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(和文)	高密度星団中での中質量ブラックホール形成に対する質量分離の効果
(英文)	The Effect of Mass Segregation on the Formation of Intermediate Massive Black Holes in Dense Star Clusters

研究分担者

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(1) このプロジェクト（同様の過去のプロジェクトも含む）での成果

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今年度中に出版された論文、国際会議集録、国際会議、学会、研究会発表、その他出版物（印刷中、投稿中の場合はその旨を記載すること）

※ 評価資料として利用いたしますので、様式・順序は任意ですが、学術論文については題名、著者、発行年月、雑誌名、巻、ページが記載されていること。

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項目の説明の文章などは消去して報告内容を記述しても構いません。

成果の概要

(必要に応じてページを加えて下さい。)

Written in separate papers :

- Ardi, E., Baumgardt, H., Mineshige, S. The influence of initial mass segregation on the runaway merging of stars (2007)

The influence of initial mass segregation on the runaway merging of stars

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ABSTRACT

We have investigated the effect of initial mass segregation on the runaway merging of stars. The evolution of multi-mass, dense star clusters was followed by means of direct N-body simulations of up to 131.072 stars. All clusters started from King models with dimensionless central potentials of $3.0 \leq W_0 \leq 9.0$. Initial mass segregation was realized in two ways : (1) by varying the minimum mass of a certain fraction of stars whose distances were closest to the cluster center or (2) whose total energies were lowest. We found that initial mass segregation decreases the central relaxation time and thus supports the formation of a high-mass core. However, unless the number density of stars in the center is high enough, initial mass segregation does not help the runaway stellar merger to happen. This is due to the fact that the collision rate of stars is not increased due to initial mass segregation. Our simulations show that initial mass segregation is not sufficient to allow runaway merging of stars in clusters with central densities typical for star clusters in the Milky Way.

Subject headings: stellar dynamics — globular clusters: general — methods: n-body simulations

1. Introduction

The discovery of point-like, ultra-luminous X-ray sources with luminosities larger than $L_X > 10^{40}$ ergs s^{-1} by the *Chandra* satellite, corresponding to a few hundred $M_\odot$ black holes (BHs) if the sources are not beamed and accrete at the Eddington rate (Matsumoto et al. 2001; Kaaret et al. 2001), could be a first hint for the existence of so called intermediate-mass black holes (IMBH). IMBHs would bridge the gap between stellar-mass BHs which form as the end-product of normal stellar evolution and the supermassive BHs observed at the centers of galaxies. Detection of IMBHs has also been reported for a number of globular clusters like M15 and G1 (Gerssen et al. 2003; Gebhardt et al. 2005), although numerical models of Baumgardt et al. (2003a,b) have shown that the observational data for these clusters can be explained without a central BH.

How IMBHs can form is still an open question. Ebisuzaki et al. (2001) proposed a scenario in which IMBHs form through successive merging of massive stars in dense star clusters. In a dense enough cluster, mass segregation of massive stars is faster than their stellar evolution. The massive stars sink into the center of the cluster by dynamical friction and form a dense inner core. In the inner core, massive stars undergo a runaway merging process and a very massive star forms with a mass exceeding several 100 solar masses. This massive star eventually collapses into a BH, which continues to grow by tidally disrupting passing stars.

Direct N-body simulations of star clusters with up to 65536 stars by Portegies Zwart & McMillan (2002) showed that runaway merging can cause the formation of a star with up to 0.1% of the total cluster mass before it turns into an IMBH.

Furthermore, Portegies Zwart et al. (2004) found that the star cluster MGG-11 in the starburst galaxy M82, whose position coincides with a ULX, can form an IMBH if its initial central concentration was high enough. An initial concentration $W_0 \geq 9.0$ was required for runaway growth through collisions to form an IMBH. Unfortunately, such a high concentration leads to a central density $\rho_c \geq 10^6 M_\odot/\mathrm{pc}^3$ which is rarely seen in present-day star clusters, implying that the formation of IMBHs in star clusters is a very rare event.

One possible way which would allow runaway collisions to occur in clusters with lower central density is the assumption of initial mass segregation. The tendency for massive stars to form preferentially near the cluster center is expected as a result of star formation feedback in dense gas clouds (Murray & Lin 1996) and from competitive gas accretion onto protostars and mutual mergers between them (Bonnell & Bate 2002). Observational evidence for initial mass segregation in globular clusters as well as in open clusters has also been reported (Bonnell & Davies 1998; de Grijs et al. 2004).

In the present study we want to explore whether or not initial mass segregation could lower the density required for runaway collisions. We also want to examine the conditions under which initial mass segregation does not help to form IMBHs. For this purpose, we perform N-body simulations of star cluster starting from different initial conditions which are described in detail in the next section. Results and analysis of our simulations are shown in section 3 while the discussion and conclusions are presented in section 4.

2. Details of numerical simulations

We have conducted a number of N-body simulations, using the collisional N-body code NBODY4 (Aarseth 1999) on the GRAPE-6 special purpose computers provided by ADC - CfCA NAO Japan, to follow the evolution of multi-mass star clusters. All simulations are run to 3 Myrs by which time we assume that the runaway stars are turned into BHs and stop the simulations.

Our clusters contain 131.072 stars initially, distributed according to a Salpeter IMF with minimum mass and maximum mass set equal to 1.0 $M_{\odot}$ and 100 $M_{\odot}$ respectively. Two stars are assumed to 'collide' if the distance between them becomes smaller than the sum of their radii. We assume that the total mass of both stars ends in the merger product. We examine the evolution of King (1966) models with central concentration $3.0 \leq W_0 \leq 9.0$. Details of the simulated clusters without initial mass segregation are presented in table 1.

In order to examine the effect of initial mass segregation, we study two scenarios. In the first scenario, we vary the minimum mass m_{min} within lagrangian radii containing 5 % of the total cluster mass (R_{005}). Increasing the minimum mass m_{min} within R_{005} (from 1 $M_{\odot}$ for a normal cluster to a higher mass for clusters with initial mass segregation) will consequently decrease the number of stars within this shell. This scenario allows massive stars to start their life in the cluster center. It is proposed to meet observations which show that massive stars are preferentially formed near the cluster center (Bonnell & Davis 1998, de Grijs et al. 2004). The initial half-mass radius and total cluster mass are chosen similar to what Portegies Zwart et al. (2004) chose to fit the observed parameters of MGG-11, namely $r_h = 1.3$ pc and $M = 3.5 \times 10^5 M_{\odot}$. Details of the runs are given in Table 2.

In the second scenario, we choose a certain fraction of stars (whose total mass is 5 % – 20 % of total mass of the cluster) with the lowest total energy and then vary the minimum mass of them, while keeping the total cluster mass constant. The number of stars is again lower than in a normal cluster. Compared to the first scenario, the second scenario brings stars even closer to the center since stars located in the center at any one time could still have high energies and spent most of their life outside the center. Hence support for runaway collisions should be stronger in the second scenario.

We also vary the half-mass radius of the clusters to see the effect of different central densities. Table 3 reports details for clusters with initial mass segregation.

3. Results and Analysis

3.1. Clusters without initial mass segregation

We run five cluster models without initial mass segregation as shown in Table 1. Each cluster contains 131.072 stars, but has different W_0. Four of them are set to have the same half-mass radius, which is 1.3 pc, to mimic MGG-11. In addition, we also examine a $W_0 =$ 7 cluster with a smaller half-mass radius of $r_h = 0.5$ pc. The central density of each cluster refers to the density within the 5% lagrangian radius of the cluster. For clusters with the same r_h, the central density is higher for clusters with higher central concentration W_0.

We also calculate the central relaxation time of the cluster to study the influence of this parameter on the occurrence of runaway merging. The central relaxation time $T_{rel,c}$ is defined as (Spitzer 1987):

$$T_{\rm rel,c} = \frac{\sigma_{3D}^3}{4.88\pi G^2 (\ln 0.11N) n \langle m \rangle^2}, \tag{1}$$

where σ_{3D}, n and $\langle m \rangle$ are the three-dimensional velocity dispersion, number density and average stellar mass at the cluster center.

Our simulations of MGG-11 like clusters (with $r_h = 1.3$ pc) show (see Table 1) that only the star cluster with the highest central concentration ($W_0 = 9.0$, corresponding to a central density of 3.4 $\times 10^6$ $M_\odot/\mathrm{pc}^3$), experiences runaway merging. This result is in good agreement with the one found by Portegies Zwart et al. (2004). Our result again proves that high central density is required to allow the runaway merging. Collisions among massive stars also occur in the lower density clusters but none of them experiences subsequent collisions leading to a super-massive star.

Fig. 1 depicts the evolution of lagrangian radii of models 1–3. The inner shells of $W_0 = 9$ cluster (model 1) suffer strong contractions due to the high central density. Core collapse happens in this cluster at $t \approx 0.7$ Myrs. On the other hand, inner shells of $W_0 = 7$

cluster (model 2) contract very slowly. Even until 3 Myrs, the contraction is not strong enough to produce core collapse. Consequently, no runaway merging occurs in this cluster. Evolution of inner shells of $W_0 = 7$ cluster however looks different when we decrease r_h to 0.5 pc (model 3). Mild contraction brings the cluster to collapse. Core collapse occurs at $t \approx$ 2.5 Myrs and it supports the runaway merging to happen. Although the runaway merging started later than in the $W_0 = 9$ cluster, three collisions are enough to form a few hundreds $M_{\odot}$ super massive star (see Table 1). A half-mass radius of $r_h = 0.5$ pc, corresponding to a central density and relaxation time of 4.4 $\times 10^6$ $M_{\odot}/\mathrm{pc}^3$ and 2.5 $\times 10^5$ Myrs, can be viewed as the boundary that allows runaway merging to occur for $W_0 = 7$ clusters.

Table 1 shows that runaway mergings occur in the two clusters with very high central density and low central relaxation time (model 1 and model 3). Central density 3.4 $\times 10^6$ $M_{\odot}/\mathrm{pc}^3$ and central relaxation time 1.5 $\times 10^5$ Myrs (both values correspond to $W_0 =$ 9.0, $r_h = 1.3$ pc) seem to be the critical limits which allow normal clusters to experience runaway stellar merging.

We also find that the central relaxation time (see column 6 on Table 1) mainly depends on the number density in the center, where $T_{\mathrm{rel,c}} \propto n^{-1}$ (see eq. 1). Other parameters such as velocity dispersion σ and average mass $\langle m \rangle$ contribute almost same value for all clusters (in average $\sigma \approx 27.5$ pc/Myr and $\langle m \rangle \approx 2.64$ $M_{\odot}$), while the number density varies from $2.8 \times 10^6/\mathrm{pc}^3$ to $5.5 \times 10^4/\mathrm{pc}^3$. As runaway merger occurs in a normal cluster with a low central relaxation time, a high number density in the cluster center is required to support runaway stellar merger.

3.2. Clusters with initial mass segregation

In models 6 – 8, we introduce initial mass segregation by filling the 5 % lagrangian radius (R_{005}) with massive stars whose mass is higher or equal than the mass m_{min} written in the 6th column of Table 2.

As we keep the mass within the R_{005} lagrangian radius constant, introducing initial mass segregation by increasing m_{min} means to increase the average mass of stars $\langle m \rangle$ and lower the total number of stars (see the 3rd column of Table 2). The increase of $\langle m \rangle$ in the core consequently decreases the central relaxation time $T_{rel,c}$. As the central parts of the clusters relax faster, the clusters may evolve faster and core collapse may happen earlier. One therefore may expect that runaway merging could occur in these clusters.

However, Fig. 2 shows that models 6 and 8 do not experience the core collapse until 3 Myrs. Our simulations also show that no runaway mergers occur in these clusters. The reason why the runaway mergers could not happen here, is that the massive stars which start their life inside the core do not constantly stay in the core. Due to the density and the velocity structure, some of them move out of the core.

The outmoving of massive stars from the core of model 6 is shown in Fig. 3. This figure depicts the evolution of lagrangian radii of massive stars whose masses are higher than 30 $M_{\odot}$ that started their life inside the R_{005}. Total mass of these massive stars is indicated by M_{R005}. The lagrangian radii of 10 % until 100 % of this total mass are presented. The upper figure shows the change of lagrangian radii up to the first 0.05 Myrs. The bold line indicates the initial radius of the core (R_{005}). We can see that soon after the cluster evolved, some of these massive stars leave the core. At t=0.01 Myrs, total mass of massive stars which still reside inside the core is only 40 % of its total mass M_{R005}. It continues decreasing to 30 % at t=0.05 Myrs. Bottom figure shows that up to t=3 Myrs, the core contains only about 30 % – 40 % of total mass of these massive stars.

In models 9 – 15, initial mass segregation is introduced by choosing stars which have the lowest total energy, up to 5 % – 20 % of the total mass of the cluster (see M_{IMS} on column 5 of Table 3). We then replace these with massive stars whose masses are higher than m_{min}. Their coordinates and velocities are randomly chosen from the stars with lowest total energy.

The central density and central relaxation time are measured for the region inside the 5 % lagrangian radius R_{005}. Since the massive stars are not as strongly concentrated toward the cluster center, the mean mass of stars within the core is not very high. Therefore the central relaxation time in this case (2.8 $\times 10^4$ Myrs $\leq T_{rel,c} \leq$ 5.2 $\times 10^5$ Myrs) is not as low as that in the table 2 (9.1 $\times 10^2$ Myrs $\leq T_{rel,c} \leq$ 2.9 $\times 10^3$ Myrs), but still shorter than models without initial mass segregation (1.5 $\times 10^5$ Myrs $\leq T_{rel,c} \leq$ 5.2 $\times 10^6$ Myrs). One may predict that the central relaxation time in this case is short enough that the massive low-energy stars would spiral into the core and create a high-mass core. Once in the cluster core, these massive stars could collide with each other and promote runaway merging.

Nevertheless, our simulations do not show runaway merging. One reason is that, although the massive stars are spiraling into the cluster center, but a high-mass core which support runaway mergers could not be form yet until 3 Myrs. Reducing the half-mass radius r_h from 1.3 pc to 0.5 pc in order to increase the central density could not help runaway merger to occur as well.

An interesting result, comparing two clusters of W_0=7 and r_h=0.5 pc in model 15 and model 3, shows different result on the runaway stellar merging. Model 15 does not experience runaway merging while it happens in a normal cluster in model 3. Figs. 4 and 5 depict the difference of their evolution of lagragian radii. The outer shells of the cluster with initial mass segregation (model 15) expand faster, while the inner shells contract faster then shells of normal cluster. It may happen because the the equipartition of energy works more effectively than the one in the normal cluster. However, the normal cluster shows prominent core collapse on about 2.5 Myrs while the cluster with IMS does not collapse yet.

The physical difference between cluster model 15 and model 3 is the number of stars (see column 3 of Tables 3 and 1). Initially, the 5 % lagrangian radius of model 15 contains 1215 stars compared to 6521 stars in model 3. The low number density of stars inside the core could be one possible reason that reduces the possibility for runaway merging to occur since reducing the number of stars inside the core also reduces the likelihood for collisions.

Based on the evolution of the number density of star in models 15 and 3, we calculate the collision rate N_{Coll} by using equation (8-122) of Binney & Tremaine (1987)

$$N_{Coll\star} = 4\sqrt{\pi} n\sigma (2\ R_\star)^2 + 4\sqrt{\pi} G M_\star n (2\ R_\star)/\sigma. \tag{2}$$

Here $N_{Coll\star}$ is average number of physical collision that a star suffers per unit time, n and σ indicate the number density of stars and velocity dispersion, $R_\star$ and $M_\star$ denote radius and mass of colliding stars, and G is the gravitational constant. The first term is derived from the kinetic theory for inelastic encounters and the second term represents the enhancement in the collision rate by the gravitational attraction of the two colliding stars.

The average number of collision per unit time in a cluster N_{Coll} is obtained by multiplying $N_{Coll\star}$ with the number of stars inside the cluster. Therefore

$$N_{Coll} = N_{Coll\star} N_{Star} = N_{Coll\star} n V \tag{3}$$

where N_{Star} is the number of stars and V is the cluster volume. Substituting $N_{Coll\star}$ with the one written in eq. 2, we see that

$$N_{Coll} \propto n^2. \tag{4}$$

We use the theoretical prediction of the collision rate (eqs. 2 and 3) to follow the growth of number of collisions per unit time in models 3 and 15. $N_{Coll\star}$ is calculated by considering mass and radius of each star as $M_\star$ and $R_\star$ and then summed up for all stars to obtain the N_{Coll}. These theoretical calculations are then compared with the collision rate we find in our simulations. The result is presented in Fig. 6. Both theoretical and simulation results show that collision rate of model 3, which experiences the runaway stellar merger, is higher than the one in model 15. The number density of stars n indeed has an important role in the process of runaway stellar merging, as $N_{Coll} \propto n^2$.

Theoretical prediction of collision rate is over estimate the simulation results by factor ≈ 2. This may due to some assumptions (i.e. mass and radius of collide stars are same) and idealizations (i.e. distribution function of velocity is Maxwellian) used in the derivation of that collision rate equation, while in the simulations we use mass spectrum and different star radii according to a certain mass-radius relation.

4. Discussion and Conclusions

We have followed the evolution of multi-mass, dense star clusters with dimensionless central potentials of $3.0 \leq W_0 \leq 9.0$. Our simulations show a good agreement with Portegies Zwart et al. (2004) in that in clusters without initial mass segregation central concentration $W_0 \geq 9.0$ or central densities $\rho_c = 10^{\,6.5} M_\odot/\mathrm{pc}^3$ are required for runaway mergings to occur. Examining clusters with lower central concentration, $W_0 \leq 7.0$, we found that two criteria take an important role to allow the process of runaway mergers to happen: the central density of the clusters and the number density of stars in the cluster center. Our results show that IMBHs would form through runaway mergers of stars if physical parameters of clusters satisfy both criteria : having high central density and high number density of stars as roughly shown in Fig. 7. Here central density and number density refers to initial condition on the region inside 5 % lagrangian radii.

Initial mass segregation could increase the average mass of stars within the cluster center and thus decrease the central relaxation time. Initial mass segregation also allow to form a high-mass core. However, unless the number density of stars in the center is high enough, initial mass segregation could not support the runaway stellar merger to happen.

A comparison with the compilation of galactic globular clusters data by Harris (1996) shows no cluster has central density exceeding $\rho_c = 3.4 \times 10^6\ M_{\odot}/\mathrm{pc}^3$ (see Fig. 8). About 4 % have central density exceeding $\rho_c = 4.3 \times 10^5\ M_{\odot}/\mathrm{pc}^3$. The rest ones having central density less than $\rho_c = 4.3 \times 10^5\ M_{\odot}/\mathrm{pc}^3$. If these densities are representative of the densities with which the clusters formed, then runaway merging would not have happened in any of these clusters. Other possibilities of forming IMBHs like the merging of many stellar mass black holes (Miller & Hamilton 2002) also need extreme initial conditions like very massive clusters (Gültekin et al. 2004; Rasio et al. 2006). This may suggest that IMBHs would be rarely found in star clusters within the Milky Way.

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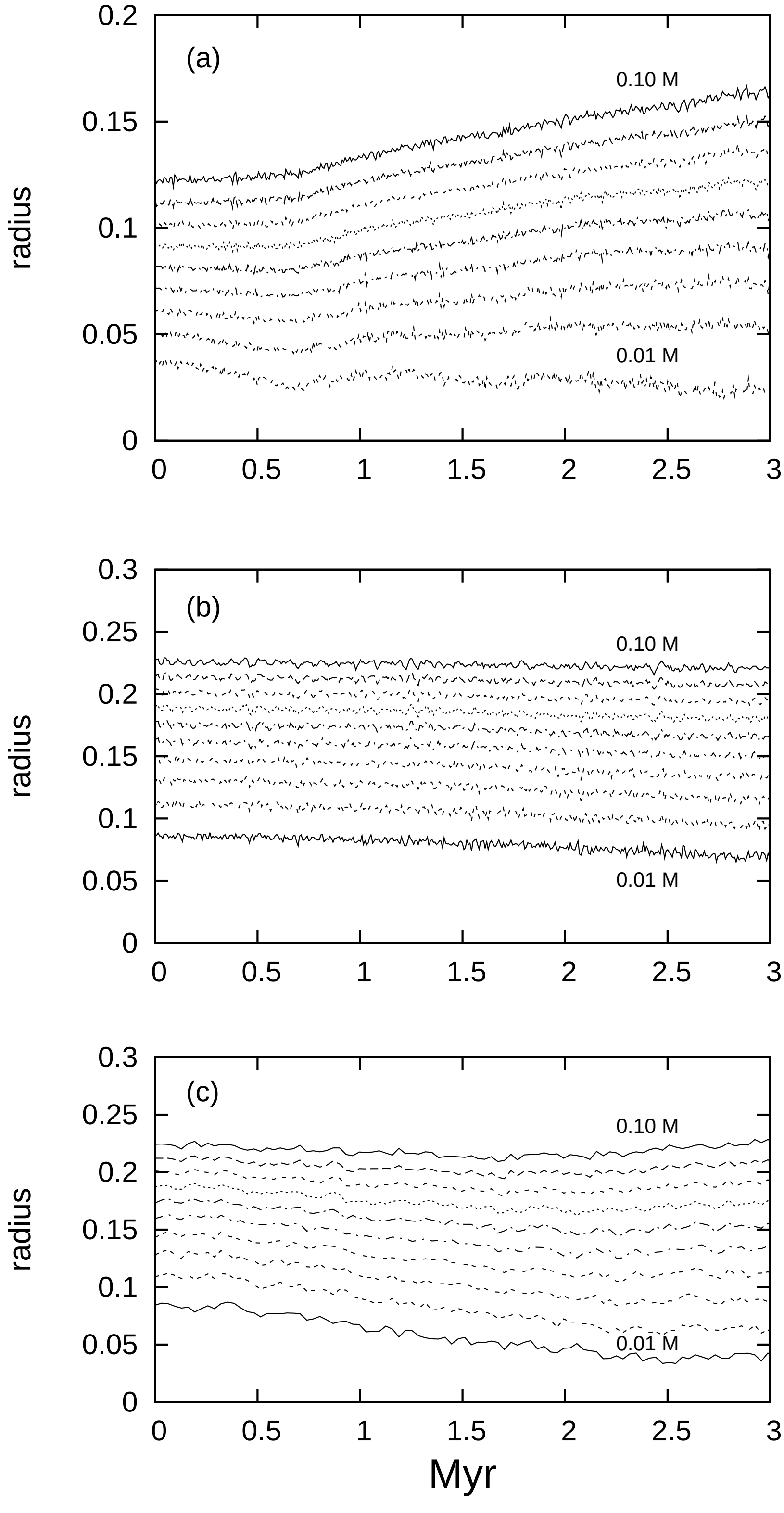

Fig. 1.— Evolution of lagrangian radii of inner shells containing 1 % – 10 % of total mass cluster (from the bottom to the top) of cluster (a). model 1 (b). model 2 and (c). model 3.

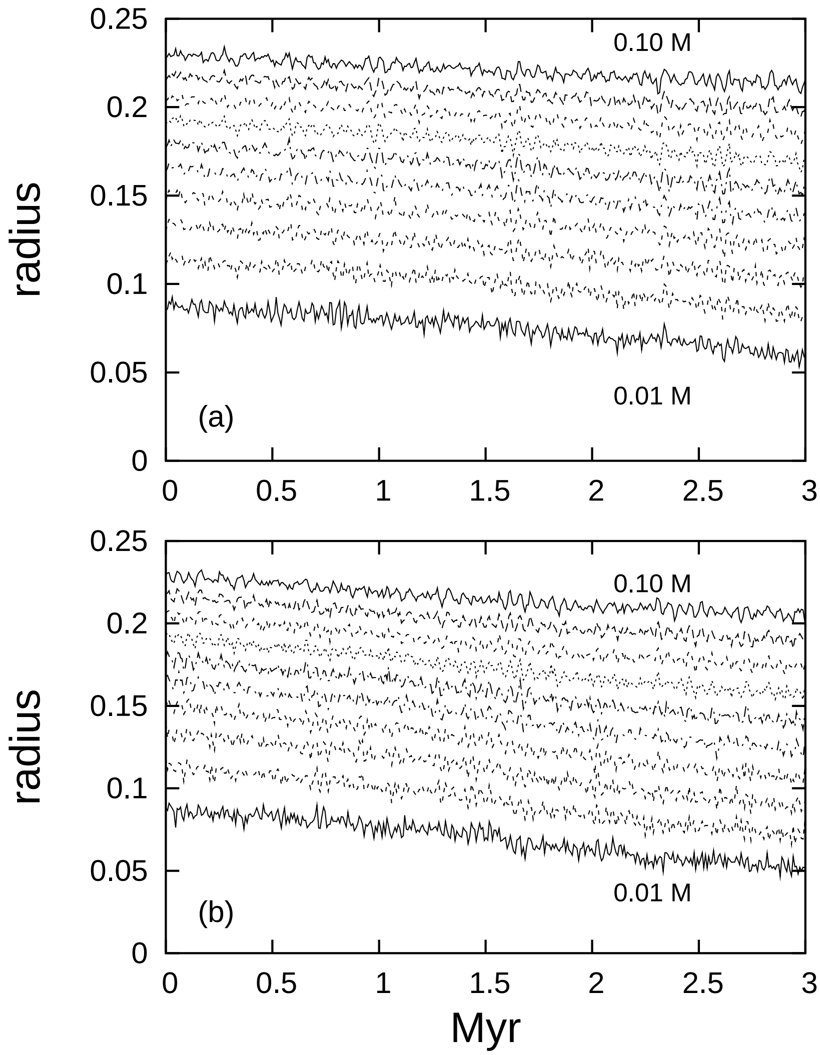

Fig. 2.— Evolution of lagrangian radii of inner shells containing 1 % – 10 % of total mass cluster (from the bottom to the top) of cluster (a). model 6 (b). model 9.

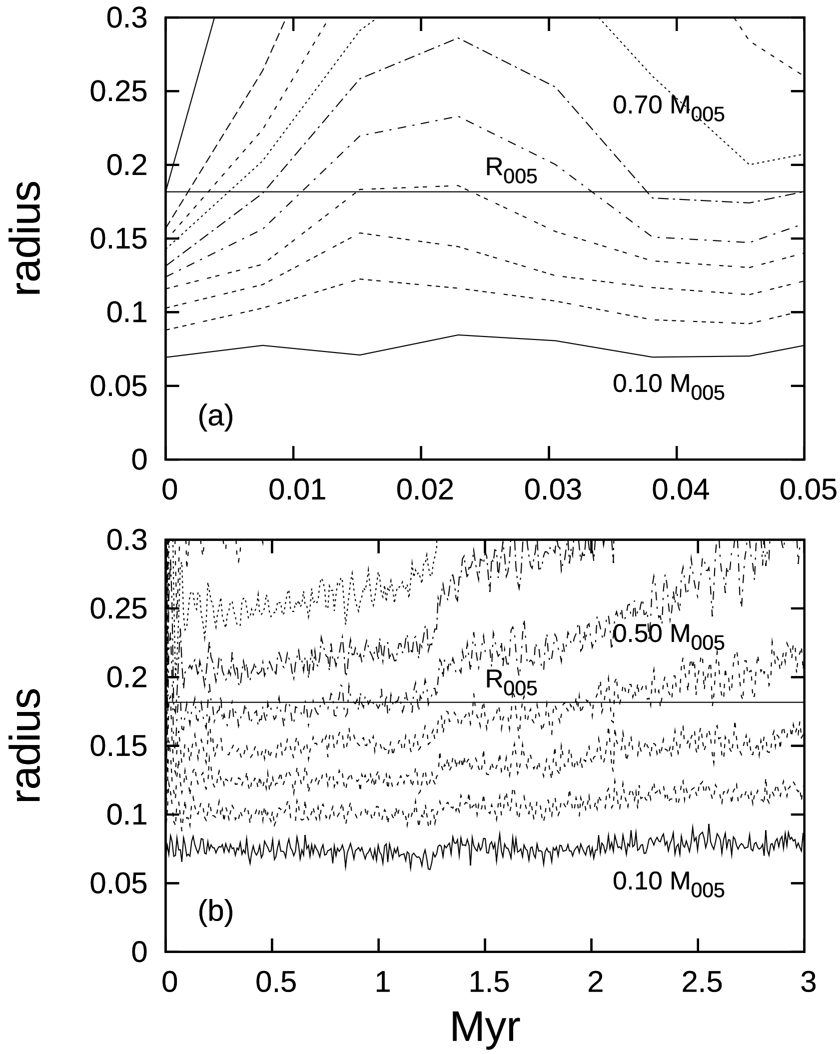

Fig. 3.— Evolution of lagrangian radii of massive stars ($m_{min} = 30M_{\odot}$) which start their life inside the core (R_{005}) (a). up to the first 0.05 Myrs (b). until 3 Myrs. Total mass of these massive stars is indicated by M_{005}.

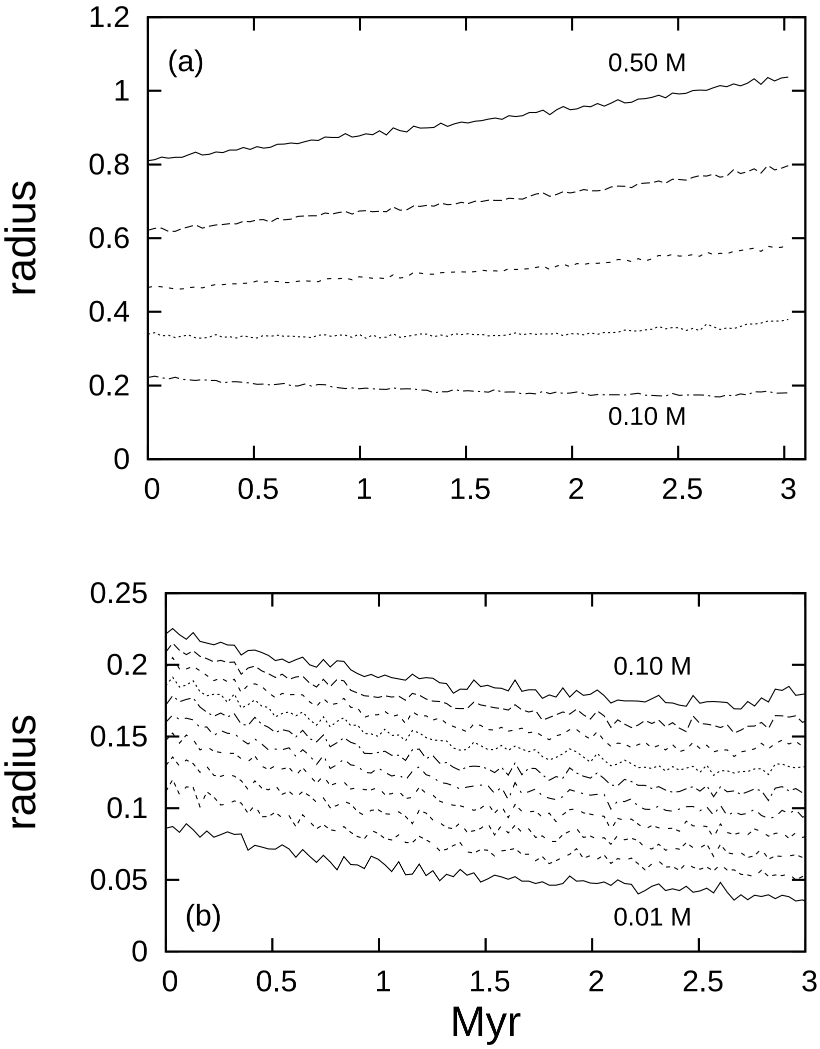

Fig. 4.— Evolution of lagrangian radii of cluster model 15 containing (a). 10 % – 50 % of total mass cluster (b). 1 % – 10 % of total mass cluster. Initial mass segragation is applied in this cluster by replacing the lowest-total-energy stars up to 20 % of total mass cluster by massive stars with $m_{min} = 30\ M_{\odot}$.

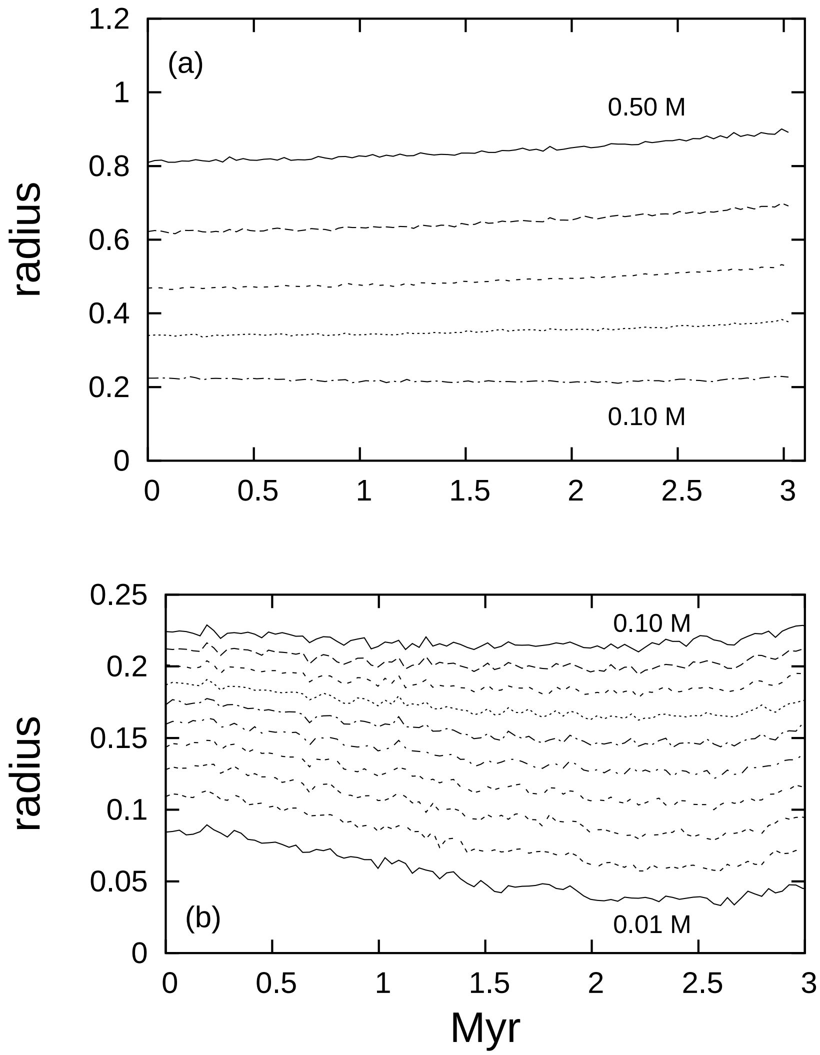

Fig. 5.— Evolution of lagrangian radii of cluster model 3 containing (a). 10 % – 50 % of total mass cluster (b). 1 % – 10 % of total mass cluster. The central density and half-mass radius of this cluster are same with the cluster model 15 but no initial mass segregation introduced in this cluster.

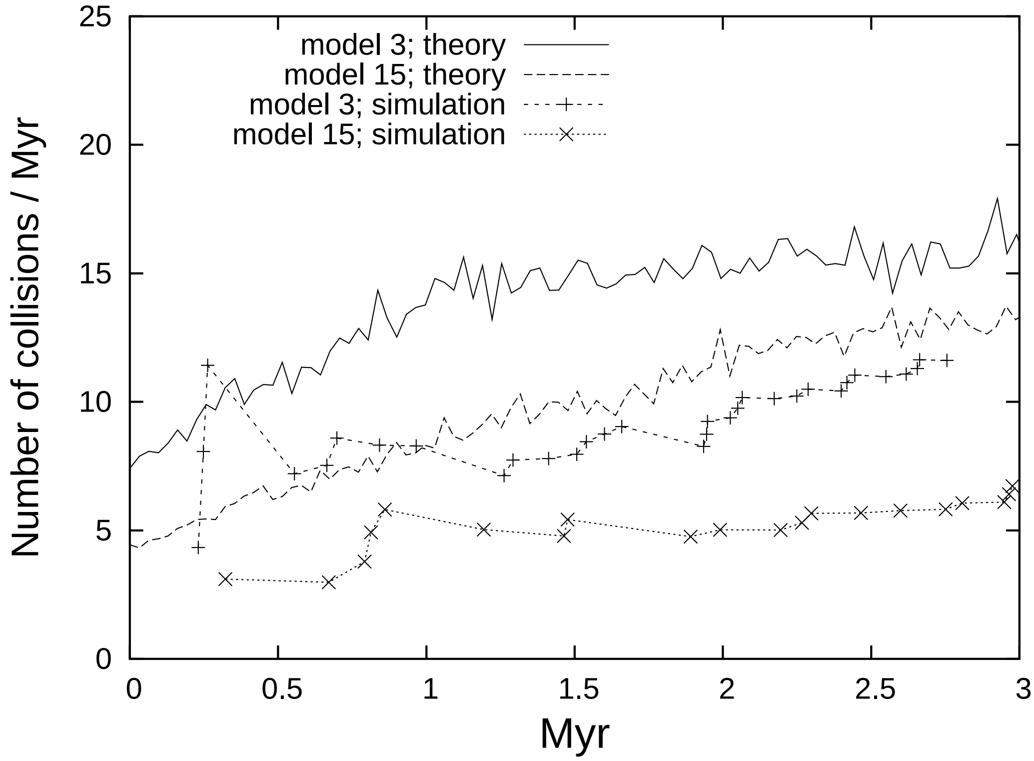

Fig. 6.— Collision rate inside inner shells of cluster models 15 and 3 obtained from simulations compared to theoretical prediction of collation rate on inelastic encounter.

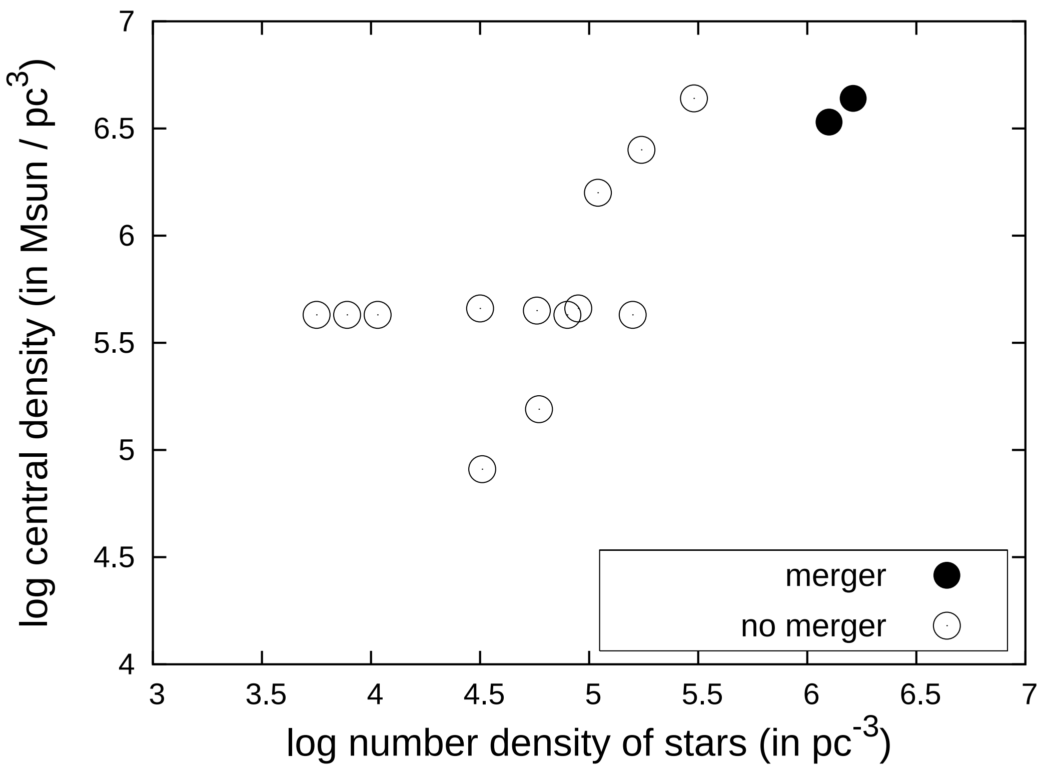

Fig. 7.— A plot of log central density – log number density of stars shows that both physical criteria should be very high to allow runaway merger to occur.

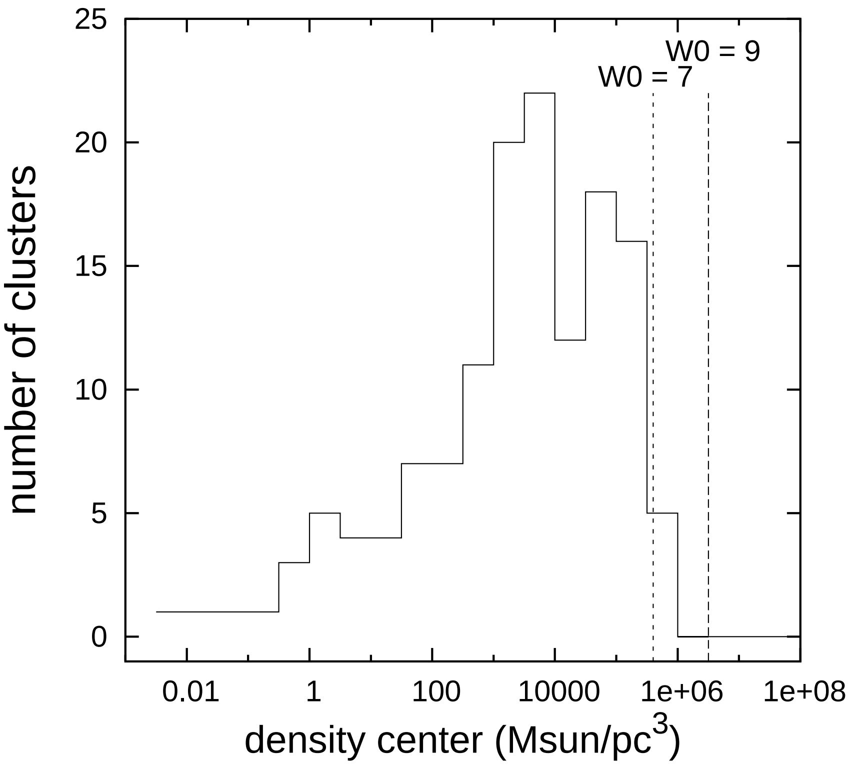

Fig. 8.— Distribution of central density of clusters in the Milky Way.

Table 1: Properties of simulated clusters without IMS

1	2	3	4	5	6	7	8	9	10	11
$Model$	W_0	N_{Star}	r_h	$logDensity$	$logT_{\rm rel,c}$	T_{rm}	$Coll$	$\langle T_{coll} \rangle$	M_{RS}	RM
			(pc)	$(M_\odot/pc^3)$	(Myr)	(Myr)		(Myr)	$(M_\odot)$	(Y/N)
1	9.0	131072	1.3	6.53	5.17	0.54	96	0.03	2786	Yes
2	7.0	131072	1.3	5.63	5.93	-	-	-	-	No
3	7.0	131072	0.5	6.64	5.39	2.55	3	0.18	258	Yes
4	5.0	131072	1.3	5.19	6.39	-	-	-	-	No
5	3.0	131072	1.3	4.91	6.72	-	-	-	-	No

Note. — 1: The first column indicates the cluster model, followed by the central concentration W_0 in the 2nd column. The number of stars in the cluster and the halfmass radius are given in the 3rd and 4th columns, respectively. The 5th column shows the logarithm of central density followed by the logarithm of the central relaxation time. The 7th column gives the time when runaway merging starts followed by the total number of collisions. The average time between collisions is shown in the 9th column, followed by the mass of the runaway star produced at the end of the runaway merging process. The last column shows whether runaway merging happens or not.

Table 2: Properties of clusters with IMS using the first scenario

1	2	3	4	5	6	7	8	9
$Model$	W_0	N_{Star}	r_h	M_{IMS}	m_{min}	$logDensity$	$logT_{\rm rel,c}$	RM
			(pc)	$(r \leq R_{005})$	$(M_\odot)$	$(M_\odot/pc^3)$	(Myr)	(Y/N)
6	7.0	124420	1.3	0.05	30.0	5.63	3.44	No
7	7.0	124305	1.3	0.05	50.0	5.63	3.26	No
8	7.0	124201	1.3	0.05	90.0	5.63	2.96	No

Note. — 2: The first and second columns indicate the cluster model and the central concentration W_0. The 3nd column shows the number of stars in the cluster followed by the half-mass radius in the 4th column. The 5th column gives the fraction of total mass of cluster (which is contained within the 5 % lagrangian radius) where the first scenario of IMS is applied. We choose some of these stars randomly and assign them with new masses which are larger than the minimum mass indicated in the 6th column. The logarithm of central density and the logarithm of the central relaxation time are given in the 7th and 8th columns. The last column shows whether runaway merging happens or not.

Table 3: Properties of simulated clusters with IMS using the second scenario

1	2	3	4	5	6	7	8	9
$Model$	W_0	N_{Star}	r_h	M_{IMS}	m_{min}	$logDensity$	$logT_{\rm rel,c}$	RM
			(pc)	(lowest$\mathrm{E_{tot}}$)	$(M_\odot)$	$(M_\odot/pc^3)$	(Myr)	(Y/N)
9	7.0	124420	1.3	0.05	30.0	5.66	5.69	No
10	7.0	124297	1.3	0.05	50.0	5.63	5.72	No
11	7.0	118805	1.3	0.10	30.0	5.65	5.46	No
12	7.0	106669	1.3	0.20	30.0	5.66	4.96	No
13	7.0	106669	0.7	0.20	30.0	6.20	4.66	No
14	7.0	106669	0.6	0.20	30.0	6.40	4.56	No
15	7.0	106669	0.5	0.20	30.0	6.64	4.44	No

Note. — Same with table 2 except the 5-th column here indicates the fraction of total mass of the cluster which contains of stars with lowest total energy. As we apply the second scenario for the IMS, some of these stars are randomly choosen and attributed with new masses, which are larger than a minimum mass given in the 6-th column.