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Deformation of Solid Earth: Effect to Milankovitch Cycle

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Deformation of Solid Earth: Effect to Milankovitch Cycle

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In past researches, no one has paid attention to the effect of solid earth deformation on the insolation variation due to the long period change of orbit and tilt of rotational axis according to gravitational perturbation among other planets—i.e. Milankovitch cycle. However, in view of the long time scale($O(10^9)$years), secular change of the earth-moon distance and rotational speed of the earth would have caused the great effect on Milankovitch cycle. Besides, in shorter time scale($O(10^4) \sim O(10^5)$years), amount of ice sheet varies in accordance to glacial and interglacial period and this fact may have some influence to the dynamics of the rotational axis. Here we concentrate on the latter problem and show the possibility of long period oscillation of the earth's obliquity which is derived from orbital calculation including deformation of solid planet, and indicate the probable feedback effect to the solar insolation variation on the earth.

Motivation and Background

Solar insolation on to the surface of the planet has been said to vary periodically and cause long period climate change. Time scale of this cycle is $10^4 \sim 10^6$ years, and it is sure that this cycle was a pacemaker of the glacial/ interglacial cycle in Quaternary on the earth (Hays et al., 1976). This is called Milankovitch cycle and its mechanism has been theorized quantitatively from so early times because it consists of classical celestial mechanics about point masses and rigid bodies. Such system was easy to calculate even at the age when computers were not available. In addition to that, it was geologists and climate researchers that have been interested in Milankovitch cycle and glacial/interglacial cycle, but few of them tried to recalculate Milankovitch theory. Hence the theory of orbital parameter variation is thought to be completed by present (Smart, 1953; Sharaf & Budnikova, 1967; Berger, 1976; Berger, 1978; Berger, 1988).

"Rigid body planet" means that the solid planet does not deform at all, but actually, observational and theoretical evidences begin to indicate that the earth is not a rigid body rather viscous fluid in longer time scale, and deforms because of growth and consumption of ice sheet, in other words, glacial/interglacial cycle (Farrell, 1972; Peltier, 1974; Wu & Peltier, 1982; Rubincam, 1984).Deformation of the planet have significant influences on the rotation and the axial tilt of the planet (Lambeck, 1980). So we have to consider here at least two problems about the deformation of the solid earth:(1)growth and consumption of ice sheets, i.e. glacial/interglacial cycle, (2)dynamic evolution of the earth-moon system. Time scale of the former problem is $10^4 \sim 10^6$ years, and the latter 10^9 years. In this paper we have mainly investigate about the former and made some qualitative discussion about the latter.

Our concept on the earth system is summarized in Figure 1. Until now, it is thought that the change of the axial tilt controls the variation of the surface distribution of the solar insolation and causes growth and consumption of ice sheet. Hence the deformation of the earth will exert a feedback effect on this process. Solid planet deform because of the growth and the consumption of ice sheets, which gives some perturbation to the dynamics of the axial tilt of the planet. Intention of the present research is to inspect the existence of this feedback process and recalculate the insolation variation including this process. This possibility was first advocated by Rubincam (1990) for Mars. He suggested the existence of this feedback process and implied the possibility that the present obliquity of Mars(25.19°) has been determined by the long period (4.6×10^9 years) secular change.

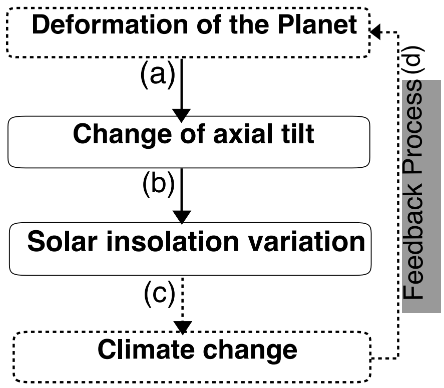

Fig.1. Whole concept of this research. Deformation of the planet changes the axial tilt, which directly influences solar insolation variation. Then the glacial/interglacial cycle is invoked that gives some feedback effect to deformation of the planet. This paper mentions only to the processes expressed by solid lines. Processes of dashed lines will be done in the near future.

Motion of Rotational Axis

To solve the problem, we have to first consider the equation of motion of the rotational axis.

When averaged over both diurnal and seasonal cycles the solar torques acting on the oblate figure of the earth cause its spin axis(characterized by the unit vector $\boldsymbol{s}$) to precess about the instantaneous orbit normal $\boldsymbol{n}$ at a rate give by Bills(1990)

$$\frac{ds}{dt} = \frac{\alpha}{(1-e^2)^{\frac{3}{2}}}(\boldsymbol{s}\cdot\boldsymbol{n})(\boldsymbol{s}\times\boldsymbol{n}) \tag{1}$$

where

$$\alpha = \frac{3n^2}{2D}\frac{C-A}{C}\left(1+\frac{M_m}{M_s}\left(\frac{a_s}{a_m}\right)^3\right) \tag{2}$$

is a rate constant which depends on the orbital mean motion n, the axial spin rate D, polar and equatorial moment of inertia C and A. Here M_m and M_s are the mass of the

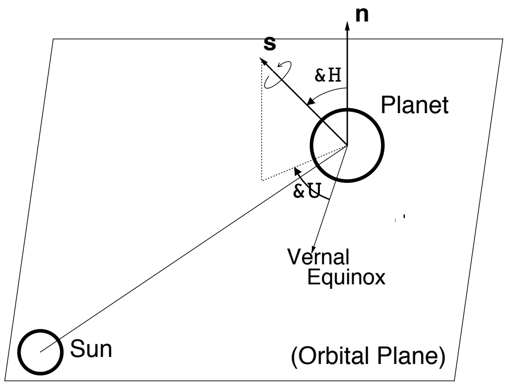

Fig.2. Relationship between θ, ϕ and s, n. Usually ϕ is measured from the orientation of vernal equinox. Thick dashed arrow shows the movement of the planet around the sun.

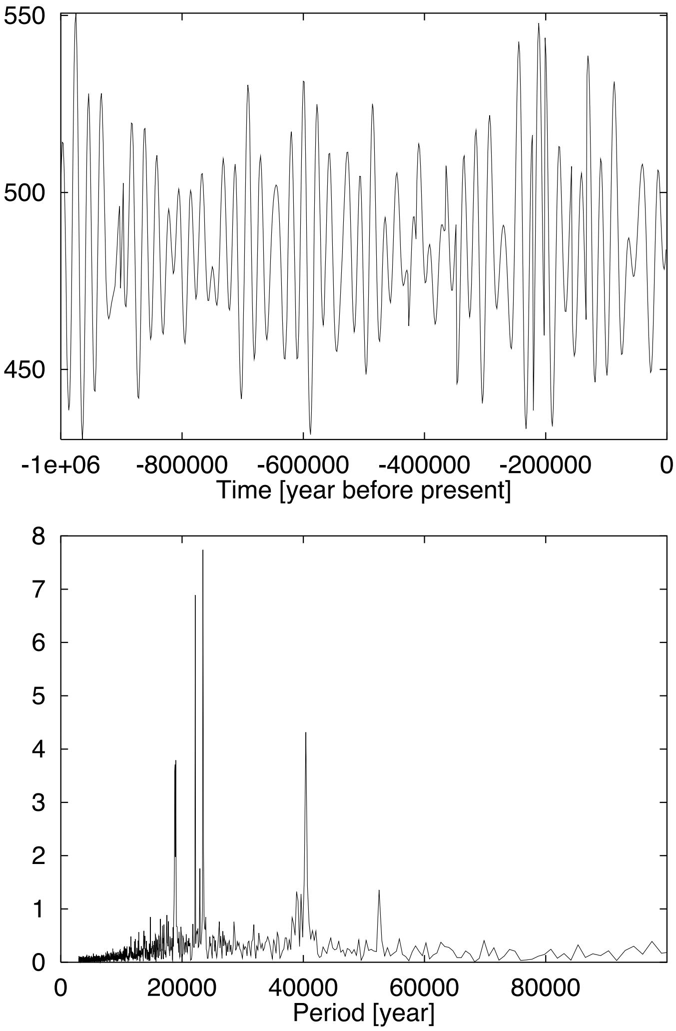

Fig.3. (Upper) daily average solar insolation curve of the earth (65°N, summer solstice). Unit of vertical axis is $\mathrm{W/m^2}$. (Lower) power spectrum of the upper data. Notice on the peak of about 19000 and 23000 years (due to C_1, the precession term) and 41000 years (due to C_0, the obliquity term).

moon and the sun, a_s is the solar distance, and a_m is the earth-moon distance. e is the orbital eccentricity. α is the sum of contribution from the sun

$$\alpha_s = \frac{3G}{2D}\frac{C-A}{C}\frac{M_s}{{a_s}^3}$$

and from the moon

$$\alpha_m = \frac{3G}{2D}\frac{C-A}{C}\frac{M_m}{{a_m}^3}$$

where G is the gravitational constant. Equation(1) is derived from the assumption of a rigid body planet.

As mentioned above, deformation of the planet has not been considered; α has been considered constant in the past researches (Berger, 1976; Sharaf & Budnikova, 1967). It is because n, D, a_s, a_m were thought not to vary very much in the time scale of Milankovitch cycle. But in fact, α changes in shorter time scale due to glacial/interglacial cycle (Figure 5).

In the present paper we implemented the integration of equation(1), taking account of the arbitrary change of α. In fact the change of α is determined by the process (c) in Figure 1, but this process is only known to be nonlinear and fairly difficult to accurately explain (Hays et al., 1976; Pollard, 1983). Besides it is not still clear whether warming of the planet surface due to the increase of incoming insolation is in favor or against the growth of ice sheets (Källén et al., 1979; Wetherald & Manabe, 1975). This process should be investigated independently as another research.

Our final goal is to get insolation variation time series. For this purpose we need to calculate the obliquity θ and the spin axis precession angle ϕ of the earth. Relationship between θ, ϕ and $\boldsymbol{s}, \boldsymbol{n}$ are schematically shown in Figure 2. It is evident that θ and ϕ are expressed as follows:

$$\theta = \cos^{-1}(\boldsymbol{s}\cdot\boldsymbol{n}), \quad \phi = \sin^{-1}\left(\frac{s_y}{\sin\theta}\right)$$

Here we use ordinary orbital parameters e (eccentricity), I (orbital inclination), Π (longitude of perihelion with respect to the fixed vernal equinox), Ω (longitude of the ascending node) and set

$$h = e\sin\Pi, \quad k = e\cos\Pi \qquad (3)$$

$$p = 2\sin\frac{I}{2}\sin\Omega, \quad q = 2\sin\frac{I}{2}\cos\Omega \qquad (4)$$

(Brouwer & van Woerkom, 1950; Bretagnon, 1974; Ward, 1974). These equations express the orbital evolution of each planet. As a solution of above equations we use the result of Laskar(1988).

For the convenience to integrate the equation(1), we first rewrite the components of the orbit normal in terms of the variables p and q of equation(4) following Bills(1990)

$$n_x = p\xi, \quad n_y = -q\xi, \quad n_z = \eta$$

where

$$\eta = \cos I = 1 - \frac{p^2+q^2}{2}$$

$$\xi = \cos\frac{I}{2} = \sqrt{1 - \frac{p^2+q^2}{4}}$$

Using this relation, we find

$$\boldsymbol{s}\cdot\boldsymbol{n} = \xi(ps_x - qs_y) + \eta s_z$$

$$\boldsymbol{s}\times\boldsymbol{n} = \begin{pmatrix} \eta s_y + q\xi s_z \\ -\eta s_x + p\xi s_z \\ -\xi(qs_x + ps_y) \end{pmatrix}$$

Thus equation(1) can be written in the form

$$\frac{d}{dt}\begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \alpha \frac{\xi(ps_x - qs_y) + \eta s_z}{\left(1-(h^2+k^2)\right)^{\frac{3}{2}}} \begin{pmatrix} \eta s_y + q\xi s_z \\ -\eta s_x + p\xi s_z \\ -\xi(qs_x + ps_y) \end{pmatrix} \quad (5)$$

which can be easily integrated. As a check of the accuracy of the numerical solutions, the spin vector $\boldsymbol{s}$ should remain a unit vector. In the integrations reported below, the norm of $\boldsymbol{s}$ derived from unity by less than one part in 10^5 at the end of a 10^7 year integration.

From the solution of equation(5) we can get the average semi-annual insolation amount (Milankovitch, 1930)

$$\begin{cases} Q_s = R_0(B_0 + C_0 - C_1) \\ Q_w = R_0(B_0 - C_0 + C_1) \end{cases} \quad (6)$$

for Northern hemisphere and

$$\begin{cases} Q_w^* = R_0(B_0 - C_0 - C_1) \\ Q_s^* = R_0(B_0 + C_0 + C_1) \end{cases} \quad (7)$$

for Southern hemisphere, where

$$R_0 = \frac{S_0}{n\sqrt{1-e^2}} \quad (8)$$

$$C_0 = \sin|\varphi| \sin\theta \quad (9)$$

$$C_1 = \frac{4e\sin\tilde{\omega}}{\pi}\cos\varphi \quad (10)$$

S_0 is the solar constant at the mean orbital radius, B_0 is called the symmetric term (contribution of the yearly averaged insolation), C_0 the obliquity term (contribution of the obliquity variation) and C_1 the precession term (contribution of the precession variation). φ is the latitude on the earth. For example daily average insolation at 65°N at the summer solstice and its power spectrum is given as Figure 3.

There is one point we should notice on about equation(6) $\sim$ (10). Π is the longitude of perihelion with respect to the fixed vernal equinox and $\tilde{\omega}$ is the longitude of perihelion with respect to the moving vernal equinox, so we can not get the value of $\tilde{\omega}$ through equations (3) $\sim$ (5). To get $\tilde{\omega}$ the relationship between $\tilde{\omega}, \Pi, \phi$ is needed. It is schematically shown in Figure 4. From the spherical trigonometric formulation we get

$$\tilde{\omega} = \Pi + \psi_g = \Pi + \left(\phi - \tan^{-1}(\cos I \tan\Omega)\right)$$

Then the next question is: how does α change with time?

Deformation of the Solid Planet

Here we concentrate our notice on the components of α, especially $\frac{C-A}{C}$. It is quite reasonable because $\frac{C-A}{C}$ expresses the degree of deformation of the planet. For further approximation we use "flattening" $f(\equiv \frac{a-c}{a})$ instead of $\frac{C-A}{C}$, where a, c is the semi-major and the semi-minor axis of the planet. This assumption simulates the nature very well in a case of constant density spheroid and $a \simeq c$(nearly sphere) (Varga, 1975; Stacy, 1977; Turcotte & Schubert, 1982).

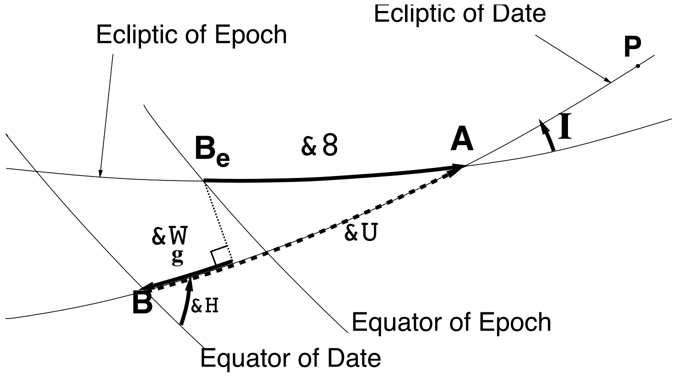

Fig.4. Relationship between $\phi, \tilde{\omega}, \Pi$. A is the ascending node, Be is the vernal equinox of epoch, B is the vernal equinox of date, P is the perihelion, I is the orbital inclination, ψ_g is the general precession in longitude, θ is the obliquity. AP is the argument of perihelion and $\Pi = \Omega +$ AP.

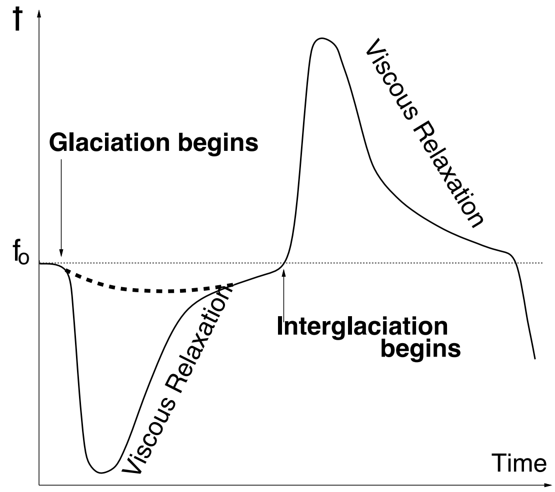

Fig.5. Schematic diagram of the time series of flattening f. Glaciation decreases f and interglaciation increases f abruptly, viscous relaxation process works inversely on both cases. From the Quaternary researches time scale of the occurrence of interglaciation(collapsing of ice sheet) is much shorter than that of glaciation, so f might trace the process expressed by thick dashed line in the glacial period.

Temporal change of f ($\propto \alpha$) with time is expected as shown in Figure 5. Complete three dimensional numerical model of the earth's interior is necessary to calculate accurate change of f (Cathles, 1975; Wu & Peltier, 1982; Rubincam, 1984). In that sense Figure 5 shows only a symbolic image of f, but we can at least say that f oscillates around its average value f_0 in accordance with glacial/interglacial cycle. So we apply a first order consideration at this stage of research.

From the researches of Quaternary, sea level has been changed about 100 meters during the last ice age (Tooley, 1987). This fact indicates the glacial/interglacial cycle has actually changed $\frac{C-A}{C}$ of the planet. Sea level change of 100 meters is equivalent to nearly 30 meters change of solid earth radius. Then we get the amplitude of f

$$\Delta f \simeq \frac{2\Delta a}{a} = \frac{2\times 30(\mathrm{m})}{6400(\mathrm{km})} \simeq 10^{-5}$$

We know f_0 is about $\frac{1}{300}$ so the amplitude of $f(\propto \alpha)$ is $10^{-5}/\frac{1}{300} \simeq 0.3\%$. On the other hand there is no clue

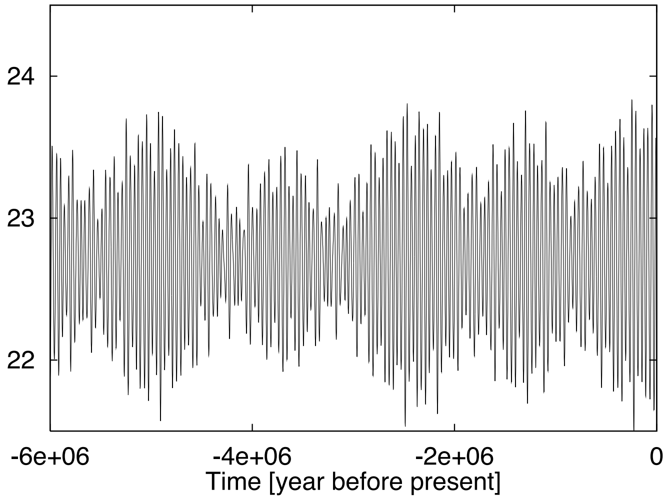

Fig.6. Obliquity oscillation of the earth. This is the standard result(i.e., the result that corresponds to the past researches). Unit of the vertical axis is degree. Notice that the average value($\sim$ 22.8°) is constant all through the integration time. Thought this average value seems somewhat low compared to other papers(Berger 1974, Ward 1979), it is not essential in the discussion here which is focused on the frequency domain.

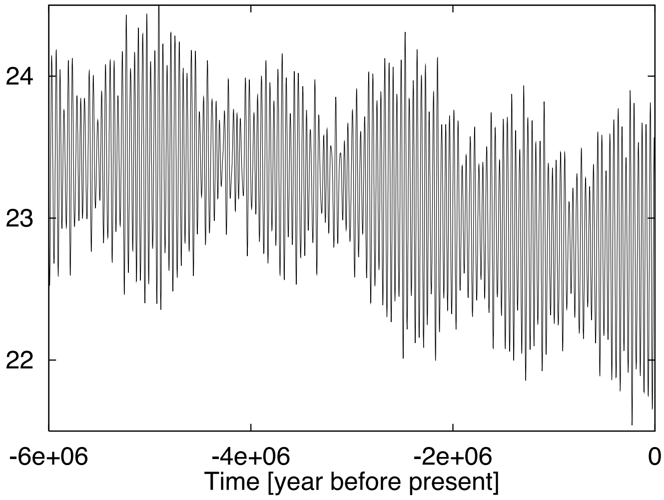

Fig.7. Obliquity oscillation of the earth like Figure 6, but this time flattening f is forced to oscillate by the period of 41000 years(main frequency of obliquity oscillation). There seems evidently very long period trend above the standard result. It is thought resonance between the eigenfrequency of the system and that of forced oscillation.

about viscous relaxation time constant. However viscous relaxation time lag ζ does make sense in a closed system such as shown in Figure 1, so in this case(forced oscillation of the earth's figure) ζ has no realistic meaning. We set $\zeta = 0$ here.

From the above discussion, we set α

$$\alpha = \alpha_0 \left(1 + \Delta\alpha \sin(\omega t + \zeta)\right) \tag{11}$$

where α_0 is the equilibrium value(corresponding to f_0), $\Delta\alpha$ is the maximum amplitude($\simeq$ 0.3% of α_0), ω is the oscillation period(parameter). We take sinusoidal form oscillation instead of saw-type oscillation shown in Figure 5 for the convenience of numerical calculation.

Our final purpose is to get insolation variation time series, so, substituting (11) into (5), integrating (5) by using fourth-order Runge-Kutta method and translating $\boldsymbol{s}$ into θ and ϕ and apply them to (6) $\sim$ (10). Results are discussed in the next section.

Results and Discussion

Obliquity of the earth have been known to oscillate in a period about 41000 years(Figure 6 and Figure 8) (Milankovitch, 1920; Vernekar, 1972; Berger, 1976). Our calculations also confirm this fact. Frequency of deformation of the solid earth ω is a parameter and we don't know exact value of it. But notice that ω is invoked by glacial/ interglacial cycle, and this fact suggests the possibility that typical frequency of obliquity $\theta(\sim 2\pi/41000(\text{years}))$ and ω are similar to each other. So as possible parameter value we set $\omega \simeq 2\pi/41000$, and got astonishing result shown in Figure 7 and Figure 8. When we set ω at this value there appears evidently very long period oscillation on the obliquity time series. The period of this oscillation reaches up to 15 million years, which has never been pointed out in the past researches. At other values of ω than $2\pi/41000$, this low frequency dose not appear at all. On the other hand spin axis precession angle ϕ is rather dull about the effect of ω and we could not find any long

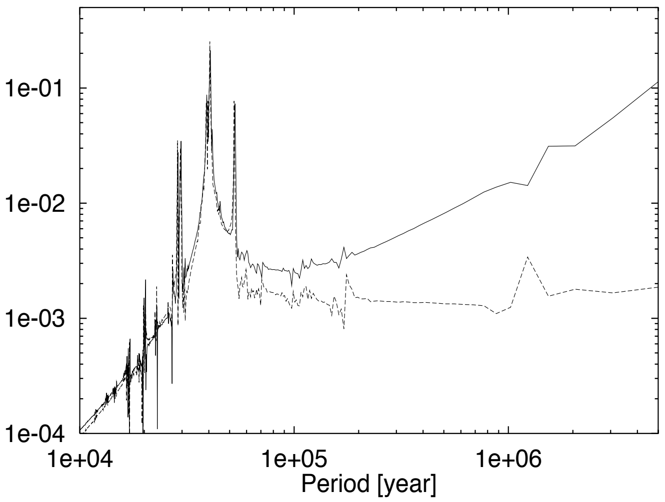

Fig.8. Power spectrum of obliquity oscillation of the earth. Solid line corresponds to Figure 4 and dashed line Figure 4. Be cautious that in both axis data are plotted in log scale unlike other papers. It is intended to emphasize the long period component.

period oscillation in the time series of ϕ. Insolation variation curve using this result is shown as Figure 9.

What does this fact mean? Actually the answer to this question is—we don't know. This extraordinary long period oscillation is the product of the resonance between the eigenfrequency of the system and the frequency of forced oscillation(both have typical period of 41000 years). However, whether this long period trend has any reality or not is still not clear. What is important here is that the system we consider is not a closed one. To close the system in Figure 1 we have to solve another two problems: first, the process of ice sheet growth and consumption (process (c)); second, quantitative calculation of deformation of the solid earth due to glacial/interglacial cycle (process (d)). The latter is relatively easy and many researchers are committed in this problem. Detailed models of the viscoelastic earth and the methods of space-time convolution for solving the dynamic deformation of the planet

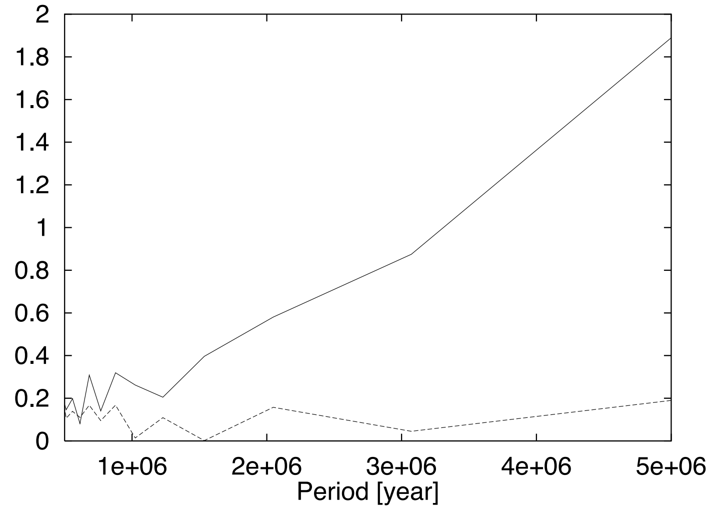

Fig.9. Solid line is the long period trend of the solar insolation variation (the situation is same as Figure 3) corresponding to Figure 8 upper. Dashed line is the same plot as Figure 8 lower (standard result). Notice that the range of both axis isdifferent from Figure 8 (especially the horizontal axis is from 5×10^5 to 5×10^6 years) in order to emphasize the long period oscillation pattern.

are available(Peltier, 1974; Wu & Peltier, 1982; Cathles, 1975). On the other hand the former is fairly difficult, and this difficulty is derived from nonlinearity which the earth's climate system has (Moriyama, 1986; Källén et al., 1979), so it is necessary to simplify the whole system to close the feedback loop and to get ultimate results.

We have to comment a word about the effect of the earth's deformation on Milankovitch cycle in longer time scale ($O(10^9)$ years). Dynamic evolution of the earth-moon system has been researched for a long time. These researches have made clear that the distance between the earth and the moon is becoming larger and rotational speed of the earth is becoming slower because of tidal friction. Besides, oblateness of the earth was much larger in ancient times than now, for f of the viscous self-gravitational liquid satisfies the relationship $f = \frac{5a^3D^2}{4Gm}$, where m is the mass of the liquid (Turcotte & Schubert, 1982). Using the observational data (Williams, 1990) and the theory (Turcotte et al., 1977) to get the value of a_m and D, we can expect the typical period of ancient Milankovitch cycle as Table 1. But this is merely a qualitative discussion and there are many problems to achieve to the final goal (for example, in the calculation of making Table 1 we ignore the inclination of the moon's orbit against the earth's orbit: Initial values of each calculation(present, 0.65Ga ago, 2.5Ga ago) are all the same for the convenience of numerical integration.). They remain as future subjects.

Finally we suggest another interesting application of this theory. As compared to the earth, variations of orbital element of Mars is fairly large (see Table 2) (Murray et al., 1973; Ward, 1974; Ward, 1979). In addition to that Mars has no large satellite such as moon and the oblateness of its figure is enormous. As resulted from these facts, axial tilt of Mars and its variation amplitude are quite large(see Figure 10). Besides, the climate system of Mars is expected to be very simple because of the lack of ocean, so the influence of Milankovitch cycle will be overwhelming. There are also ice sheet called "polar cap" which con-

	−2.5Ga	−0.65Ga	Present
length of day (h)	19.7	21.9	24.0
flattening $f(\times 10^{-3})$	4.98	4.03	3.35
frequency of θ (Ka)	2.4	3.8	4.1

Table 1. Rotational speed, flattening and the excellent frequency of the obliquity oscillation at the present, 0.65Ga ago, 2.5Ga ago. Physical values of the earth-moon system is from Williams (1990) and Turcotte (1977). Actually this table shows only a qualitative trend and the each value is quite rough estimation. 2.5Ga ago is the age of banded iron formation and 0.65Ga ago is thought to be in a great ice age.

sists of solid CO_2, and maybe glacial/interglacial cycle exists(Carr, 1987). Several accurate numerical models of atmosphere and energy balance of Mars is submitted(Fanale, 1975; Tanaka & Abe, 1991), it is possible to build a full model of climate system of Mars. Such a models is far from realization in the case of the earth. If examining the reality of above feedback mechanism it may be better to choose Mars as object rather than the earth.

Conclusion and Future Prospect

Although reality is not still to be examined, possibility of long period oscillation of the earth's obliquity were indicated. If it is true, variation of solar insolation theory—Milankovitch cycle—must be modified very much and so the researches about glacial/interglacial cycle in Quaternary.

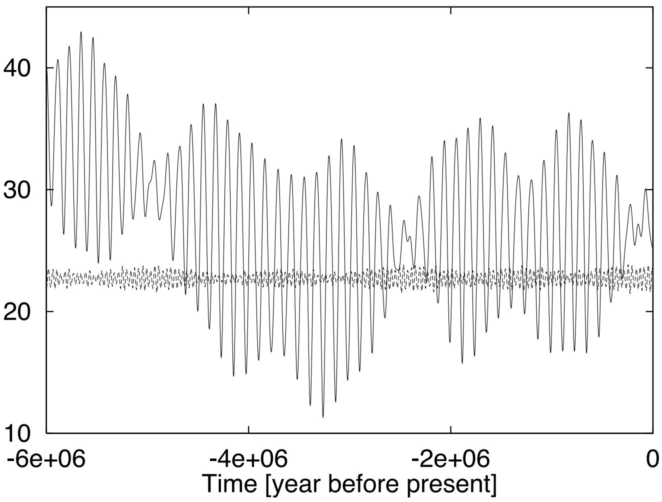

Fig.10. Obliquity oscillation of Mars in contrast to the earth. Solid line express the obliquity of Mars, dashed line the earth. Unit of the vertical axis is degree, but it is noticeable that the range of vertical axis is about ten times larger than Figure 5 and Figure 6. As mentioned in text, orbital element variation of Mars is quite large and it is reflected on the huge amplitude of the obliquity oscillation.

In the past, climate research was a patent of meteorology and geology. But actually climate system contains almost all phenomena on the planet from the interior structure to the orbital element variation. It will be more and more important from now on to have such "planetary" viewpoint in climate research. This paper is a reconnoitering scout for this purpose.

Acknowledgments. The author has benefitted from several stimulating discussions with Yutaka Abe, Masanao Abe, Teru-

	Mars	Earth
eccentricity e	0.0934	0.0167
inclination $I(^\circ)$	1.851	0.002
flattening f	6.117×10^{-3}	3.353×10^{-3}

Table 2. Orbital elements, flattening of the earth and Mars. Data are from Turcotte(1982) and Ward(1979).

yuki Nakajima, Takafumi Matsui, and Kiyoshi Kuramoto. Detailed and constructive reviews by Yozo Hamano and Kooiti Masuda have considerably improved the presentation of this paper. The calculation were performed on the SUN4/50 of Department of the Earth and Planetary Physics, Faculty of Science, University of Tokyo. This paper was presented at the 24th ISAS Lunar and Planetary Symposium on August 4, 1992.

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