Numerical characteristics of core-collapse supernova simulations

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Q: How does the explosion occur? What is observed?
From 1D to 3D
Multi-messenger Astronomy of CCSNe

Comparison between theory and observation is a hot topic.

- Multi-messengers
  - GW and $\nu$ => Kuroda
  - Nucleosynthesis (Ni synthesis) => Nakamura
  - Shock breakout => Suzuki
  - Supernovae => Nakamura (?)

- Thermal & Ni -> Co -> Fe

- Supernova!
Successful 3D simulations and the Caveats

- Takiwaki+12, 14, Horiuchi+14
  Newtonian, ignoring some ν-reactions

- Meloson+15a,b
  Phenomenological GR
  Extremely light progenitor
  Exotic ν-reaction

- Mueller+15
  Artificial convection is included

There is no perfect model!
Is MM-astronomy a house built on sand?

House, Castle: Interesting comparison with observation
Base: The explosion mechanism

10 years ago, we did not observe explosion.

Without base, we can’t build a house.

Now, we find some explosion models with a lot of controversy and caveat.
We have a house whose base is unstable.

To make a castle, firm base is necessary.
We should give conclusive results on the explosion mechanism.
FAQ on supernovae simulation

Q1: Energy conservation
Q2: Convergence on resolution
Q3: Uncertainty of initial setups
Q4: Uncertainty of input physics

Briefly introduce Q1-Q3. My main topic is Q4.
Q1 Energy Conservation

Gravitational Energy is dominant: $\sim 10^{53}$ erg
Typical Explosion: $10^{51}$ erg

Perfect conservative formalism is difficult.

\[
\frac{\partial}{\partial t} \left[ e + \rho \Phi + \frac{1}{8\pi G} \left( \frac{\partial \Phi}{\partial x^i} \right)^2 \right] + \frac{\partial}{\partial x^i} \times \left[ (e + P + \rho \Phi) v^i + \frac{1}{4\pi G} \Phi \frac{\partial^2 \Phi}{\partial t \partial x^i} \right] = 0
\]

New scheme starts from

\[
\frac{\partial (e + \rho \Phi)}{\partial t} + \frac{\partial (e + P + \rho \Phi) v^i}{\partial x^i} = \rho \frac{\partial \Phi}{\partial t}
\]

\[
\Rightarrow \quad \frac{\partial e}{\partial t} + \frac{\partial (e + P + \rho \Phi) v^i}{\partial x^i} - \frac{\partial \rho v^i}{\partial x^i} \Phi = \rho \frac{\partial \Phi}{\partial t}
\]

Consider spatial and time difference of $\Phi$!

Error is $10^{50}$ erg at bounce, after that < $10^{49}$ erg.
Both are less than the typical explosion energy of $10^{51}$ erg!
Q2 Conservation on resolution

Radice+2015

Required Resolution
Large scale: \( \sim 2\degree \) <= our study
Turbulent Structure: \( \sim 0.1\degree \)

Caveat: convergence is worse for nearly-critical models
Q3 Uncertainty of initial setups

Initial models are made by other simulations.

Compactness is important

\[ \xi_M \equiv \frac{M/M_\odot}{R(M)/1000 \text{ km}} \]

\[ \Rightarrow \text{Nakamura's talk} \]

Strange behavior at 18-22M_s!
Statistical argument is possible.
Nakamura+15, Ugliano+12, Ertl+15

Now the method are limited in 1D.
3D effect is phenomenologically included.
Update of the effects is on progress (e.g. Meakin and Arnett).

Applications for CCSNe is done by Couch+13.
FAQ on supernovae simulation

Q4: Uncertainty of input physics

Q4-1: Equation of State
Q4-2: Neutrino Reactions (Melson+15, Sullivan+16)
Q4-3: Method of Radiation Transport (Sumiyoshi+14, Nagakura+15)
Q4-4: General Relativity (Kuroda+14, Mueller+10)
Q4-5: Other exotic effect (Sterile v, axion etc)
Evolution of the shock

Softer EOS shows larger shock radius.

Soft vs stiff comparisons for different EOS models, showing the impact on shock radius over time.
Basic idea to connect EOS and Explosion

1. The PNS gradually shrinks by the gravity.
2. $E_{\text{grav}}$ is released.
3. $E_{\text{thermal}}$ is increased.
4. The $L_\nu$ and sonic waves are emitted from the surface of PNS.

Soft EOS releases large energy and makes the PNS dense, that produce strong acoustic wave.

Softer EOS is preferable to the explosion.
Neutrino Luminosity

LS(K220): Soft EOS => rapidly shrink => Large \( L_\nu \)
Shen: Stiff EOS => slowly shrink => small \( L_\nu \)

\[
L_{\text{eff}} = \frac{1}{2} \sum_{i=\nu_e,\bar{\nu}_e} L_i \left( \frac{T_i}{4.5\text{MeV}} \right)^2
\]

(Sumiyoshi+2005 and Fisher+ 2013 show similar results.)
Sonic Wave

Strong sonic wave is reflected at the PNS! (It is a little bit hard to see, but) softer EOS make stronger sonic wave.

(Couch 2013 and Suwa+ 2013 show similar results.)
In 3D, contribution of sonic wave is relatively small. The difference of EOS is also not so significant.

$$100 \times 10^{49} \text{erg/s}$$  

$$20 \times 10^{49} \text{erg/s}$$
From $10^{10}$ to $10^{13}$ g/cm$^3$, nuclei becomes larger and larger. Before $10^{14}$ g/cm$^3$, pasta phase appears. After that nuclear matter becomes uniform.

<table>
<thead>
<tr>
<th>Density g/cm$^3$</th>
<th>Part of EOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{10}$</td>
<td>&quot;Fe&quot;</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>Heavy nuclei, n,p</td>
</tr>
<tr>
<td>$10^{13}$</td>
<td>Pasta</td>
</tr>
<tr>
<td>$10^{14}$</td>
<td>Uniform matter</td>
</tr>
</tbody>
</table>

Which part of EOS is important?

Surface of PNS
Parametrized Equation of State

Employing LS’s parameter sets, we construct number of equation of states.

Non-uniform matter: LS $K=220\text{MeV}$
Uniform matter: LS changing $K=180, 230, 280\text{[MeV]}$ and $S=25, 30, 35\text{[MeV]}$.

Collaboration with Togashi and Sumiyoshi

$Ye=0.3$, $s=1.0$
Evolution of the shock

Divergence due to non linear evolution of 2D smeared out the difference of equation of state at uniform density.
Previous study: Change stiffness at all range
   => Evolution of Shock depends EOS

This study: Change stiffness at high density
   => Evolution of Shock does not depend EOS

Stiffness affects the shock indirectly.
Summary

- Multi-D simulations are going to succeed. Study of multi-messenger astronomy become active.
- Explosion mechanism is still uncertain. To make a firm model for MM-astronomy, all relevant things should be considered.
- In this talk, we consider Effect of EOS. EOS significantly change the dynamics of the shock, cuz v-emission and propagation of sonic wave. In 3D, effect of sonic wave becomes weak.
- We found low density EOS is relevant for the effect. Stiffness of nuclei indirectly change the low-density EOS.
To make the firm conclusion

Every little things should be considered since that could affect SNe. It seems to be complex and boring. To make firm base, such an effort is inevitable.

http://revcharlieholt.com/what-am-i-doing-with-my-life/
http://sedeslav.deviantart.com/art/castle-on-the-rock-96976197
Can we distinguish EOS by $\nu$ obs.

Super Kamiokande detects anti-$e^- \nu$ preferably.

The effect of EOS is relatively smaller compared to that of progenitor.