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Determining the stiffness of the equation of state using low T/W dynamical instabilities in differentially rotating stars

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概 要

We investigate the nature of low T/W dynamical instabilities in a variety range of a stiffness of the equation of state in differentially rotating stars by means of both linear perturbation and three-dimensional hydrodynamical simulation. Here T is the rotational kinetic energy, while W the gravitational binding energy. We find the unstable normal mode of differentially rotating stars by solving the eigenvalue problem along the equatorial plane of the star. We also figure out that the physical mechanism of low T/W dynamical instability can be qualitatively understood by a scattering problem of a sound wave between corotation and surface by the corotation barrier. Therefore we can draw a picture that existing pulsation modes become unstable due to amplified reflection of sound waves from corotation. The feature in eigenfrequency and eigenfunction of the unstable mode in the linear analysis roughly agrees with that in three-dimensional hydrodynamical simulations in Newtonian gravity. Moreover, the nature of the eigenfunction that oscillates between corotation and the surface for unstable stars requires reinterpretation of the pulsation modes in differentially rotating stars. Finally we propose a manner to constrain the stiffness of the equation of state by the direct detection of mode decomposed gravitational waveforms.

We have investigated the unstable features of low T/W dynamical instabilities in differentially rotating stars in terms of a wide range of stiffness of the equation of state. We have adopted normal mode analysis and a scattering problem from the corotation barrier in the equatorial plane for our study, and compare the results with those of three dimensional hydrodynamic simulations.

We find unstable normal modes for low T/W dynamically unstable stars in the linear analysis, and they are qualitatively confirmed by the amplification by the scattering of sound waves between corotation and surface due to corotation. Although the timescale is in agreement in a qualitative level, the criteria has good agreement to the results of both numerical simulation and normal mode analysis. We do not find any additional modes to the well-known f- and p-modes in the linear analysis, but the stability of the system can be changed when corotation barrier appeared in the effective potential. The resonance frequency in both the cylindrical and the spheroidal models of the linear analysis agrees with that of hydrodynamic simulation when approaching to non rotating star. The fact confirms us that our models are efficient for finding low T/W dynamically unstable stars.

We also find that the eigenfunction of the modes have a similar behaviour to the well known f- and p-modes. Once corotation exists inside the stars, the perturbed enthalpy

表 1: Equilibrium configuration of differentially rotating stars

model	n	Ω_c/Ω_e[1]	T/W
I	1	26.0	6.09×10^{-2}
II	1.5	26.0	6.76×10^{-2}
III	2	26.0	7.29×10^{-2}
IV	3	26.0	7.21×10^{-2}

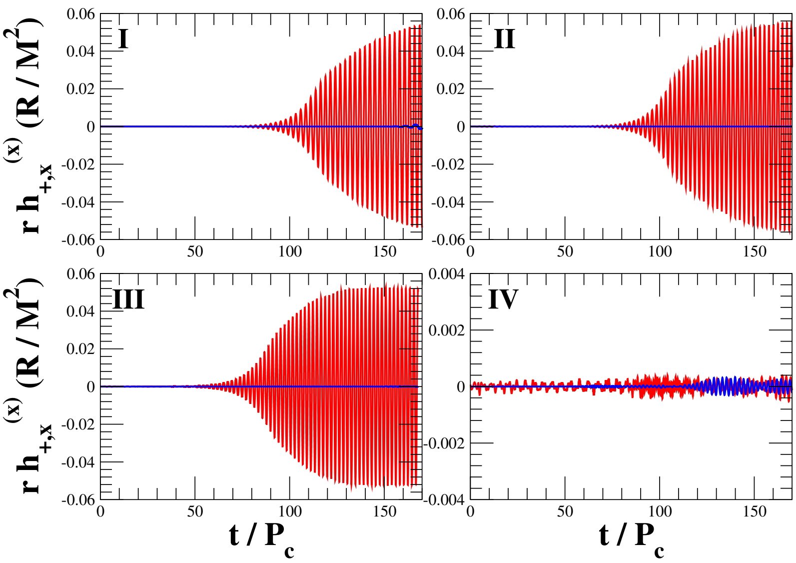

図 1: Gravitational waveform for four low T/W dynamically unstable stars observed from the principal axis in the equatorial plane of the equilibrium star (the label represents the equilibrium models in Table 1). Red and blue lines represent $+$ ($m = 2$ diagnostic feature which contains a quasi-periodic oscillation) and $\times$ ($m = 1$ diagnostic feature which contains an amplified oscillation) modes, respectively.

oscillates between corotation and the surface. This may indicates that the perturbed enthalpy is affected by corotation barrier, and therefore cannot cross corotation radius. This feature requires reinterpretation of the pulsation modes in rotating stars when corotation singularity exists inside the stars.

Finally we are able to constrain the stiffness of the equation of state by the direct observation of mode decomposed gravitational waves from low T/W dynamically unstable stars. Investigating the dominancy of the m-mode in normal mode analysis in cylindrical model, the threshold of $m = 2$ is around $\Gamma \approx 1.50$. Using the above fact, we are able to constrain the stiffness of the equation of state by focusing the ratio between $m = 1$ and $m = 2$ of the gravitational waveform (e.g. Figure 1).

A more detailed discussion is presented in Saijo (2018) [2].

[1] Ω_c: Central angular velocity; Ω_e: Equatorial surface angular velocity
[2] Submitted to Physical Review D