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Numerical experiments to inspect the long-term stability of the planetary motion (−1)

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Numerical experiments to inspect the long-term stability of the planetary motion (-1)

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Stability of the solar system has been a terribly challenging problem over several hundred years. We performed long-term numerical experiments to inspect the stability of the solar system by using a fast computation algorithm, mixed variable symplectic integrator. Our results show that the inner planets, as well as outer planets, exhibit the macroscopic stability over the timespan of $\pm 2.5 \times 10^8$ years; variational amplitudes and mean values of the orbital elements are almost the same during the integration period, though the system appear chaotic in the viewpoint of Lyapunov characteristic exponent. From now on we have to investigate *how* stable the solar system is, and *what* made the solar system so stable, by using some kinds of semi-analytical numerical simulations.

Introduction

Although stability of the solar system has been discussed over several hundred years, since the era of Newton, we do not yet have any definite answer whether it is stable or not. It is partly due because the definition of the phrase "stability" is quite vague when it is used on the problem of planetary motion in the solar system. Actually it is pretty difficult to give a rigorous definition of the stability of the solar system, and as Poincaré showed a century ago, there is no quasi-periodic solution of the equations of motion of the planets over the infinite timespan. Besides, the planetary motion of the solar system is recently being reputed chaotic (Sussman & Wisdom, 1992; Laskar, 1990), and there occurs terrible confusion that "since planetary system is chaotic, it is not stable."

However, there seems to be no explicit relationship between the stability and the chaotic character of the planetary motion (Kinoshita & Nakai, 1995). And more, only we want to know is the stability of the finite timespan, $\pm 5 \times 10^9$ years. So in this paper, we only concentrate on the variational width of the orbital elements, especially eccentricity e and inclination I. For example, if the variational width of the eccentricity Δe (maximum e − minimum e during the period T_1 from the present) is not so different from Δe after a long interval T_2 ($T_2 \gg T_1$), and the mean value of e does not vary much during T_2, we say that the motion of the planet is stable (of course the inclination I must satisfy such conditions). In this paper, we put the total integration period $T_2 = \pm 2.5 \times 10^8$ years, and performed numerical experiments of the gravitational few-body problem to inspect the stability of the solar system planets. In the case of outer planets it is already well done (*cf.* Kinoshita and Nakai (1995), $\pm 5.5 \times 10^9$-year calculation has been done), and it end up with that the outer planetary system is completely stable in the timespan of the age of the solar system (Fig. 1). However, the case of inner planets has not been fully inspected due to its heavy task of numerical computation.

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Methods

Physical models we have used to describe the planetary system is the simplest one; planets are represented by point masses, and only classical gravitational force is considered. We took nine planets from Mercury to Pluto into account. Effect of the moon is restricted and bunched into the mass of the earth. Initial conditions are taken from the development ephemeris of JPL, DE245 (*cf.* Standish (1990)). Detailed configuration of our numerical experiments are summarized in Table 1.

As the numerical integration method to solve the equations of motion, we utilized MVS, mixed variable symplectic integrator (Kinoshita *et al.*, 1991; Wisdom & Holman, 1991). MVS utilizes the separation character of Hamiltonian $H = H_{\mathrm{Kepler}} + H_{\mathrm{interaction}}$, and is quite suitable integration method for the actual planetary motion which is very close to the Keplerian problem (i.e. perturbation between planets is very small). It is an order of magnitude faster than the former integration methods. Low-pass filtering to smooth out the short periodic terms (Quinn *et al.*, 1991) is not used.

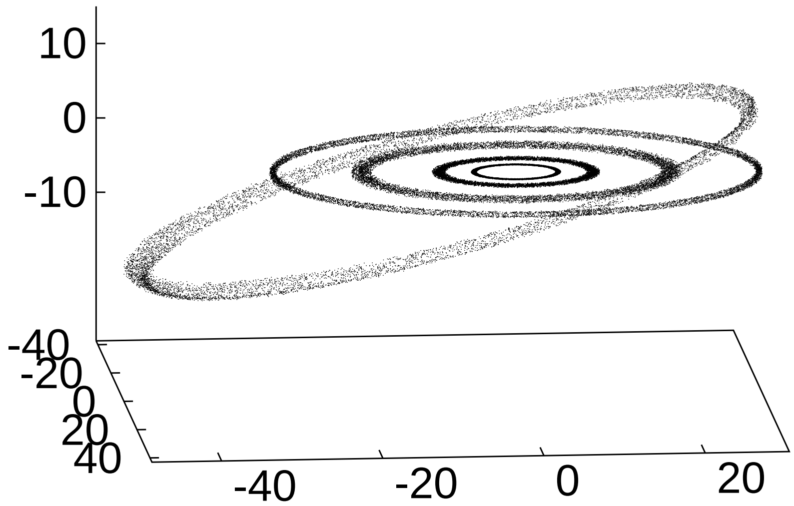

Figure 1. Plots of the orbits of outer five planets during past $\pm 5.5 \times 10^9$ years (Kinoshita & Nakai, 1995). Ascending node of Pluto is fixed on the x-axis in this figure. From innermost, Jupiter, Saturn, Uranus, Neptune, and Pluto respectively. Spatial unit of each axis is AU. Time interval of each plot is 2×10^7 days (about 54757 years).

Table 1. Detailed configuration of our numerical experiments.

Objects	nine planets (Mercury to Pluto) and Sun
Physical models	point masses, only classical gravitation
Earth-moon system	point mass on baricenter of the earth-moon system
Integration period	about $\pm 2.5 \times 10^8$ years
Integration method	MVS, second order
Time step	10 days
Initial conditions	DE245
Solver of Kepler equation	Halley's method, third order
Output interval	200000 days (about 547 years)
Output filtering	none
Machines	JCC JP4/133 (PowerPC 604 133MHz)
Language	ANSI C
Total calculation time	about 6 months
Data amount	about 2Gbyte

Results

Results of our calculations are shown form Figure 2 to 10. In a word, planetary systems, both outer and inner, are quite stable in the sense we mentioned before. It is typically shown in the frequency domain, Figure 7 to 10. Though you can see some peaks in the high frequency (short periodic) region, typical frequencies which characterize the quasi-periodic motion of the planets indicate little changes of over the integration period. In addition, they well agree with the results of the secular perturbation theory of Laskar (1985, 1986, 1988).

Error estimation

Total energy and angular momentum are very well preserved when we use the symplectic integrators. In our calculations, relative error of total energy $\frac{dE}{E_0} = O(10^{-10})$ and relative error of angular momentum $\frac{dL}{L_0} = O(10^{-14})$. As for the error estimation of the longitude, we performed a time-reversal numerical experiment for 300000 years. A measure calculation is done using a time step of 0.125 days, and the result of time-reversal test for this measure calculation is $\Delta l_{\mathrm{Earth}} \simeq 0.0015°$ on the mean longitude of the earth, which can be extrapolated to $\Delta l_{\mathrm{Earth}} \simeq 25°$ for 5 Gyr calculation. Using this result, we can estimate the longitude error of main experiments for 5 Gyr calculation as $\Delta l_{\mathrm{Earth}} \simeq 20$ rotations. Similarly, longitude error of Pluto can be estimated as $\Delta l_{\mathrm{Pluto}} \simeq 12°$, which is much better than that of Kinoshita and Nakai (1995) where $\Delta l_{\mathrm{Pluto}} \simeq 60°$.

Although error of 20 rotations seems rather large, what we are interested in is not the accurate positions of the orbits but the variational amplitudes of the elements. In this viewpoint, longitude error of several tens of rotations may be tolerable in our calculations.

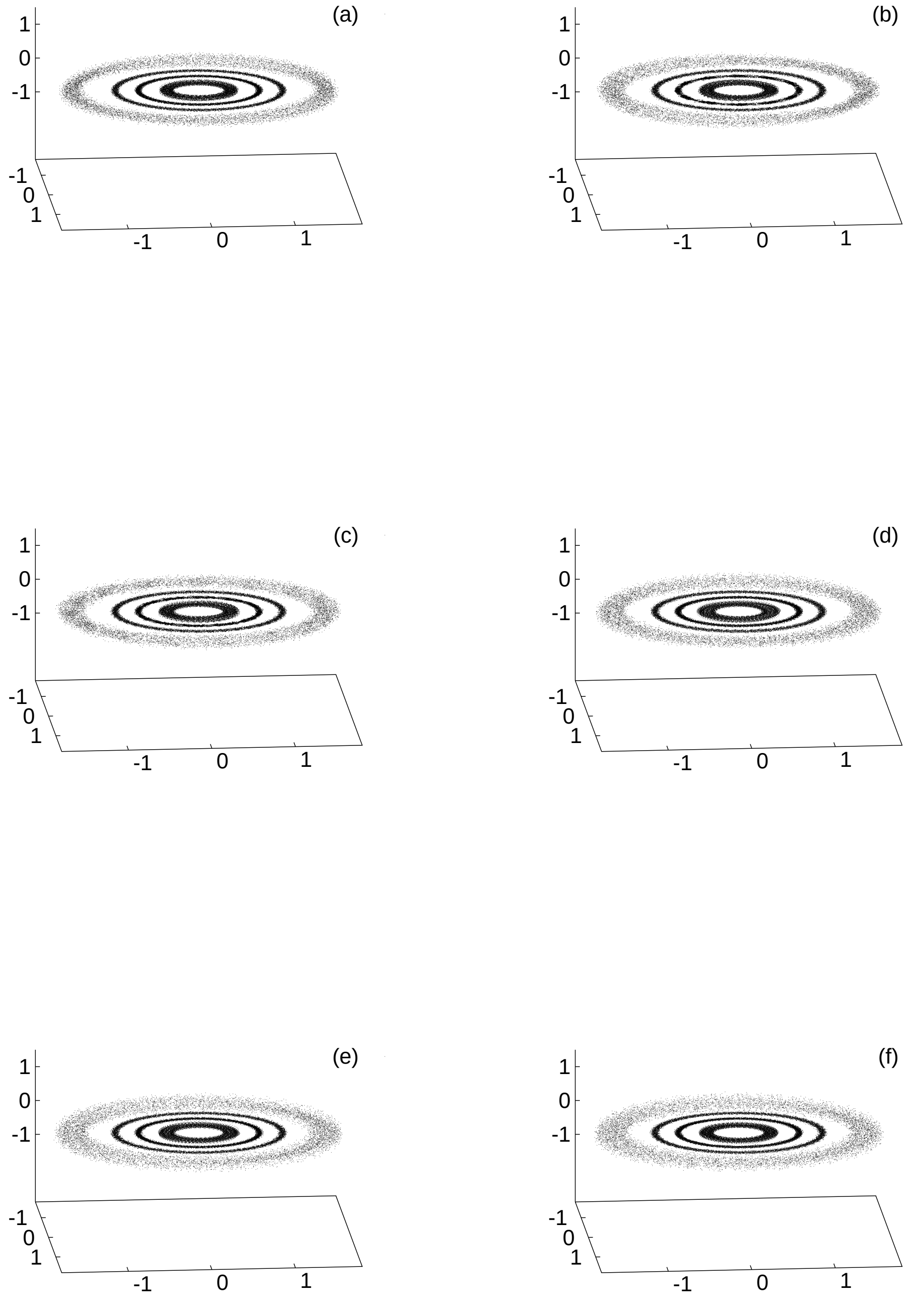

Figure 2. Plots of inner four planets for each five million years. From innermost, Mercury, Venus, Earth, and Mars, respectively. (a)+50 million years after present, (b)present, (c)−50 million years ago, (d)−100 million years ago, (e)−150 million years ago, (f)−200 million years ago. Spatial unit of each axis is AU. Time interval of each plot is about 20000 years.

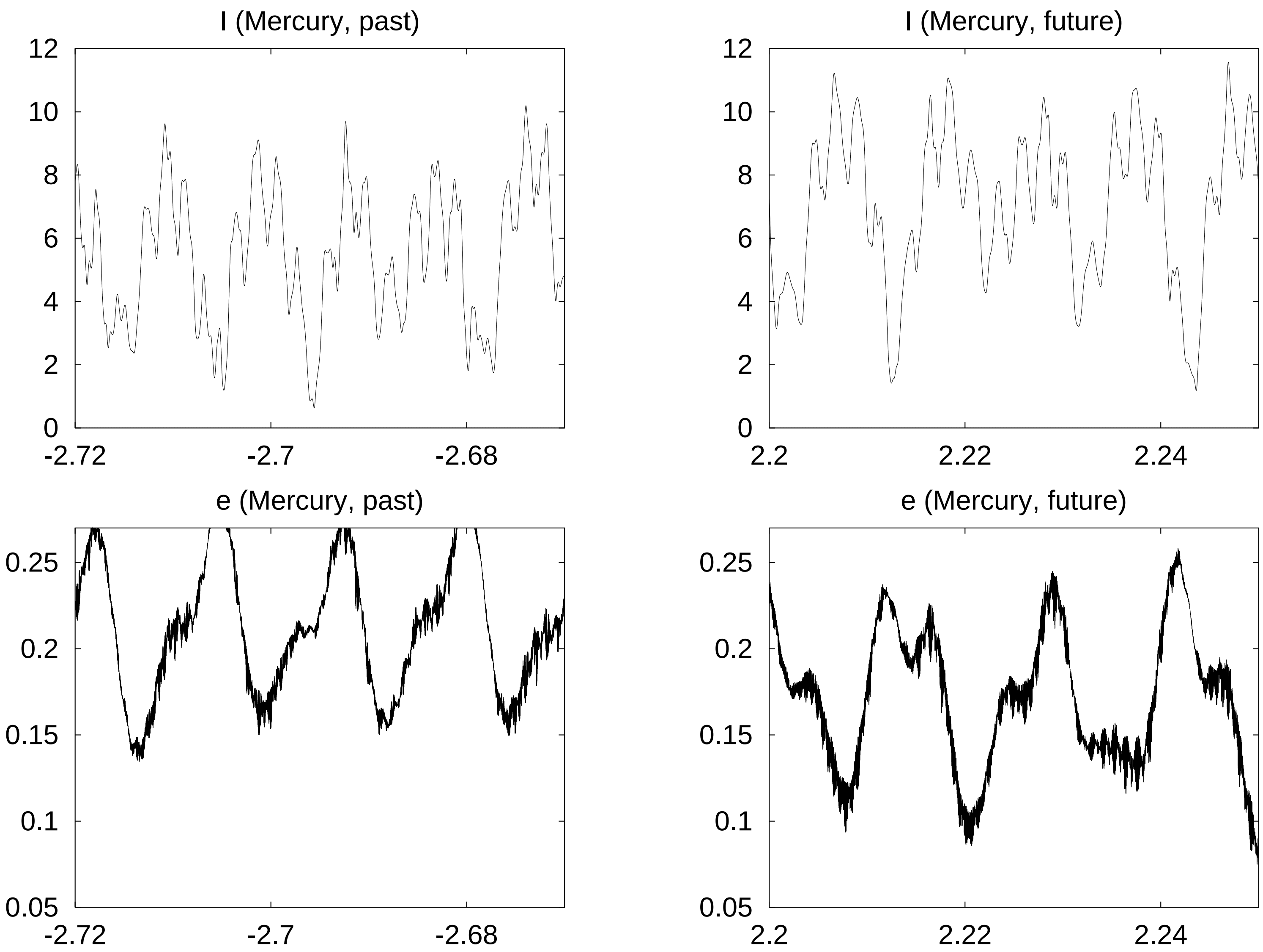

Figure 3. Plots of eccentricity and inclination of Mercury, 220 myr future and -270 myr past. Time unit is 10^8 years, and the unit of inclination I is degree.

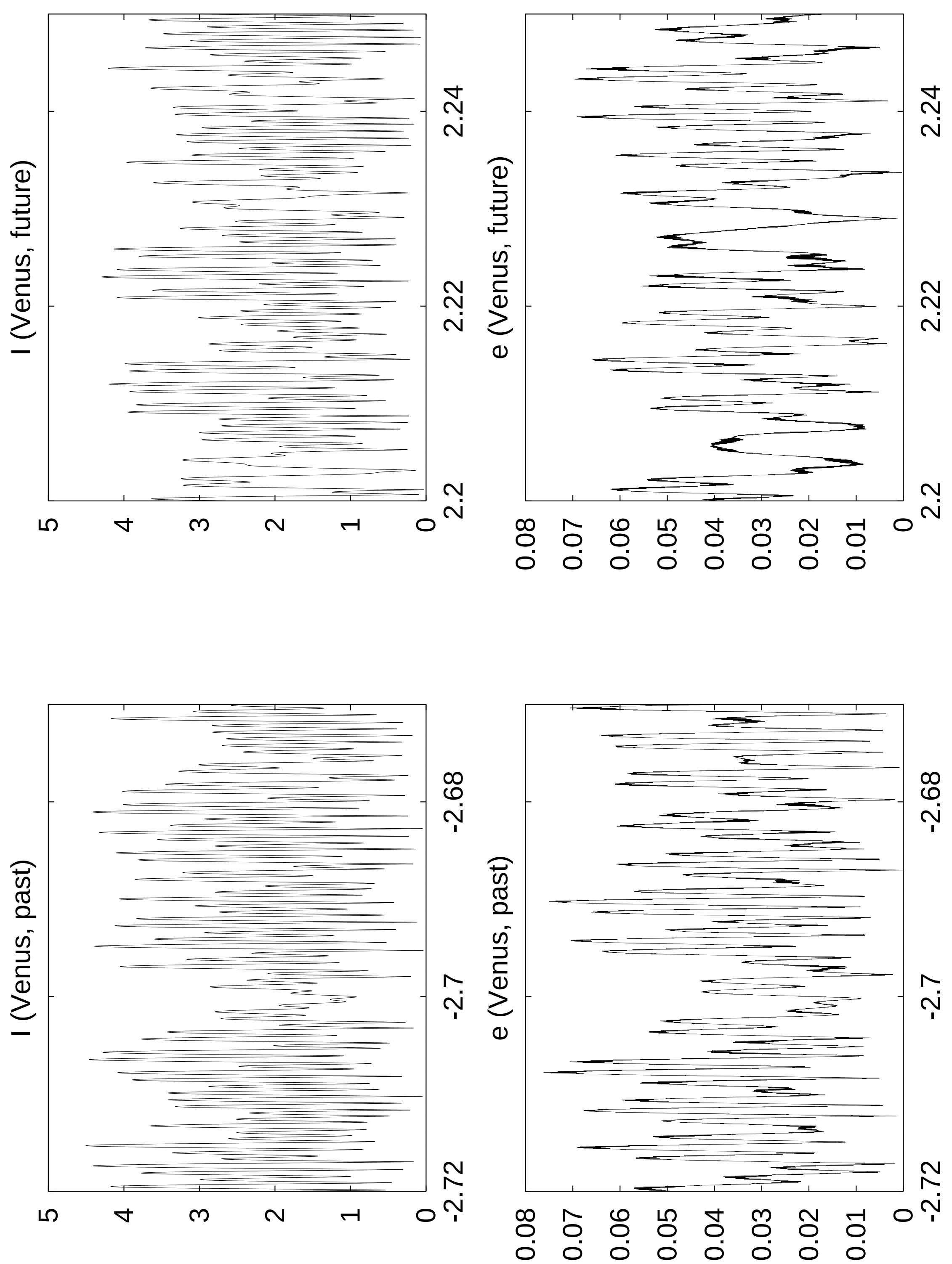

Figure 4. Plots of eccentricity and inclination of Venus, 220 myr future and -270 myr past. Time unit is 10^8 years, and the unit of inclination I is degree.

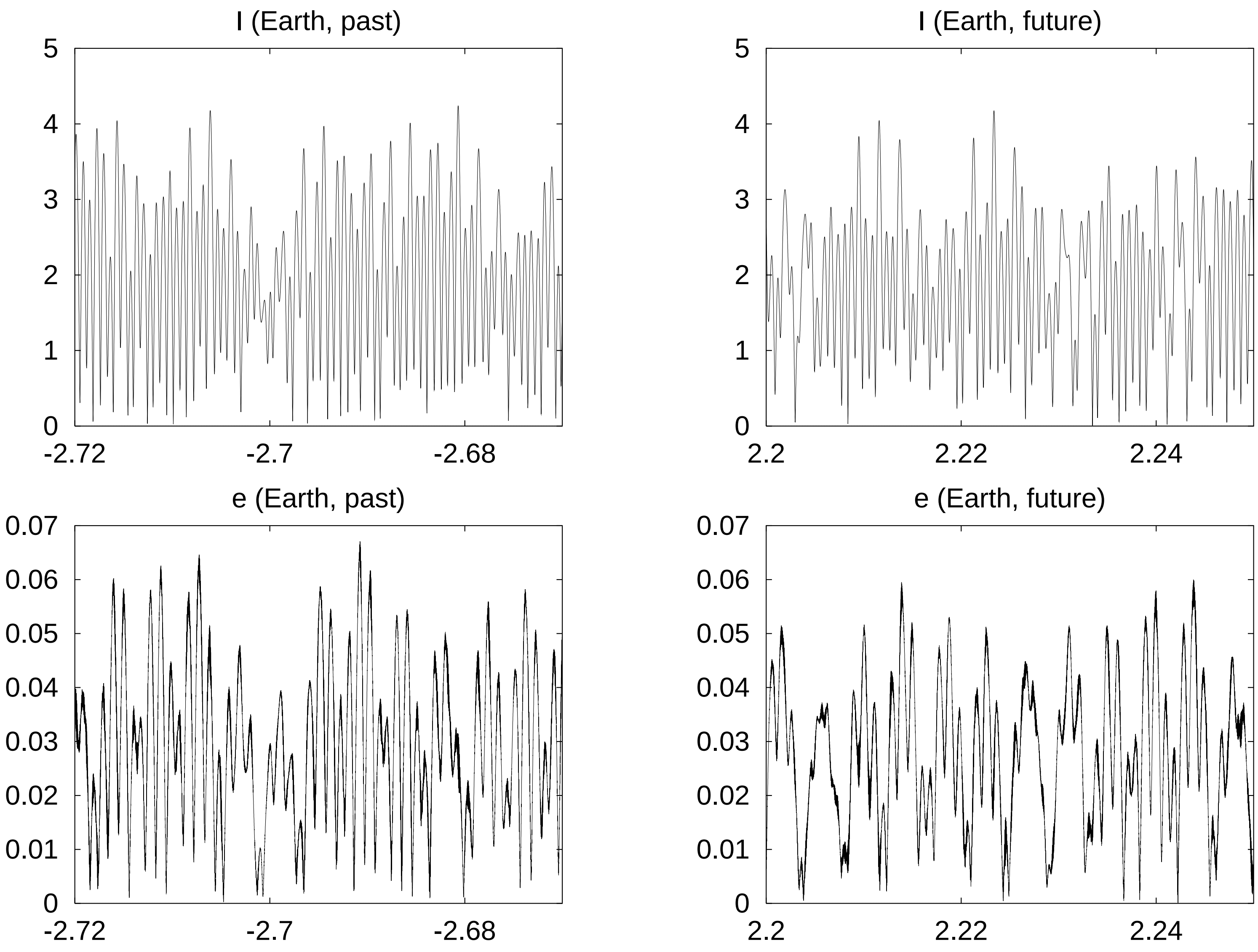

Figure 5. Plots of eccentricity and inclination of Earth, 220 myr future and −270 myr past. Time unit is 10^8 years, and the unit of inclination I is degree.

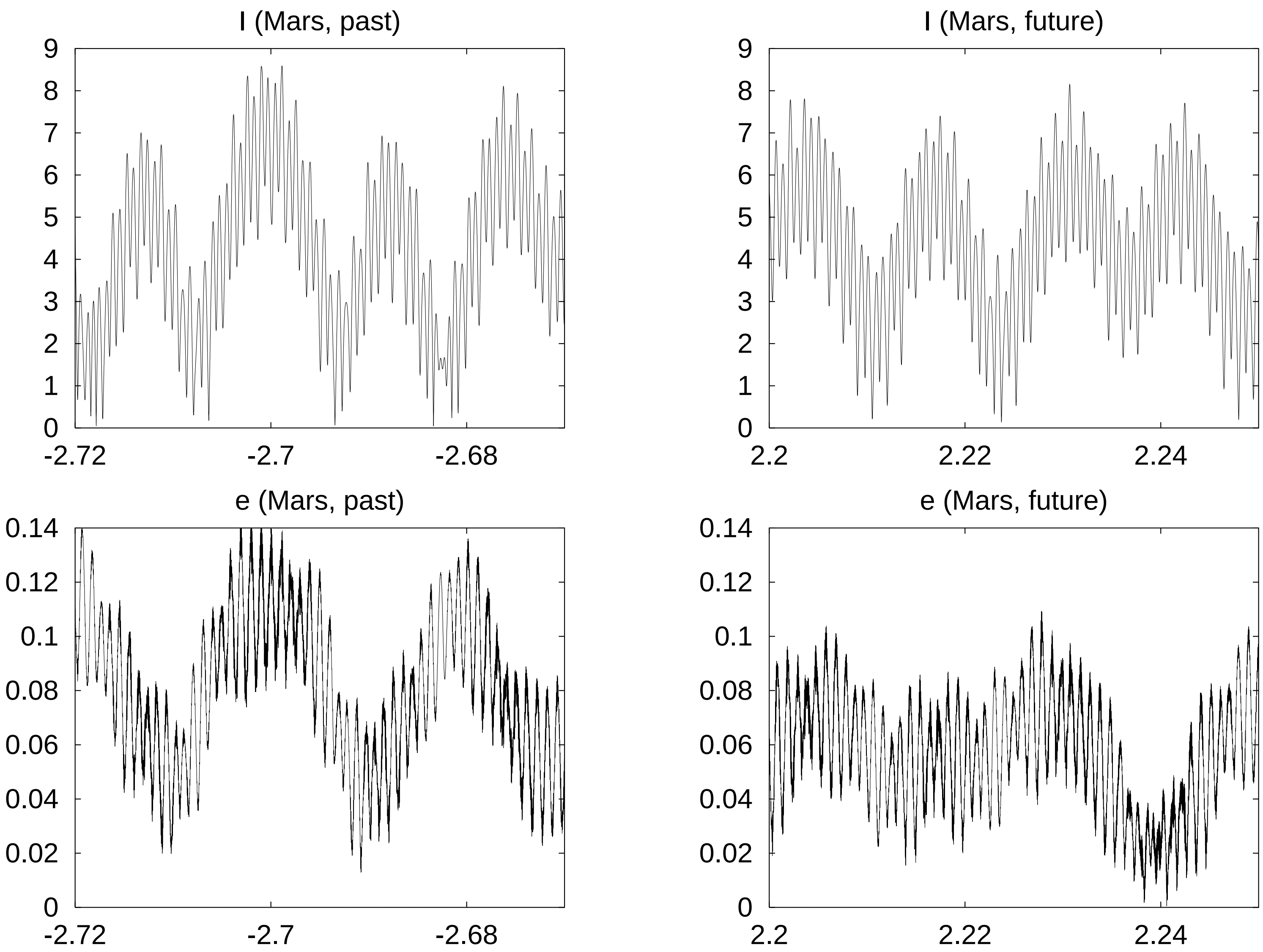

Figure 6. Plots of eccentricity and inclination of Mars, 220 myr future and -270 myr past. Time unit is 10^8 years, and the unit of inclination I is degree.

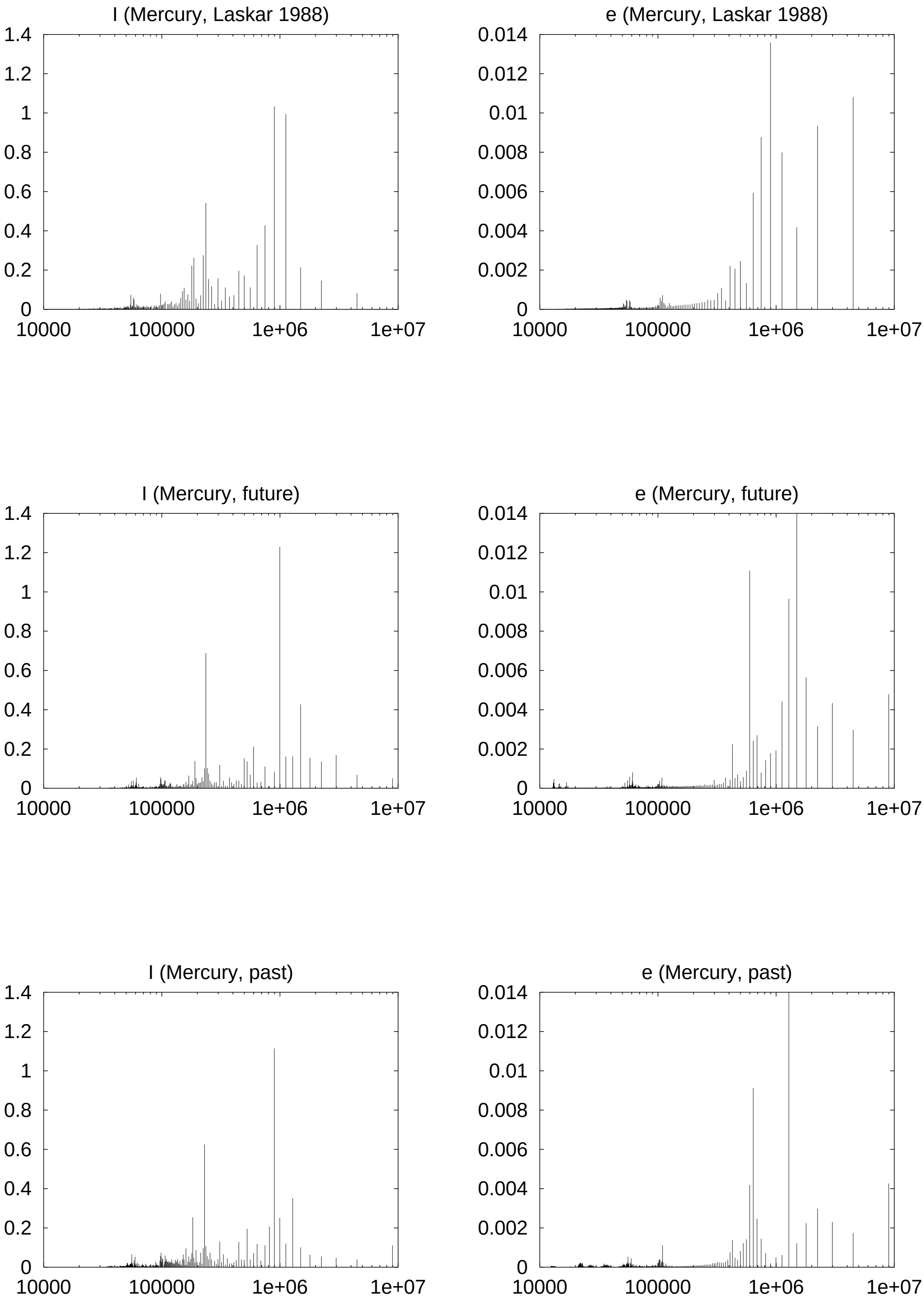

Figure 7. Spectrum diagram of eccentricity and inclination of Mercury. Horizontal axis denotes period (years). Top ones are the results of secular perturbation theory of Laskar (1988). Middle ones are +220 myr future, and bottom ones are −270 myr past. Duration of data acquisition is about five million years.

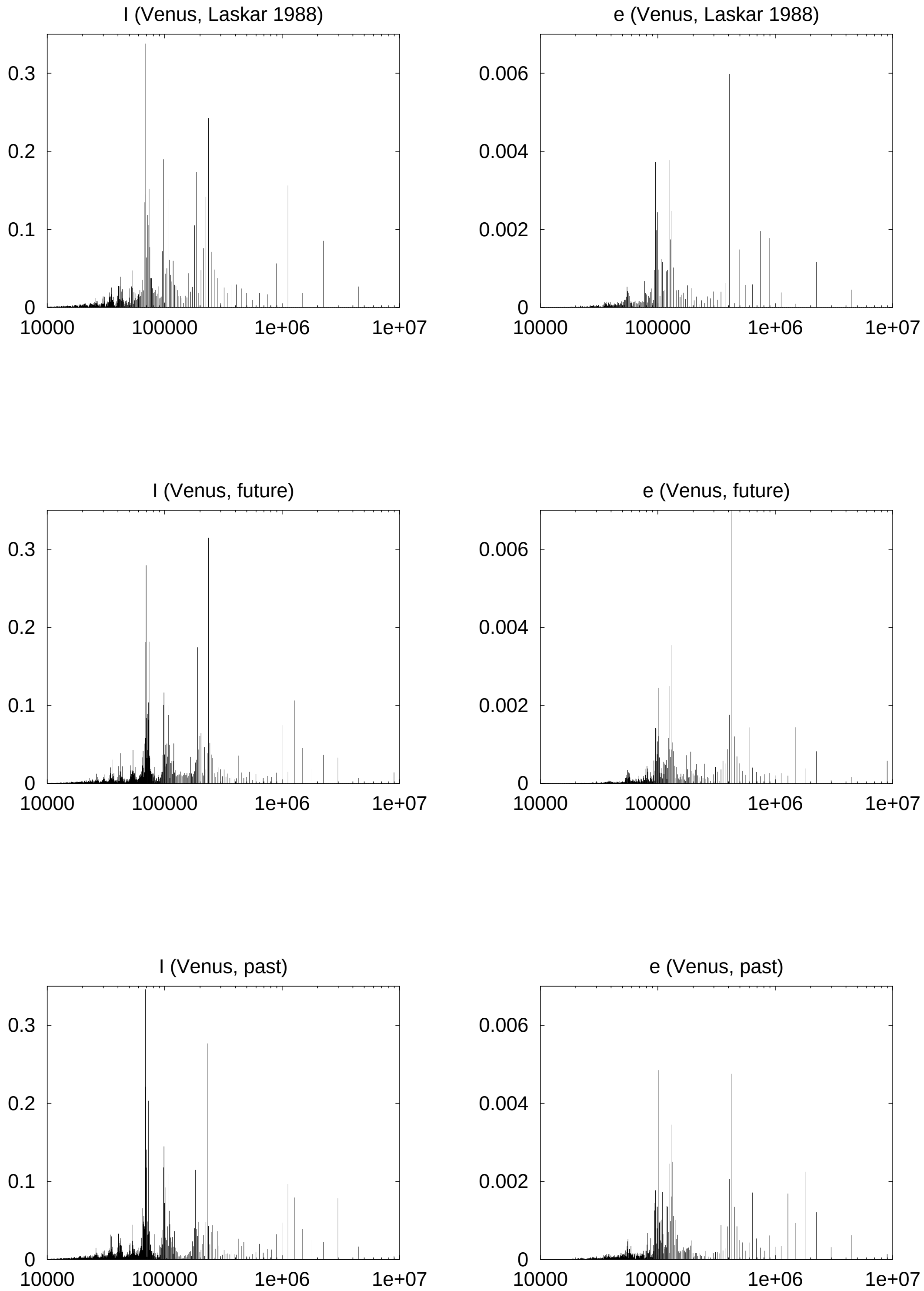

Figure 8. Spectrum diagram of eccentricity and inclination of Venus. Horizontal axis denotes period (years). Top ones are the results of secular perturbation theory of Laskar (1988). Middle ones are +220 myr future, and bottom ones are −270 myr past. Duration of data acquisition is about five million years.

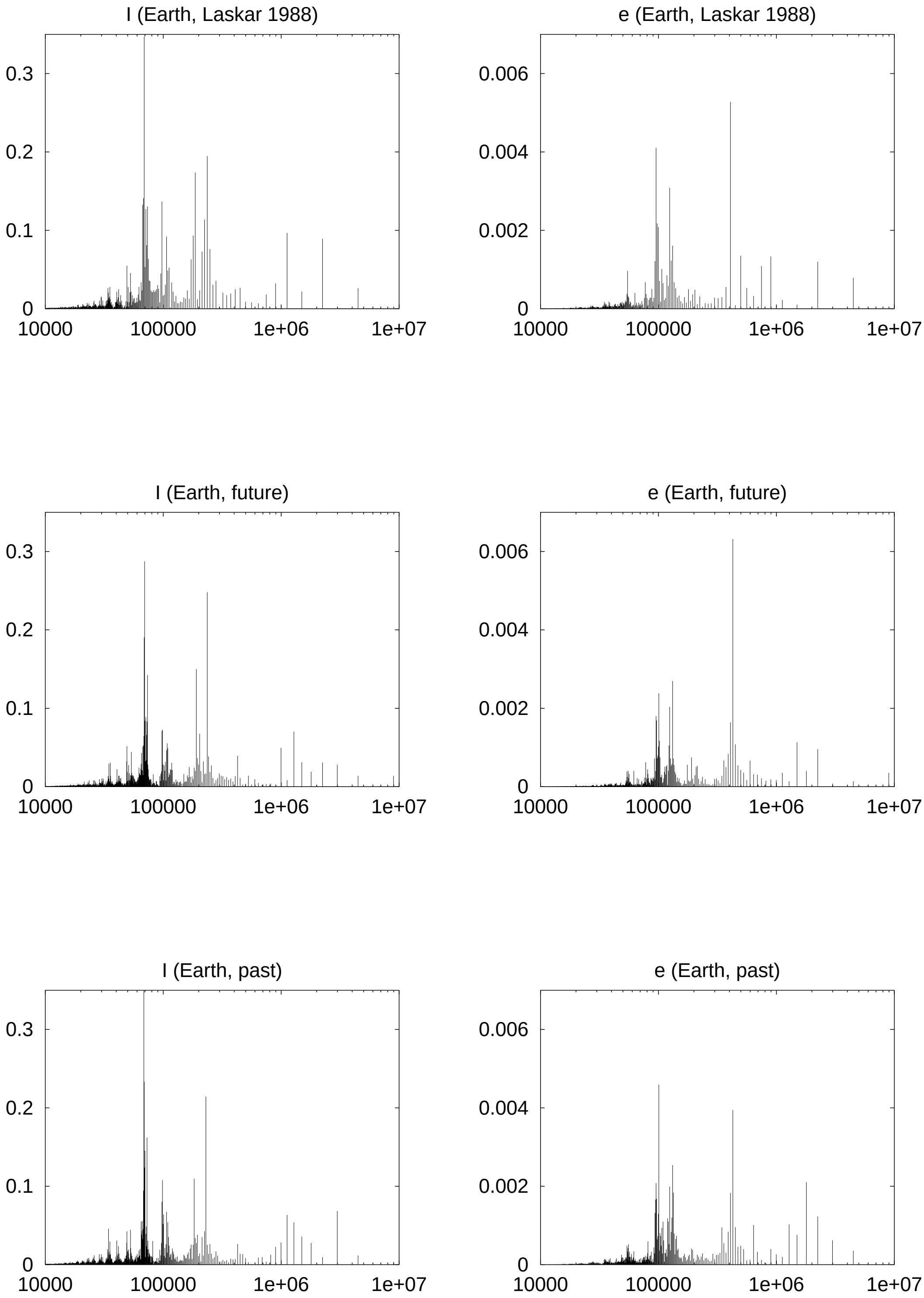

Figure 9. Spectrum diagram of eccentricity and inclination of Earth. Horizontal axis denotes period (years). Top ones are the results of secular perturbation theory of Laskar (1988). Middle ones are +220 myr future, and bottom ones are −270 myr past. Duration of data acquisition is about five million years.

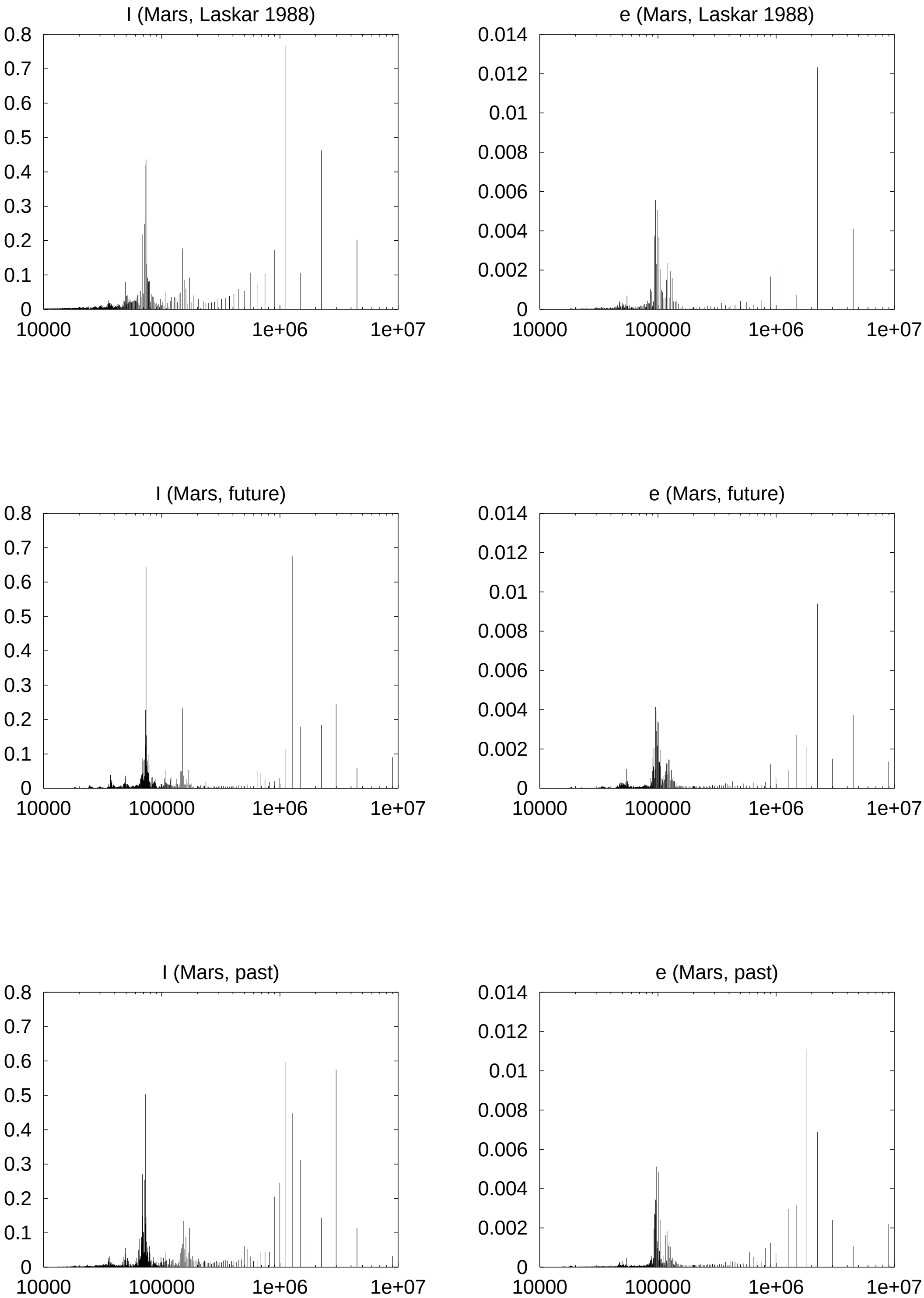

Figure 10. Spectrum diagram of eccentricity and inclination of Mars. Horizontal axis denotes period (years). Top ones are the results of secular perturbation theory of Laskar (1988). Middle ones are +220 myr future, and bottom ones are −270 myr past. Duration of data acquisition is about five million years.

Discussion

From the long-term numerical calculation described here, we can conclude that the planetary motions (both inner and outer) are macroscopically stable, at least in a timespan of $\pm 2.5 \times 10^8$ years. We have done two kinds of experiments using different kinds of initial condition, and the results are almost the same; though the system is chaotic in the viewpoint of Lyapunov characteristic exponent, variable ranges and the mean values of each orbital element are virtually the same during the integration period, i.e., the system is macroscopically stable.

From now on we have to tackle with remaining problems. First, physical models must be improved toward more realistic ones. Effect of the general relativity, tidal force from the moon should be included in the next calculation. Second, it is necessary to speed up the numerical computation. Since a $\pm 2.5 \times 10^8$-year calculation took over half a year using the fastest workstation we can use here, $\pm 5 \times 10^9$-year calculation requires several years, which is not durable. In addition, we must not degrade the accuracy of the calculation. Though these two are ambivalent tasks for any numerical experiments, it is necessary to achieve them to decode the dynamical evolution of the solar system.

There are some important points to notice. From these calculations, what we can get is the information of the ancient (or future) figure of the solar system under the *present* boundary conditions. We do not know *how* stable it is, nor *why* it became so stable, even if the calculational results show the macroscopic stability over a very long timespan. We are now planning to perform new experiments to inspect how stable the planetary system is by using the semi-analytical secular perturbation theory (Kozai, 1985; Nakai & Kinoshita, 1985; Yoshikawa, 1989). And more, our ultimate interest reaches to the question why the solar system became so stable, or in another word, what the making process of such a stable planetary system was. Answers to these question are beyond our hands for now, but numerous researches are now started in various ways (*cf.* Gladman (1993)). It is also quite significant to solve the problem of the stability of the planetary system in view of the existing probability of terrestrial planets on which life can survive and evolve.

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