N 体シミュレーションの学校 講義2
参考になる文献やサイト (講師が目を通したもの)

最終更新 2026 年 2 月 2 日

(リンクや書誌情報に誤りを発見されたら学校主催者へお知らせください)

講義の前半部分

(常) 微分方程式の一般論入門

- 千葉 (2021) 力学系の入門にも好適
- Boyce *et al.* (2021) 欧米では定評のある教科書
- 琉球大学が公開している講義資料

数値計算の誤差

- 日本 NAG 公開している資料 (打ち切り誤差と丸目誤差)
- 皆本 (2002) (丸め誤差)
- IEEE754 (Wikipedia 英語版) (丸め誤差)
- 筑波大学が公開している講義資料 (丸め誤差)

汎用的な解法

- 本学校の過去の資料のひとつ
- Hairer *et al.* (1993) 定番の教科書 1
- Hairer and Wanner (1996) 定番の教科書 2
- 戸川 (1998) 本講義の前半に関して最も頻繁に参考にした
- 京都大学が公開している講義資料
- 東京大学が公開している講義資料
- 東北大学が公開している講義資料

修正オイラー法・Runge–Kutta 法など

- 九州大学が公開している講義資料
- Qiita 上の記事
- 京都大学が公開している講義資料
- Wikipedia 英語版

天体力学に於ける多段法 (対称型) の応用例

- Quinlan and Tremaine (1990)
- Kinoshita and Nakai (1996)
- Fukushima (1999)
- arXiv:astro-ph/9901136 この方法の不安定性について

講義の後半部分

シンプレクティク数値積分法の概説

- Yoshida (1992, 1993)
- 吉田 (1994, 1995, 1997, 2001) 本講義の後半に関して最も頻繁に参考にした
- Donnelly and Rogers (2005) 計算の過程が分かりやすい入門用文献
- プラズマ・核融合学会誌上の特集記事群 1
- プラズマ・核融合学会誌上の特集記事群 2
- プラズマ・核融合学会誌上の特集記事群 3
- UC San Diego が公開している講義資料 2021 年度の本校受講者による情報
- 神戸大学が公開している講義資料
- 京都大学が公開している講義資料
- University of Rochester が公開している講義資料
- LibreTexts Mathematics 上の記事
- 慶應義塾大学が公開している講義資料
- Kamberaj (2020)

解析力学一般

- Goldstein *et al.* (2002)
- 山本義隆 and 中村孔一 (1998a,b) 日本語教科書としてはこれを挙げる
- Lichtenberg and Lieberman (1992) 正準形式による力学系の記述
- Boccaletti and Pucacco (1996, 1998) 正準形式による N 体系の記述

正準変換・シンプレクティク形式

- 神戸大学が公開している講義資料 1
- 神戸大学が公開している講義資料 2
- 神戸大学が公開している講義資料 3
- 大阪大学が公開している講義資料
- 個人ブログ上の記事
- Wikipedia 英語版

Leapfrog, velocity Verlet

- Wikipedia 英語版
- Qiita 上の記事
- Hairer *et al.* (2003)

Campbell–Baker–Hausdorff–Dynkin 公式

- Oteo (1991)
- Casas and Murua (2009)
- Achilles and Bonfiglioli (2012)
- Bonfiglioli and Fulci (2012) 包括的な書籍
- Van-Brunt and Visser (2015)

Wisdom–Holman symplectic map

- Wisdom (1982, 1983) 下記の基礎となる写像
- Wisdom and Holman (1991, 1992)
- Kinoshita *et al.* (1991) 上記とほぼ等価な算法の提示

初期値に対する誤差依存性、逐次出発・暖機出発

- Saha and Tremaine (1992)
- Ito and Tanikawa (2012)

シンプレクティク修正子

- Wisdom *et al.* (1996)
- Mikkola and Palmer (2000)
- Wu *et al.* (2003)
- Rein *et al.* (2019)

研究黎明期の主要文献

- Dragt and Finn (1976)
- Neri (1985)
- Yoshida (1990a, 1993)
- Forest and Ruth (1990)
- Sanz-Serna (1992)

適応型刻み幅を実装することによる精度向上

- Mikkola (1997)
- Mikkola and Tanikawa (1999a,b, 2013)
- Preto and Tremaine (1999)
- Emel'Yanenko (2007)

階層的刻み幅を実装することによる精度向上

- Biesiadecki and Skeel (1993)
- Skeel and Biesiadecki (1994)
- Duncan *et al.* (1998)

天体の接近遭遇や高離心率系への対応

- Levison and Duncan (1994, 2000)
- Rauch and Holman (1999)
- Chambers (1999, 2003)
- Emel'yanenko (2002)
- Mikkola and Wiegert (2002)
- Rein and Tamayo (2015)
- Wisdom (2017)

剛体の自転運動への応用

- Touma and Wisdom (1994)
- van Zon and Schofield (2007)
- Laskar and Vaillant (2019)

誤差と信頼性に関する議論

- Fukushima (2001)
- Wisdom (2015)
- Hernandez *et al.* (2020)

関連するその他の話題

- Yoshida (1990b) N 体系の全角運動量の厳密保存
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- Qiita 上の記事 時間反転に関する対称性
- 東京大学が公開している講義資料 微分形式、外積
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