

N 体シミュレーションの学校 講義 2

参考になる文献やサイト (講師が目を通したもの)

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(リンクや書誌情報に誤りを発見されたら学校主催者へお知らせください)

講義の前半部分

(常) 微分方程式の一般論入門

- 千葉 (2021) 力学系の入門にも好適
- 琉球大学が公開している講義資料
- 岐阜大学が公開している講義資料

打ち切り誤差

- 神戸大学が公開している講義資料
- 東海大学が公開している講義資料

丸め誤差

- 皆本 (2002)
- IEEE754 (Wikipedia 英語版)
- 筑波大学が公開している講義資料

汎用的な解法

- 本学校の過去の資料のひとつ
- Hairer *et al.* (1993) 定番の教科書 1
- Hairer and Wanner (1996) 定番の教科書 2
- 戸川 (1998) 本講義の前半に関して最も頻繁に参考にした
- 京都大学が公開している講義資料
- 東京大学が公開している講義資料
- 東北大学が公開している講義資料

修正オイラー法

- 九州大学が公開している講義資料
- Qiita 上の記事

Runge–Kutta 法

- [京都大学が公開している講義資料](#)
- [Wikipedia 英語版](#)

天体力学に於ける多段法 (対称型) の応用例

- [Quinlan and Tremaine \(1990\)](#)
- [Kinoshita and Nakai \(1996\)](#)
- [Fukushima \(1999\)](#)
- [arXiv:astro-ph/9901136](#) この方法の不安定性について

講義の後半部分

シンプレクティック数値積分法の概説

- [Yoshida \(1992, 1993\)](#)
- [吉田 \(1994, 1995, 1997, 2001\)](#) 本講義の後半に関して最も頻繁に参考にした
- [プラズマ・核融合学会誌上の特集記事群 1](#)
- [プラズマ・核融合学会誌上の特集記事群 2](#)
- [プラズマ・核融合学会誌上の特集記事群 3](#)
- [UC San Diego が公開している講義資料](#) 2021 年度の本校受講者による情報

Leapfrog, velocity Verlet

- [Wikipedia 英語版](#)
- [Qiita 上の記事](#)

正準変換・シンプレクティック形式

- [神戸大学が公開している講義資料 1](#)
- [神戸大学が公開している講義資料 2](#)
- [大阪大学が公開している講義資料](#)

Campbell–Baker–Hausdorff–Dynkin 公式

- [Oteo \(1991\)](#)
- [Casas and Murua \(2009\)](#)
- [Achilles and Bonfiglioli \(2012\)](#)
- [Bonfiglioli and Fulci \(2012\)](#) 包括的な書籍
- [Van-Brunt and Visser \(2015\)](#)

Wisdom–Holman symplectic map

- Wisdom (1982, 1983) 下記の基礎となる写像
- Wisdom and Holman (1991, 1992)
- Kinoshita *et al.* (1991) 上記とほぼ等価な算法の提示

初期値に対する誤差依存性、逐次出発・暖機出発

- Saha and Tremaine (1992)
- Ito and Tanikawa (2012)

シンプレクティック修正子

- Wisdom *et al.* (1996)
- Mikkola and Palmer (2000)
- Wu *et al.* (2003)
- Rein *et al.* (2019)

研究黎明期の主要文献

- Dragt and Finn (1976)
- Neri (1985)
- Yoshida (1990a, 1993)
- Forest and Ruth (1990)
- Sanz-Serna (1992)

適応型刻み幅を実装することによる精度向上

- Mikkola (1997)
- Mikkola and Tanikawa (1999a,b, 2013)
- Preto and Tremaine (1999)
- Emel'Yanenko (2007)

階層的刻み幅を実装することによる精度向上

- Biesiadecki and Skeel (1993)
- Skeel and Biesiadecki (1994)
- Duncan *et al.* (1998)

天体の接近遭遇や高離心率系への対応

- [Levison and Duncan \(1994, 2000\)](#)
- [Rauch and Holman \(1999\)](#)
- [Chambers \(1999, 2003\)](#)
- [Emel'yanenko \(2002\)](#)
- [Mikkola and Wiegert \(2002\)](#)
- [Rein and Tamayo \(2015\)](#)
- [Wisdom \(2017\)](#)

剛体の自転運動への応用

- [Touma and Wisdom \(1994\)](#)
- [van Zon and Schofield \(2007\)](#)
- [Laskar and Vaillant \(2019\)](#)

誤差と信頼性に関する議論

- [Fukushima \(2001\)](#)
- [Wisdom \(2015\)](#)
- [Hernandez *et al.* \(2020\)](#)

時間反転に関する対称性

- [慶應義塾大学が公開している講義資料](#)
- [Qiita 上の記事](#)

微分形式、外積

- [東京大学が公開している講義資料](#)
- [個人ブログ上の記事](#)

関連するその他の話題

- [Yoshida \(1990b\)](#) N 体系の全角運動量の厳密保存
- [鈴木 \(1995\)](#) 指数演算子の分解についての解説
- [Makino \(1991\)](#) Hermite 型積分法の最適次数と刻み幅
- [McLachlan and Quispel \(2006\)](#) 幾何学的積分法の全般
- [Hubaux *et al.* \(2012\)](#) 多分割ハミルトニアンの利用例
- [Hernandez and Dehnen \(2017\)](#) 誤差ハミルトニアンに関する詳しい議論

書誌情報

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