c \geq 10^6 M_\odot/\mathrm{pc}^3$ which is rarely seen in present-day star clusters, implying that the formation of IMBHs in star clusters is a very rare event.

One possible way which would allow runaway collisions to occur in clusters with lower central density is the assumption of initial mass segregation. Initial mass segregation, which allows massive stars to start their life in the cluster center, might be a way to lower the density requirement for the onset of runaway collisions. The tendency for massive stars to form preferentially near the cluster center is expected as a result of star formation feedback in dense gas clouds (Murray & Lin 1996) and from competitive gas accretion onto protostars and mutual mergers between them (Bonnell & Bate 2002). Observational evidence for initial mass segregation in globular clusters as well as in open clusters has also been reported (Bonnell & Davies 1998; de Grijs et al. 2004).

Dynamical evolution of young dense star clusters with initial mass segregation until the onset of core-collapse stage has been studied by Gürkan et al. (2004) by using Monte Carlo simulations. Besides decreasing the core collapse time, they found that initial mass segregation applied in clusters with $N = 1.25 \times 10^6$ stars which followed a Plummer density profile initially, results in a total mass of the collapsed core of about 0.2 % of the total cluster mass.

Motivated by results of Portegies Zwart et al. (2004), that without initial mass segregation, the dense star cluster MGG-11 could experience runaway merging only if the central density was higher than $10^6 M_\odot/\mathrm{pc}^3$, in the present study we want to explore whether or not initial mass segregation could lower the density required for runaway collisions in MGG-11 like clusters. For this purpose, we perform N-body simulations of MGG-11 like clusters starting from different initial conditions which are described in detail in the next section. Results and analysis of our simulations are shown in section 3 while the discussion and conclusions are presented in section 4.

2. Details of numerical simulations

We have conducted a number of N-body simulations, using the collisional N-body code NBODY4 (Aarseth 1999) on the GRAPE-6 special purpose computers provided by ADC - CfCA NAO Japan, to follow the evolution of multi-mass star clusters. All simulations are run for a time span of 3 Myrs by which time we assume that the runaway stars are turned into BHs and stop the simulations.

Our clusters contain 131.072 stars initially, distributed according to a Salpeter IMF with minimum mass and maximum mass equal to 1.0 $M_\odot$ and 100 $M_\odot$ respectively, which is chosen to fit the McCrady et al. (2003) observations for MGG-11. Stellar evolution is modeled according to Hurley et al. (2000). Since we only follow the first 3 Myrs of cluster evolution, stellar evolution is important only for the most massive stars. Two stars are assumed to 'collide' if the distance between them becomes smaller than the sum of their radii. We assume that the total mass of both stars ends in the merger product and do not follow the stellar evolution of the runaway stars. We examine the evolution of King (1966) models with central concentration $3.0 \leq W_0 \leq 9.0$. The initial half-mass radius and total cluster mass are chosen similar to what Portegies Zwart et al. (2004) chose to fit the observed parameters of MGG-11, namely $r_h = 1.3$ pc and $M = 3.5 \times 10^5 M_\odot$. Details of the simulated clusters without initial mass segregation are presented in Table 1.

In order to examine the effect of initial mass segregation, we study two scenarios. In the first scenario, we vary the minimum mass m_{min} within the lagrangian radius containing 5% of the total cluster mass (R_{005}). Increasing the minimum mass m_{min} within R_{005} (from 1 $M_\odot$ to $m_{min} > 1 M_\odot$ for clusters with initial mass segregation), while keeping the total cluster mass and energy constant, will consequently decrease the number of stars within this sphere. This scenario allows massive stars to start their life in the cluster center. It is proposed to meet observations which show that massive stars are preferentially formed near the cluster center (Bonnell & Davies 1998; de Grijs et al. 2004). Details of runs where mass segregation is introduced inside a certain radius are given in Table 2.

In the second scenario, we choose a certain fraction of stars (whose total mass is 5 % – 20 % of total mass of the cluster) with the lowest total energy and then vary the minimum mass of them, while keeping the total cluster mass and energy constant. The number of stars is again lower than in a normal cluster. Compared to the first scenario, the second scenario brings massive stars even closer to the center since stars located in the center at time t=0 could still have high energies and spent most of their life outside the center. Hence support for runaway collisions should be stronger in the second scenario.

We also vary the half-mass radius of the clusters to see the effect of different central densities. Table 3 reports details for clusters with initial mass segregation, using the second scenario.

3. Results and Analysis

3.1. Clusters without initial mass segregation

We run five cluster models without initial mass segregation as shown in Table 1. Each cluster contains 131.072 stars, but has different W_0. Four of them are set to have the same half-mass radius, which is 1.3 pc, to mimic MGG-11. In addition, we also examine a W_0 = 7.0 cluster with a smaller half-mass radius of r_h = 0.5 pc. The central density of each cluster refers to the density within the core radius of the cluster, which is determined with the method of Casertano & Hut (1985). For clusters with the same r_h, the central density is higher for clusters with higher dimensionless central potential W_0.

We also calculate the central relaxation time of each cluster to study the influence of this parameter on the occurrence of runaway merging. The central relaxation time $T_{rel,c}$ is defined as (Spitzer 1987):

$$T_{\rm rel,c} = \frac{\sigma_{3D}^3}{4.88\pi G^2 \ln(0.11N) n \langle m \rangle^2}, \tag{1}$$

where σ_{3D}, n and $\langle m \rangle$ are the three-dimensional velocity dispersion, number density and average stellar mass at the cluster core. Here the cluster core refers to the region inside the core radius $\rm r_{core}$.

Our simulations of MGG-11 like clusters (with r_h = 1.3 pc) show (see Table 1) that only the star cluster with the highest dimensionless central potential (W_0 = 9.0, corresponding to a central density of 3.24 $\times 10^6$ $M_\odot/\rm pc^3$), experiences runaway merging. This result is in a good agreement with the one found by Portegies Zwart et al. (2004). Our result again proves that high central density is required to allow runaway merging to occur. Collisions among massive stars also occur in the lower density cluster but none of them experiences subsequent collisions leading to a super-massive star.

Fig. 1 depicts the evolution of lagrangian radii containing 1% – 20 % of total mass of cluster models 1 – 3. Core radii ($\rm r_{core}$), which are marked by bold lines, are calculated according to Casertano & Hut (1985). The inner shells of the W_0 = 9.0 cluster (model 1) suffer strong contractions due to the high central density. Core collapse happens in this cluster at $t \approx 0.6$ Myrs. The core collapse supports the runaway merging to happen since runaway merging sets in at $t = 0.54$ Myrs, about the same time when core collapse happens (see the 10th column of Table 1.). Inner shells of W_0 = 7.0 cluster (model 2), on the other hand, contract very slowly. Even until 3 Myrs, the contraction is not strong enough to produce core collapse. Consequently, no runaway merging occurs in this cluster. Evolution of inner shells of W_0 = 7.0 cluster however looks different when we decrease r_h to 0.5 pc (model 3). Mild contraction brings the cluster to collapse. Core collapse occurs at $t \approx 2.6$ Myrs. At the same time, the first collision leading to runaway merging happens ($t = 2.55$ Myrs, see the 10th column of Table 1). Although the runaway merging started later than in the $W_0 = 9.0$ cluster, three collisions are enough to form a super-massive star with few hundred $M_\odot$ (see columns 9 and 11 of Table 1).

The two clusters which experience runaway merging (models 1 and 3), have very high central densities. The $W_0 = 7.0$ model has $\rho_c = 2.95 \times 10^6$ $M_\odot/\mathrm{pc}^3$ while the $W_0 = 9.0$ model has $\rho_c = 3.24 \times 10^6$ $M_\odot/\mathrm{pc}^3$. Runaway merging does not occur in clusters whose central densities are lower than 10^6 $M_\odot/\mathrm{pc}^3$. Therefore the critical density which allows clusters without initial mass segregation to experience runaway stellar merging should be larger than $10^6 M_\odot/\mathrm{pc}^3$. This limit holds for globular-cluster-size objects with masses of $10^5 M_\odot$.

Since in our runs the central density is varied, we find that the central relaxation time (see column 6 of Table 1) mainly depends on the number density of stars in the center, where $T_{rel,c} \propto n^{-1}$ (see eq. 1). The central relaxation time is hardly affected by the change of velocity dispersion σ and average mass $\langle m \rangle$ (on average $\sigma \approx 27.9$ km/s and $\langle m \rangle \approx 2.64$ $M_\odot$.) A high number density of stars in the cluster center seems to be required to support runaway stellar merger in a cluster without initial mass segregation.

Our result, that runaway merging does not occur in clusters with too low central density, is in good agreement with the one found by Freitag et al. (2006a,b). Fig. 1 of Freitag et al. (2006b) (which is essentially the same as Fig. 1 of Freitag et al. (2006a)) shows that a cluster with mass $3 \times 10^5 M_\odot$ and dimensionless central potential $W_0 = 8.0$ experiences runaway collisions if its N-body length unit (R_{NB}) ≤ 2 pc. This value corresponds to an initial half mass radius $R_h \leq 1.74$ pc (see sec. 2.1 of their paper where they show that $R_{NB} \simeq 1.15$ R_h for $W_0 = 8.0$). This value is not too far from the critical value we find, since our simulations show that a $W_0 = 7.0$ cluster without initial mass segregation experiences runaway collisions when its initial half-mass radius is somewhere between 1.3 pc and 0.5 pc (see models 2 and 3 in Table 1), while a $W_0 = 9.0$ cluster with initial half-mass radius 1.3 pc experiences runaway collisions.

3.2. Clusters with initial mass segregation

In models 6 – 8, we introduce initial mass segregation by replacing stars within the 5 % lagrangian radius (R_{005}) with massive stars whose masses are higher or equal than the mass m_{min} written in the 6th column of Table 2. Replacing is done by randomly selecting new positions and velocities for the massive stars from the positions and velocities of innermost stars. The number of massive stars is chosen such that the overall mass density profile remains constant.

As we keep the mass within the R_{005} lagrangian radius constant, introducing initial mass segregation by increasing m_{min} means to increase the average mass $\langle m \rangle$ of stars and lower the total number of stars (see the 3rd column of Table 2). The increase of $\langle m \rangle$ in this region consequently decreases the central relaxation time $T_{rel,c}$. The central relaxation time of these clusters should be lower than the one of a $W_0 = 7.0$ cluster without initial mass segregation (see column 6 of model 2 in Table 1). As the central parts of these clusters relax faster, the clusters may evolve faster and core collapse could happen earlier. One may therefore expect that runaway merging should now occur at lower central densities.

The top part of Fig. 2 shows that model 6 (with $m_{min} = 30\ M_\odot$) does not experience core collapse before 3 Myrs. Even increasing m_{min} up to 90 $M_\odot$, as in model 8, does not lead the cluster to experience core collapse before 3 Myrs either (see bottom part of Fig. 2). Our simulations also show that no runaway merger occurs in these clusters. The reason why runaway merging does not happen is that massive stars, which start their life in the region within R_{005} do not constantly stay there. Some of these massive stars, whose initial velocities are high enough, leave this region. Since the cluster is initially mass segregated, this outward movement of massive stars is not balanced by a sufficiently large number of massive stars moving inward, hence the average mass of stars decreases in the center. We note however that, since our clusters are started in virial equilibrium, the expansion of the high-mass stars is balanced by a corresponding number of low-mass stars moving further in, so that the central density remains constant.

The depletion of massive stars from the initial R_{005} is shown for model 6 in Fig. 3. This figure depicts the evolution of lagrangian radii of massive stars whose masses are higher than 30 $M_\odot$ and that started their life inside R_{005}. Total mass fraction of these massive stars is indicated by M_{005}. The lagrangian radii containing between 10 % up to 100 % of these stars are presented. The upper figure shows the change of lagrangian radii within the first 0.05 Myrs. We can see that within a few core crossing times ($t_{cross} \approx 8 \times 10^3$yrs) some of these massive stars leave the initial R_{005}. At t=0.05 Myrs, total mass of massive stars which still reside inside this region is only 60 % of the initial mass M_{005}. Bottom figure shows that up to t=3 Myrs, this region contains only about 30 % of total mass of these massive stars.

Increasing the minimum mass of stars whose distances are closest to the cluster center does not succeed to produce high-mass cores. In order to keep massive stars in the cluster core, we used a second scenario where initial mass segregation is realized by varying the minimum mass of a certain fraction of stars whose total energies are lowest. Since the second scenario is more favorable to create a high-mass core of massive, low-energy stars, we will base our results on this scenario.

In the second scenario, initial mass segregation was introduced by replacing stars which have the lowest total energy, up to 5 % – 20 % of the total mass of the cluster (models 9 – 15, see M_{IMS} in column 5 of Table 3) with massive stars whose masses are higher than m_{min}. The coordinates and velocities of massive stars are randomly chosen from the stars with the lowest total energy and their total number is again adjusted such to keep the overall mass density profile constant and the cluster in virial equilibrium.

In order to show that clusters are in virial equilibrium, Figs. 4 and 5 depict the evolution of lagrangian radii of all stars and those of massive (M $\geq 30\ M_{\odot}$) and less massive stars (M $< 30\ M_{\odot}$) of cluster model 11. As can be seen lagrangian radii of massive as well as less massive stars are nearly constant within the first few crossing times. This shows that the cluster is in a stable equilibrium condition after mass segregation was introduced.

The central density and central relaxation time are measured for the region inside the cluster core. Since massive stars are not strongly concentrated toward the cluster center, the mean mass of stars within the cluster core is not very high (4.77 $M_{\odot}$ – 11.82 $M_{\odot}$). Therefore the central relaxation time of clusters with r_h = 1.3 pc (6.55 Myrs $\leq T_{rel,c} \leq$ 8.69 Myrs) is not as low as that of the clusters in Table 2 (3.54 Myrs $\leq T_{rel,c} \leq$ 3.84 Myrs). One may expect that the central relaxation time should be short enough that massive, low-energy stars spiral into the cluster core and create a high-mass core. Once in the cluster core, these massive stars could collide with each other and promote runaway merging.

Nevertheless, our simulations do not show runaway merging (see models 9 – 12 of Table 3). Reducing the half-mass radius r_h from 1.3 pc to 0.5 pc in order to increase the central density (models 13 – 15, see column 4 of Table 3) does not help runaway merger to occur either.

Model 15 actually has the same initial central density and half-mass radius as model 3. Initial mass segregation is not introduced in model 3, but the cluster experiences runaway merging through three collisions (see Table 1). Fig. 6 depicts the evolution of lagrangian radii of inner shells of these two models. Both clusters experience contractions of their cores. While the contraction of model 3 is sufficiently strong to let core-collapse occur at $t = 2.6$ Myrs, the core of model 15 does not collapse until 3 Myrs and no runaway merging occurs.

By using the Monte Carlo method, Gürkan et al. (2004) studied core-collapse of star clusters with initial mass segregation. A direct comparison of their results with ours is again difficult due to differences in the adopted initial mass spectrum, density profile, number of particles (up to $N = 10^7$ for Gürkan et al. (2004)) and the method used in introducing initial mass segregation. Gürkan et al. (2004) note in the caption of Fig. 13 that stellar evolution can reverse core collapse. This agrees at least qualitatively with what we see in our runs, since for example Fig. 6 shows that, despite of similar size and density profile, model 3 goes into core collapse earlier than model 15. This could be due to the fact that core collapse in model 15, whose core contains many high-mass stars due to initial mass segregration, is delayed by the stronger mass loss from the core due to stellar evolution.

Besides the effect of stellar evolution, the difference of the evolution of models 3 and 15 may due to the difference of their collision rates. We examine the collision rate of these models by calculating the the collision rate N_{Coll} using equation (8-122) of Binney & Tremaine (1987) .

$$N_{Coll\star} = 4\sqrt{\pi} n \sigma (2\ R_\star)^2 + 4\sqrt{\pi} G M_\star n (2\ R_\star)/\sigma. \tag{2}$$

Here $N_{Coll\star}$ is the average number of collisions that a star suffers per unit time, n indicates the number density of stars, σ is the velocity dispersion of stars, $R_\star$ and $M_\star$ denote radius and mass of colliding stars, and G is the gravitational constant. The first term is derived from the kinetic theory for inelastic encounters and the second term represents the enhancement in the collision rate by the gravitational attraction of the two colliding stars.

Let us consider the region inside the core radius $\mathrm{r_{core}}$. The average number of collisions per unit time N_{Coll} is obtained by multiplying $N_{Coll\star}$ with the number of stars inside the core radius $N_{\star core}$. Therefore

$$N_{Coll} = N_{Coll\star}\ N_{\star core}. \tag{3}$$

The number density of stars inside the core radius n can be written as

$$n = N_{\star core}/V_{core}. \tag{4}$$

where $V_{core} = \frac{4\pi}{3}\mathrm{r}^3_{\mathrm{core}}$. Thus

$$N_{Coll} = N_{Coll\star}\ n\ V_{core}. \tag{5}$$

Substituting $N_{Coll\star}$ with the expression written in eq. 2, we see that

$$N_{Coll} \propto n^2. \tag{6}$$

We use the theoretical prediction of the collision rate (eqs. 2 and 5) to follow the growth in the number of collisions per unit time in models 3 and 15. $N_{Coll\star}$ is calculated by considering the mass and radius of each star and then summing up over all stars within the region inside the core to obtain N_{Coll}. Core parameters and collision rates are calculated each time N-body data was stored and are then summed up over all times.

The theoretical estimates are compared with the collision rate we find in our simulations in Fig. 7. Both theoretical and simulation results (see column 7 of Table 1 and column 9 of Table 3) show that the collision rate of model 3, which experiences runaway merging, is higher than the one in model 15. The theoretical prediction of the collision rate overestimates the simulation results by a factor ≈ 2. This may be due to assumptions (i.e. mass and radius of colliding stars are the same) and idealizations (i.e. distribution function of velocity is Maxwellian) used in the derivation of eq. 2, while in the simulations we use a mass spectrum and stellar radii according to a certain mass-radius relation.

4. Discussion and Conclusions

We have followed the evolution of multi-mass, dense star clusters with dimensionless central potentials of $3.0 \leq W_0 \leq 9.0$. Our simulations show a good agreement with the results of Portegies Zwart et al. (2004) that in MGG-11 type clusters without initial mass segregation, dimensionless central potentials $W_0 \geq 9.0$ corresponding to central number densities larger than $10^6/\mathrm{pc}^3$ are required for runaway mergers to occur. Examining clusters with lower dimensionless central potential, $W_0 \leq 7.0$, confirm this limit for runaway mergering, as shown in Fig. 8.

Initial mass segregation increases the average mass of stars within the cluster center and thus decreases the central relaxation time. It also allows to form a high-mass core. However, as long as the mass density profile is kept constant, we find that initial mass segregation does not support runaway stellar merging to happen since the collision rate is decreased.

In spite of the differences in adopted IMF, number of particles, treatment of stellar evolution and stellar collisions, our results are in line with Freitag et al. (2006a,b) (see sec. 3.1) and Gürkan et al. (2004) (see sec. 3.2).

The data of Milky Way globular cluster given by Harris (1996) provide the central luminosity density (in $L_{\odot}/pc^3$) which can be converted into a central mass density by assuming a mass-to-light ratio $M/L = 1$. Doing this, we find that about 67 % of Milky Way globular clusters have central densities $10^2\ M_{\odot}/\mathrm{pc}^3 < \rho_c < 10^5\ \mathrm{M}_{\odot}/\mathrm{pc}^3$. Only 4 % have central densities exceeding $\rho_c = 4.3 \times 10^5\ M_{\odot}/\mathrm{pc}^3$, while none has a central density larger than $\rho_c = 10^6\ M_{\odot}/\mathrm{pc}^3$, as depicted on Fig. 9. Studies of the evolution of clusters containing IMBHs by Baumgardt et al. (2004a,b) have shown that clusters with IMBHs expand due to energy generation in their cusp. Baumgardt et al. (2004b) found that the cluster expansion can be strong enough that very concentrated clusters can end up among the least dense clusters. However, Milky Way globular clusters have half-mass radii very similar to the radii of clusters which form today, like galactic open clusters or super-star clusters in interacting galaxies (see i.e. Schilbach et al. (2006); Trancho et al. (2007)), which speaks against strong expansion. If the current densities are representative of the densities with which the clusters formed, then runaway merging would not have happened in any of these clusters. In addition, the data of young star clusters in the LMC given by Mackey & Gilmore (2003) (see Table 6 of their paper) also shows that nearly all LMC clusters, including very young ones, have central densities far below the critical value needed for runaway merging. Other possibilities of forming IMBHs like the merging of many stellar mass black holes (Miller & Hamilton 2002) also need extreme initial conditions like very massive clusters (Gültekin et al. 2004; Rasio et al. 2006).

Hence it seems likely that most star clusters did not have sufficient high central densities to form IMBHs. This indicates that the formation of IMBHs in star clusters must have been a rare event.

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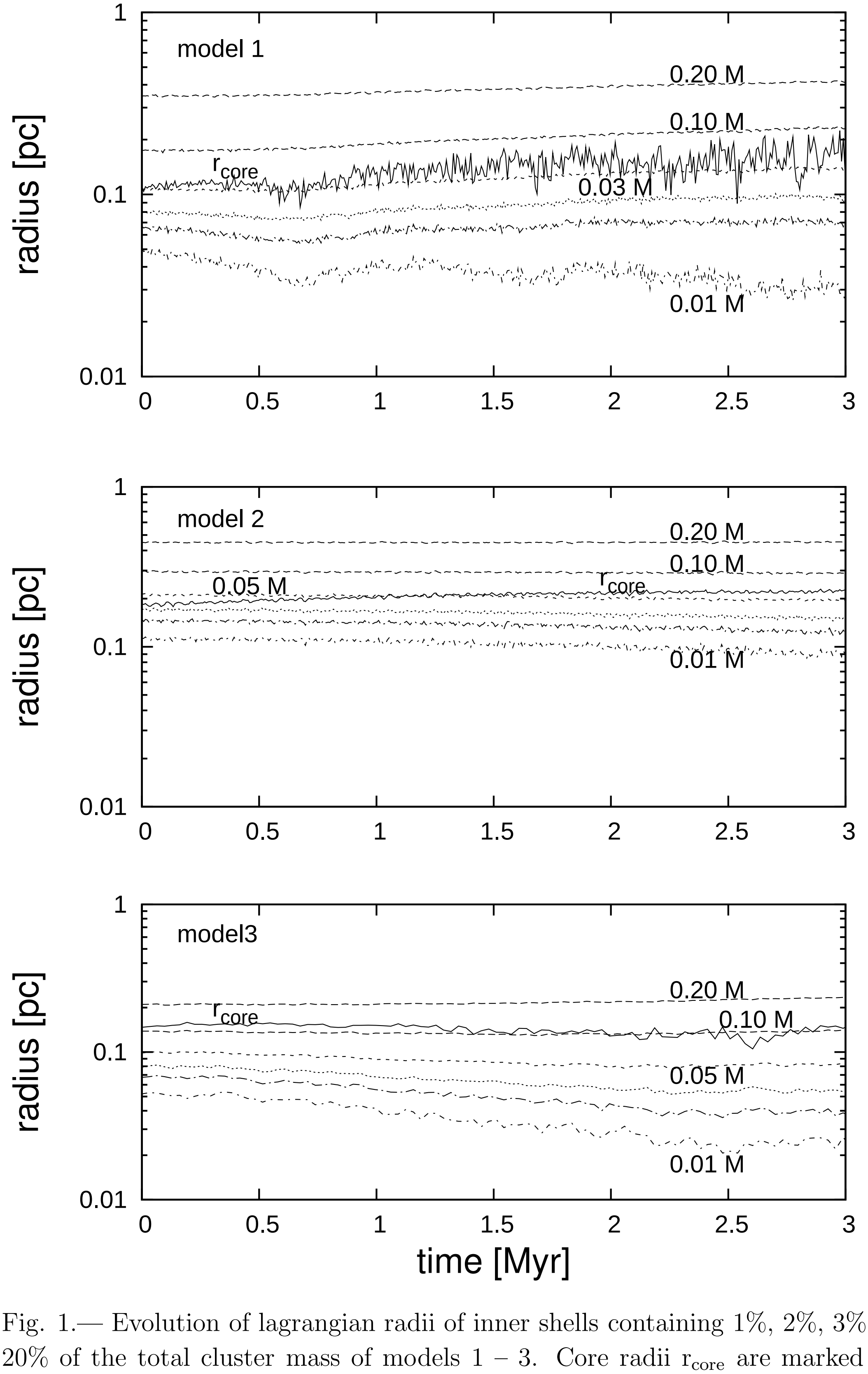

Fig. 1.— Evolution of lagrangian radii of inner shells containing 1%, 2%, 3%, 5%, 10% and 20% of the total cluster mass of models 1 – 3. Core radii r_{core} are marked by bold lines. Model 1 has short enough central relaxation time that core collapse and subsequent runaway merging of stars happen within a few Myrs. Model 2 has a higher $T_{rel,c}$ (see column 6 of Table 1) which prevents its core to collapse. Compared to model 1, model 3 has similar value of ρ_c which allows mild contractions to bring the core to collapse before 3 Myrs.

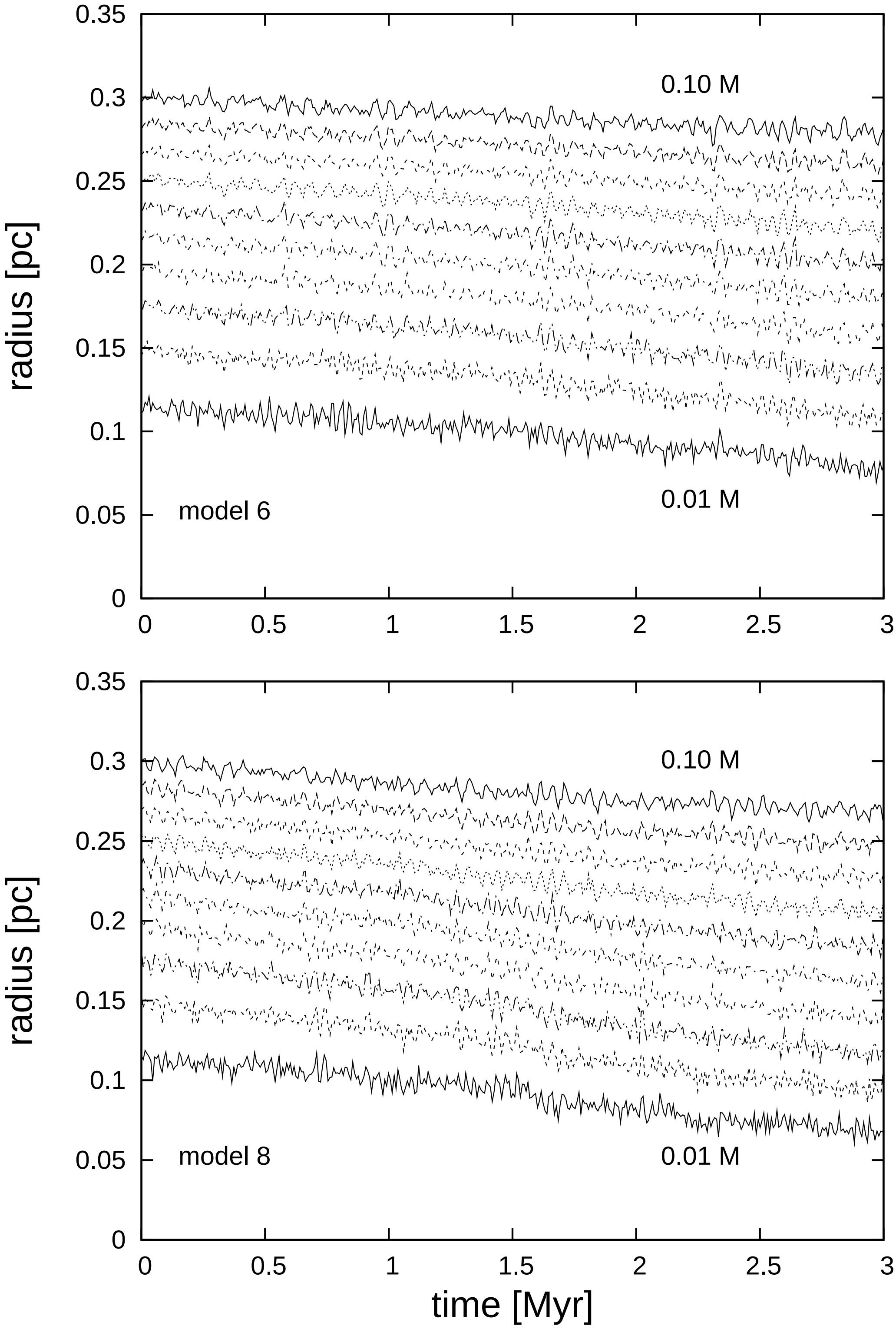

Fig. 2.— Evolution of lagrangian radii of inner shells containing 1% – 10% of the total cluster mass of model 6 (top) and model 8 (bottom). Filling the region inside the 5 % lagrangian radii R_{005} with stars more massive than 30 $M_{\odot}$ (model 6) does not support the core to collapse. Even increasing the minimum mass of stars in this region to 90 $M_{\odot}$ (model 8) does not help core collapse to happen. The reason is that a large fraction of massive stars, which start their life inside the R_{005}, move out of this region on a crossing time-scale (see Fig. 3).

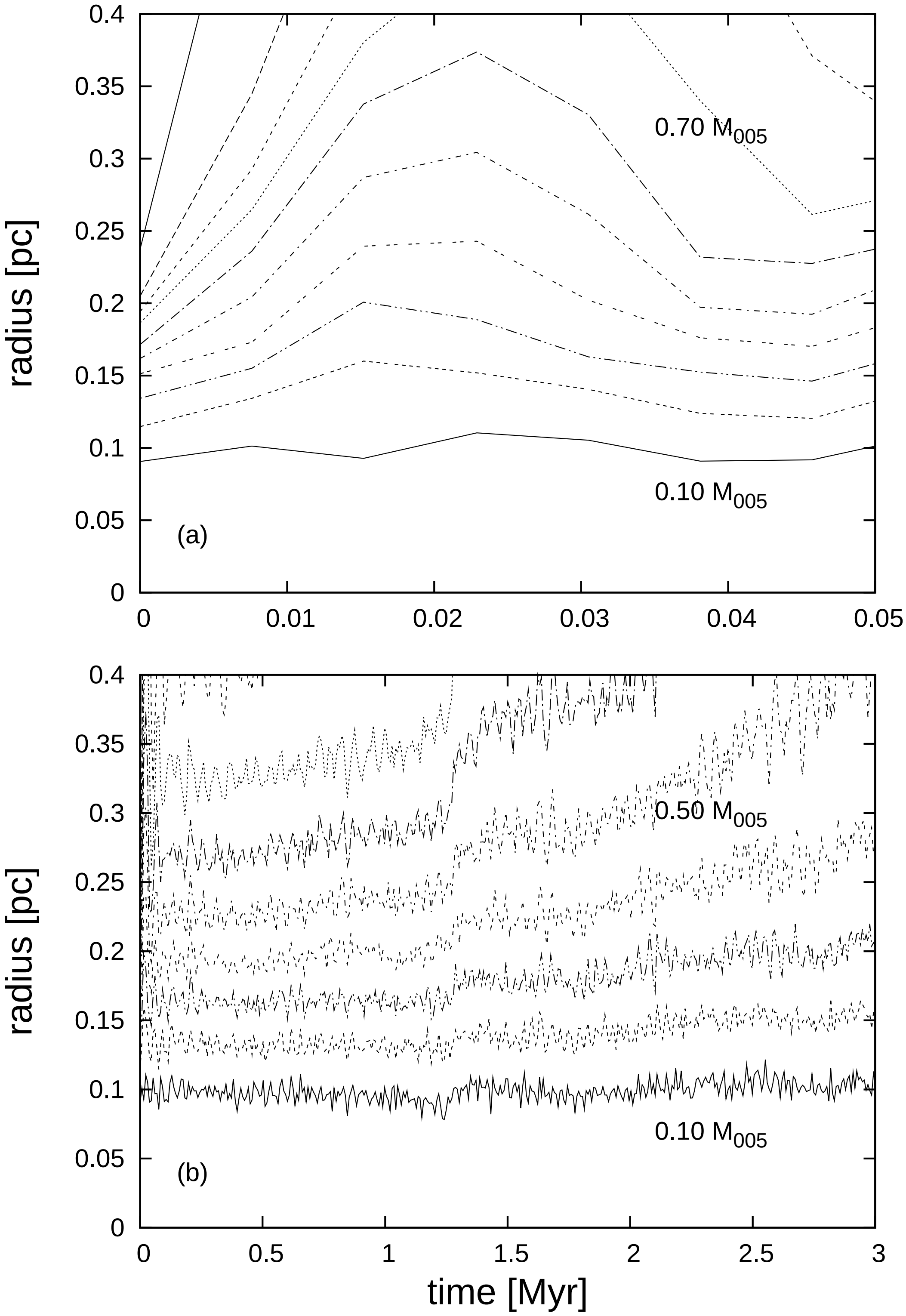

Fig. 3.— Evolution of lagrangian radii of massive stars ($m_{min} = 30M_{\odot}$) which start their life inside the 5 % lagrangian radius of the cluster model 6 (a) up to the first 0.05 Myrs, (b) until 3 Myrs. Total mass fraction of these massive stars is indicated by M_{005}. Within a crossing time-scale, some massive stars leave the region within the initial 5% lagrangian radius, which is 0.24 pc in this model, due to their high initial velocities. The escape of the massive stars is balanced by low-mass stars moving in from larger radii (which is not shown in these figures).

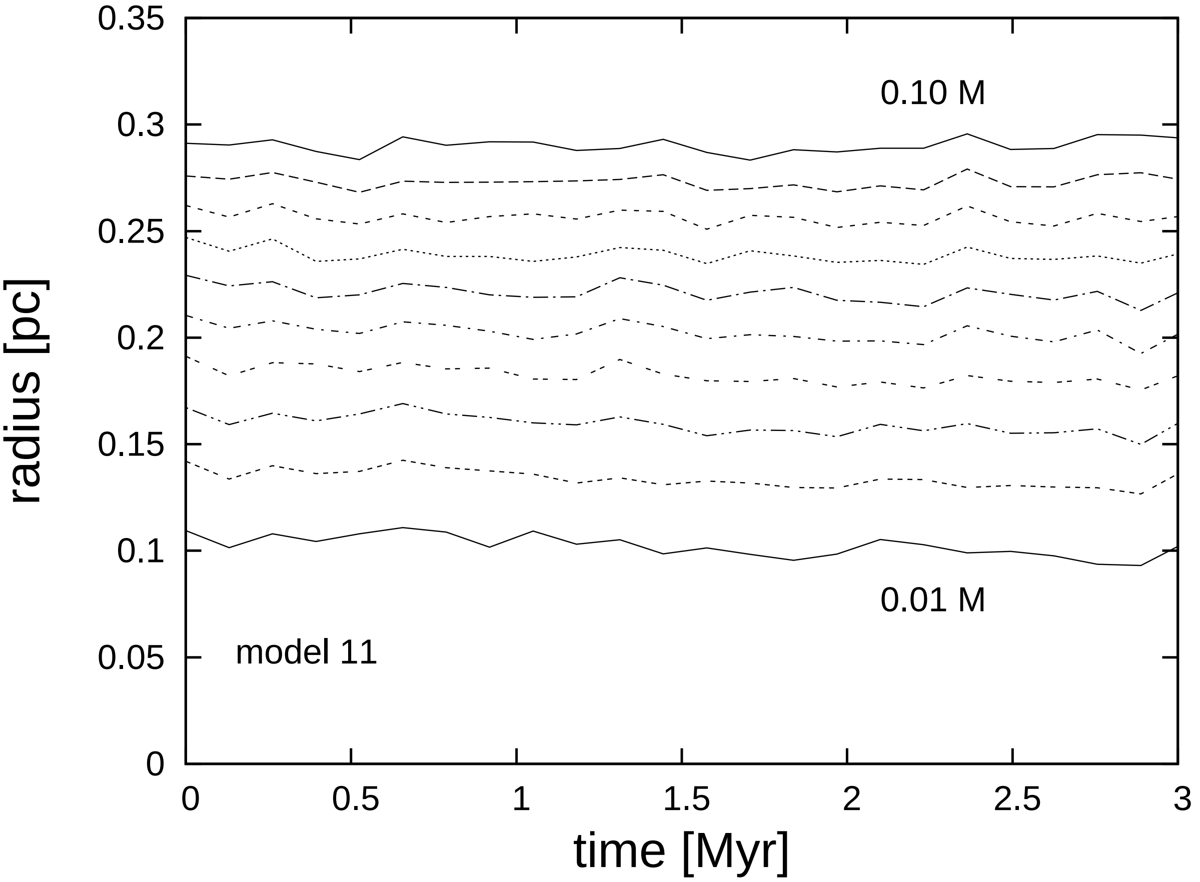

Fig. 4.— Evolution of lagrangian radii of inner shells containing 1% – 10% of the total cluster mass of model 11. Replacing 10% of the lowest total energy stars with stars more massive than 30 $M_{\odot}$ does not support the core to collapse. This happens because the high-mass core did not be form until 3 Myrs.

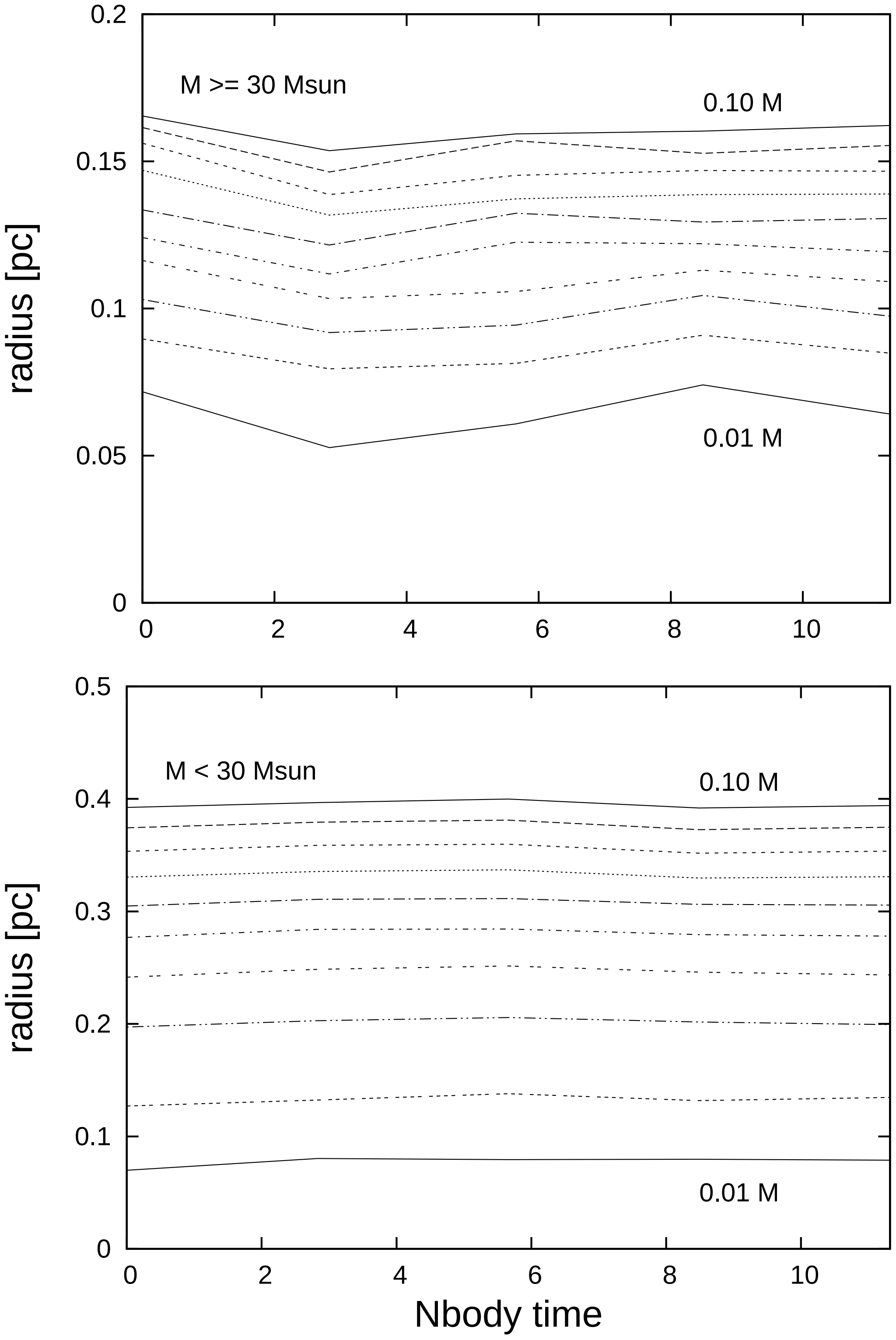

Fig. 5.— Evolution of lagrangian radii of massive stars whose masses are at least 30 $M_{\odot}$ (top) and low-mass stars whose masses are less than 30 $M_{\odot}$ (bottom) up to 10 Nbody unit. These figures depict stable evolution of shells containing 1% – 10% of the total mass of these stars in the cluster model 11.

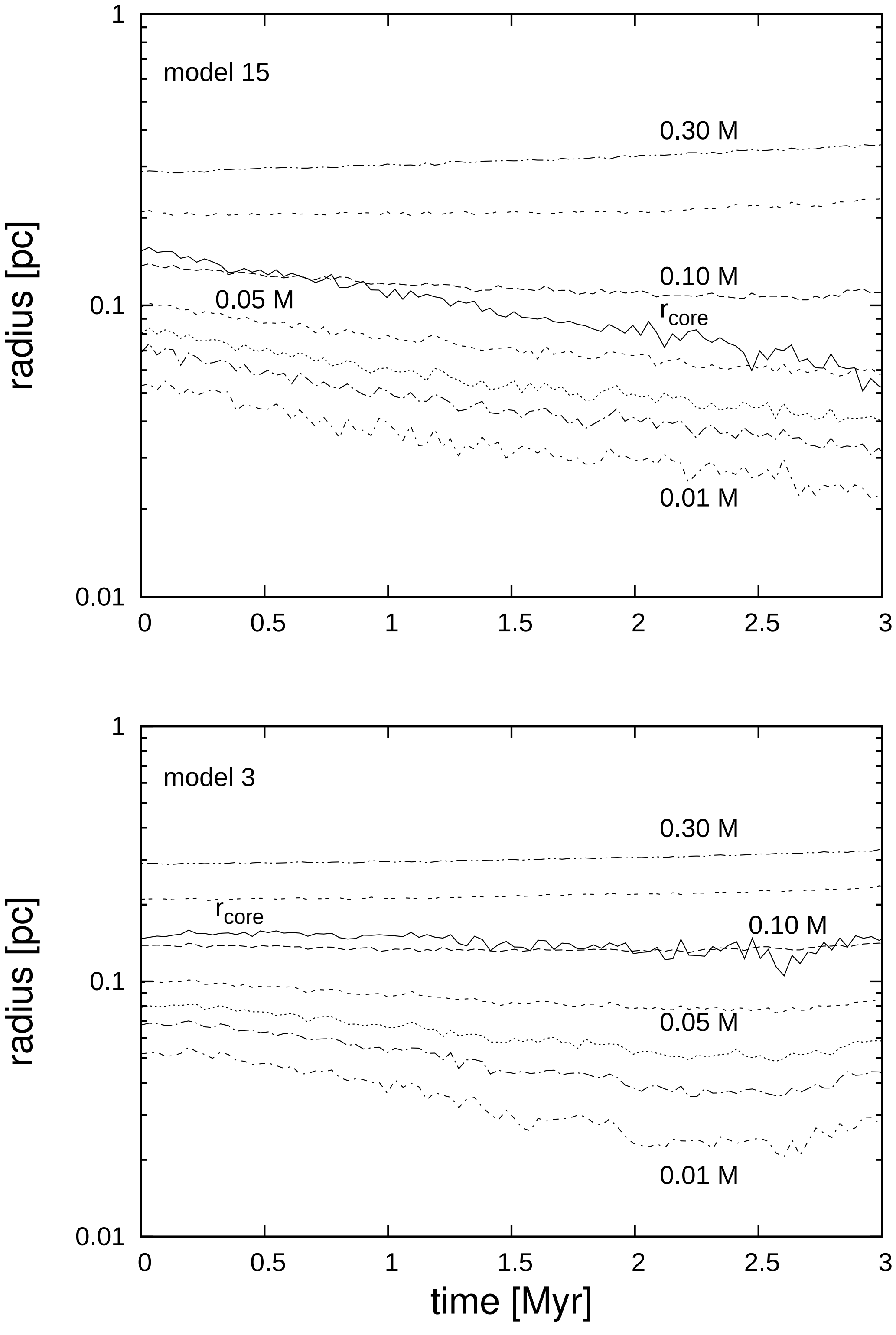

Fig. 6.— Evolution of lagrangian radii containing 1% – 30% of total mass of model 15 and model 3. Initial mass segregation is applied in model 15 by replacing 20 % of stars with the lowest total energy by massive stars with $m_{min} = 30\ M_{\odot}$. The inner shells experience contraction but no core collapse until 3 Myr. Therefore runaway merger does not occur in this cluster. The cluster model 3 has same initial density profile and same half-mass radius as model 15, but no initial mass segregation. However, mild contraction in the inner shells of model 3 is enough to let runaway mergings occur. This may happen since the number of collisions inside the inner shells of model 3 is higher than the one in model 15 (see Fig. 7).

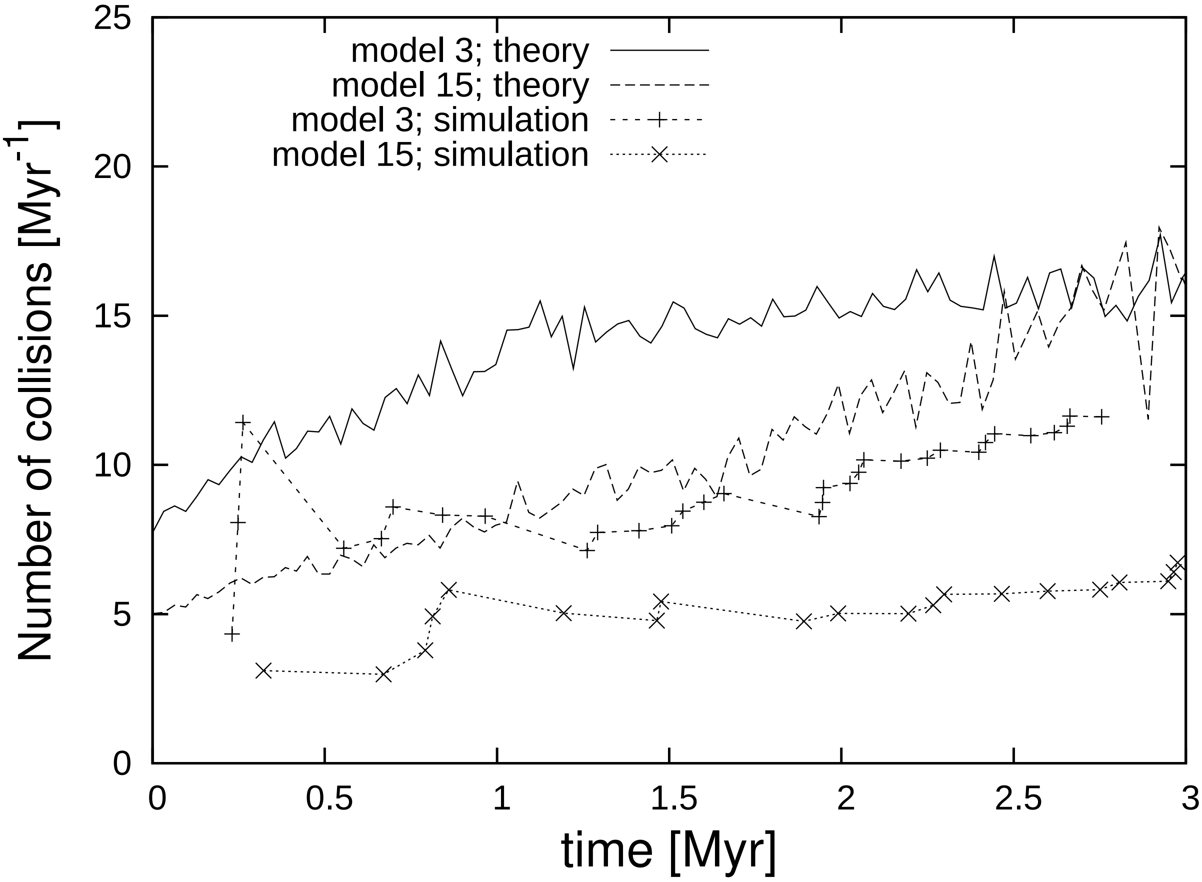

Fig. 7.— Collision rate inside inner shells of cluster models 3 and 15 obtained from simulations, compared to the theoretical prediction of collision rate based on inelastic encounters. The collision rate of the model without mass segregation (model 3) is higher than the model with initial mass segregation (model 15) because there are more stars inside the cluster core. Therefore the possibility for a runaway merger to occur is also higher.

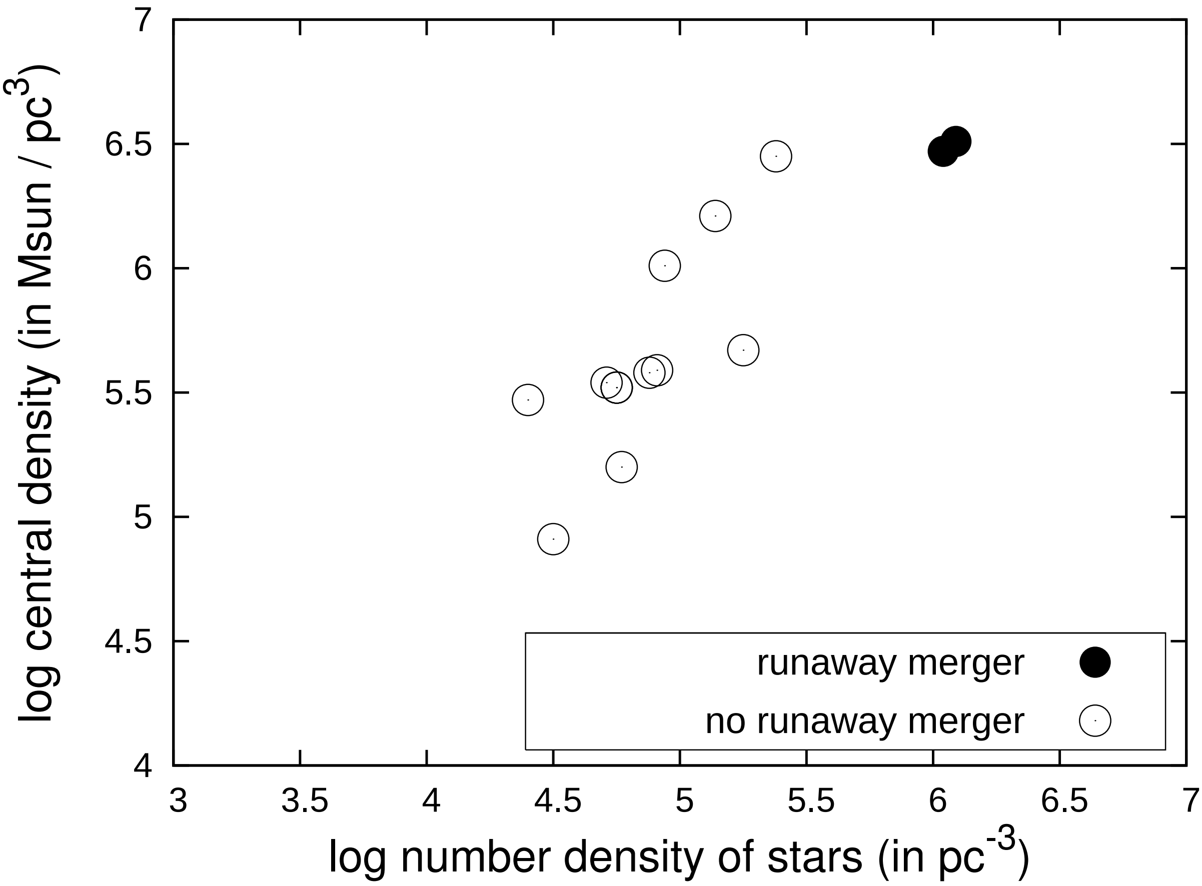

Fig. 8.— Plot of log central density vs. the log number density of stars for all calculated models. In order for runaway mergers to occur, a number density of stars larger than $10^6/\mathrm{pc}^3$ in the core is necessary.

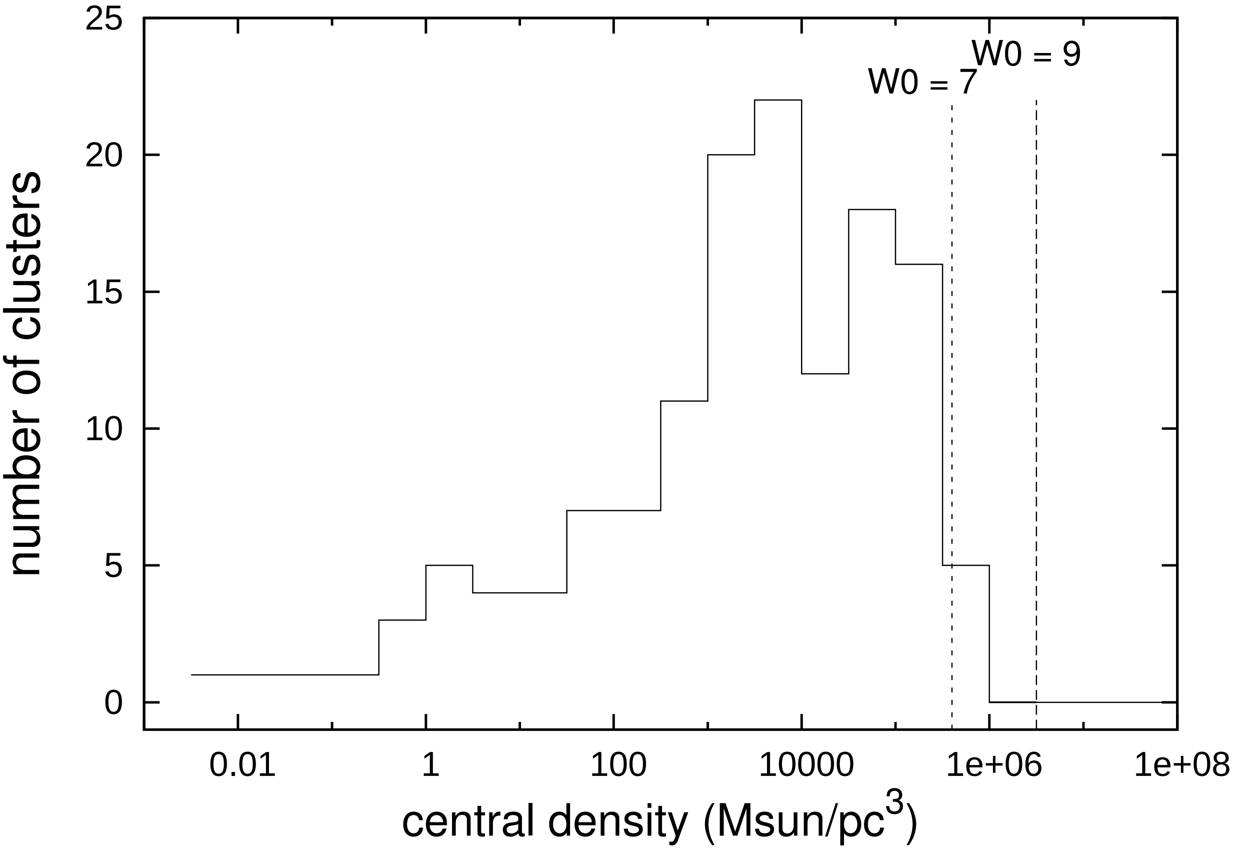

Fig. 9.— Distribution of central densities of Milky Way globular clusters. The dashed lines mark the central densities of clusters in which runaway merging occured (models 1 and 3 of Table 1). Most galactic globular clusters have central densities far below this limit, meaning that runaway merging of stars was unlikely to have occured in them.

Table 1: Properties of simulated clusters without initial mass segregation

1	2	3	4	5	6	7	8	9	10	11	12
$Model$	W_0	N_{Star}	r_h	$log\ \rho_c$	$T_{\rm rel,c}$	Col	$\langle T_{\rm col}\rangle$	$Col_{\rm rm}$	$T_{\rm rm}$	M_{RS}	RM
			(pc)	$(M_\odot/pc^3)$	(Myr)		(Myr)		(Myr)	$(M_\odot)$	(Y/N)
1	9.0	131072	1.3	6.51	1.16	104	0.03	96	0.54	2786	Yes
2	7.0	131072	1.3	5.67	5.98	5	0.60	-	-	-	No
3	7.0	131072	0.5	6.47	2.71	37	0.08	3	2.55	258	Yes
4	5.0	131072	1.3	5.20	18.36	-	-	-	-	-	No
5	3.0	131072	1.3	4.91	39.75	-	-	-	-	-	No

Note. — 1: The first column indicates the cluster model, followed by the dimensionless central potential W_0 in the 2nd column. The number of stars in the cluster and the half-mass radius are given in the 3rd and 4th columns, respectively. The 5th column shows the logarithm of central density followed by the logarithm of the central relaxation time. The 7th column gives the total number of collisions that occur up to 3 Myrs, followed by the average time between collisions. The 9th and the 10th columns indicate the number of collisions leading to runaway mergers and the time when runaway merging starts. The mass of the runaway star produced at the end of the runaway merging process is given in the 11th column. The last column shows whether runaway merging happens or not.

Table 2: Properties of clusters with initial mass segregation introduced within a certain radius

1 $Model$	2 W_0	3 N_{Star}	4 r_h (pc)	5 M_{IMS} $(r \leq R_{005})$	6 m_{min} $(M_\odot)$	7 $log\ \rho_c$ $(M_\odot/pc^3)$	8 $T_{\rm rel,c}$ (Myr)	9 Col	10 $\langle T_{\rm col} \rangle$ (Myr)	11 $Col_{\rm rm}$	12 RM (Y/N)
6	7.0	124420	1.3	0.05	30.0	5.52	3.54	3	1.00	-	No
7	7.0	124305	1.3	0.05	50.0	5.52	3.78	7	0.43	-	No
8	7.0	124201	1.3	0.05	90.0	5.52	3.84	2	1.50	-	No

Note. — 2: The first and second columns indicate the cluster model and the dimensionless central potential W_0. The 3rd column shows the number of stars in the cluster followed by the half-mass radius in the 4th column. The 5th column gives the fraction of total mass of cluster (which is contained within the 5 % lagrangian radius) where the first scenario of IMS is applied. We choose some of these stars randomly and assign them with new masses which are larger than the minimum mass indicated in the 6th column. The logarithm of central density and the logarithm of the central relaxation time are given in the 7th and 8th columns. The 9th and 10th columns indicate the total number of collisions that occur up to 3 Myrs and the average time between collisions. The 11th columns gives the number of collisions leading to runaway mergers. Here we see that none of these collisions leads to a runaway merger process. The last column shows whether runaway merging happens or not.

Table 3: Properties of simulated clusters with initial mass segregation introduced below a certain energy

1	2	3	4	5	6	7	8	9	10	11	12
$Model$	W_0	N_{Star}	r_h (pc)	M_{IMS} (lowest$\mathrm{E_{tot}}$)	m_{min} $(M_\odot)$	$log\ \rho_c$ $(M_\odot/pc^3)$	$T_{\mathrm{rel,c}}$ (Myr)	Col	$\langle T_{\mathrm{col}}\rangle$ (Myr)	Col_{rm}	RM (Y/N)
9	7.0	124420	1.3	0.05	30.0	5.59	8.24	2	1.50	-	No
10	7.0	124297	1.3	0.05	50.0	5.58	8.69	6	0.50	-	No
11	7.0	118805	1.3	0.10	30.0	5.54	8.07	1	3.00	-	No
12	7.0	106669	1.3	0.20	30.0	5.47	6.55	2	1.50	-	No
13	7.0	106669	0.7	0.20	30.0	6.01	3.25	12	0.25	-	No
14	7.0	106669	0.6	0.20	30.0	6.21	2.57	8	0.38	-	No
15	7.0	106669	0.5	0.20	30.0	6.45	1.96	20	0.15	-	No

Note. — Same as Table 2 except that the 5th column indicates the fraction of stars with lowest total energy which were replaced by massive stars. The minimum masses m_{min} of these stars are given in column 6.