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Evolution of the Earth's Obliquity and the Role of Core-Mantle Coupling

Takashi ITO[1] and Yozo HAMANO[2]

[1] *Division of Astrometry and Celestial Mechanics, National Astronomical Observatory, Tokyo 181, Japan*
[2] *Department of Earth and Planetary Physics, Faculty of Science, University of Tokyo, Tokyo 113, Japan*

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Variation in the solar insolation onto the earth due to the change of the earth's rotation is considered to play a significant role in the evolution of the climate system of the earth. However, there remain plenty of things still unknown about the dynamics of rotation and revolution of the earth, especially about the effect of the dissipative coupling between the core and the mantle for the secular change of the earth's obliquity. We have to tackle these problems one by one in the near future. In this short paper we will review the possible causes of the change in the earth's obliquity, and also we will briefly discuss the enigmatic feature of reverse climatic zonation in Precambrian and its relationship to the core-mantle dissipative coupling which was recently reported by Williams (1993).

1. Introduction

It has been long before that Adhemar (1842) and Croll (1875) discussed the solar insolation variation as a trigger of the climate change of the earth. Causes of this variation are apparently the movement of earth's rotational axis and the change of orbital elements (eccentricity, orbital inclination, argument of perihelion) due to the gravitational forcing. Mechanisms of these gravitational perturbation and insolation cycles have been theorized quantitatively from very early times of the last century (Le Verrier, 1855; Stockwell, 1873; Newcomb, 1905). In the present century, the availability of high-speed computers and the development of various technique of numerical calculation have aided to produce the theories of higher accuracy (Brouwer & van Woerkom, 1950; Bretagnon, 1974; Laskar, 1985; Laskar, 1986; Laskar, 1988; Berger & Loutre, 1991; Quinn *et al.*, 1991). Time scales and spatial scales of these changes vary in a wide range; for example, "nutation" with very small amplitudes with very short periods, "precession" with the period of about 26000 years with large amplitude, variation of the orbital elements due to the gravitational perturbation among planets, "spin-orbital resonance" with the time scale of 10^7 years, and the secular frictions of tidal or climatic, etc. (Munk & MacDonald, 1960; Lambeck, 1980).

Combination of these changes is considered to have affected the insolation variation during the major part of the earth's history. Of these changes, effect of nutation (annual, semi-annual, monthly, semi-monthly, 18.6-year periodic, $\cdots$) due to the lunar and solar attraction is small (both amplitudes and periods) enough to be easily neglected as the cause of the solar insolation variation, so we don't take them into account here (Moritz & Meuller, 1988). Precessional motion of the spin axis with the period of about 26000 years, and the periodic variation of the orbital elements with the time scale of $10^4 \sim 10^5$ years are believed to be in close relationship with the solar insolation variation and glacial/interglacial cycles in Quaternary (Hays *et al.*, 1976; Berger, 1977; Berger, 1978a; Berger, 1978b).

In the viewpoint of the distribution of solar insolation, change of the earth's obliquity (the angle between the orbital plane and the equatorial plane. See Fig. 1(b)) plays the important part

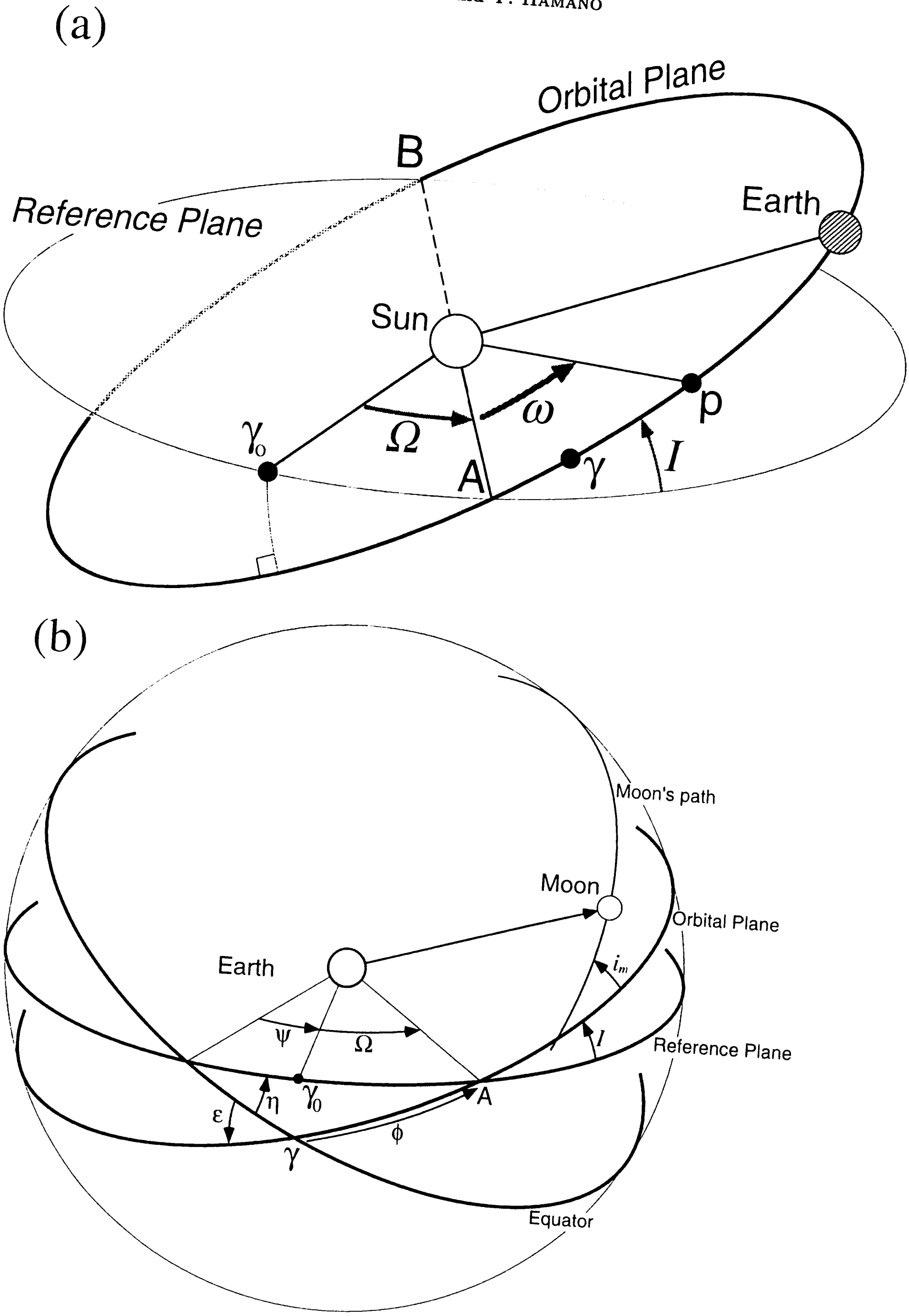

Fig. 1. Relationship between the orbital plane, reference plane, equator, and the moon's path. (a) is drawn by the Copernican (heliocentric) coordinate, and (b) by the Ptolemaic (geocentric) coordinate. In (a), Ω is longitude of the ascending node, I is the orbital inclination of the earth, ω is the argument of perihelion. A denotes the ascending node, B denotes the descending node, p denotes the perihelion, γ is the venal equinox of date, and γ_0 means the venal equinox of epoch. In (b), ε is the obliquity, ϕ is the precession angle, ψ is the luni-solar precession, i_m is the moon's orbital inclination with respect to the earth's orbital plane. η and

in causing the contrast between the high latitude region and the low latitude region (climatic zonation, *cf.* Vernekar (1972)). For example, using obliquity ε, mean motion around the sun n, orbital eccentricity e_s and longitude of perihelion with respect to the moving vernal equinox $\tilde{\omega}$, we can get the formula of the average semi-annual insolation amount Q (*cf.* Milankovitch (1930)

$$\begin{cases} Q_s = R_0(B_0 + C_0 - C_1) \\ Q_w = R_0(B_0 - C_0 + C_1) \end{cases} \tag{1}$$

for the northern hemisphere and

$$\begin{cases} Q_w^* = R_0(B_0 - C_0 - C_1) \\ Q_s^* = R_0(B_0 + C_0 + C_1) \end{cases} \tag{2}$$

for the southern hemisphere, where

$$\begin{aligned} R_0 &= \frac{S_0}{n\sqrt{1 - {e_s}^2}} \\ C_0 &= \sin|\vartheta| \sin\varepsilon \\ C_1 &= \frac{4e_s \sin\tilde{\omega}}{\pi} \cos\vartheta. \end{aligned} \tag{3}$$

S_0 is the solar constant at the mean orbital radius, B_0 is called the symmetric term (contribution of the yearly averaged insolation) which is a function of $\sin\varepsilon$. Here ϑ means a latitude on the earth. C_0 is called the obliquity term (contribution of the obliquity variation) and C_1 the precession term (contribution of precessional motion and the movement of perihelion). Variational amplitude of ε in Quaternary is about $\pm 1°$ (Berger, 1976). For the time scale of Quaternary, time averaged value of obliquity $\bar{\varepsilon}$ is considered nearly constant ($\sim 23°$). Of course on the longer time scale $\bar{\varepsilon}$ would have varied to some extent, but few researchers consider that the ancient obliquity was so largely different from the present that it could give some significant effect on the climatic zonation (Berger *et al.*, 1984; Berger, 1988). For instance, in the previous researches on this problem such as Ito *et al.* (1993), Loutre & Berger (1989) or Berger & Loutre (1993), they perform their calculation assuming that the time averaged value of the obliquity $\bar{\varepsilon}$ was constant all through the time, and according to the calculational results of the tidal evolution of the earth-moon system such as Abe *et al.* (1992), obliquity of the earth has been increased from about 18° to 23° during the past four billion years, and such small change of the obliquity is sufficiently weak to affect the climatic zonation since the geometrical effect of obliquity ε is only $\sin\varepsilon$ in the formula of solar insolation variation (3). However actually, evolutionary history of $\bar{\varepsilon}$ over the time scale of 10^8 years is totally covered with unknowns and there have been scarcely any quantitative discussions about this problem (Bills, 1990a). It is even difficult to say that the tidal friction is the only cause of the secular variation of obliquity. We have to inspect the mechanisms to change the obliquity which might have occurred in the past history. In the next section, we will review the possible physical processes to cause the secular variation of the obliquity.

2. Possible causes for the secular variation of the earth's obliquity

2.1 Tidal friction

The moon exerts a net torque acting around the earth's spin axis that tends to retard the earth's rotation, and the tidal bulge exerts a reciprocal torque that tends to accelerate the moon's orbital motion. By this mechanism, angular momentum is transferred from the rotation of the earth to the revolution of the moon, which have caused to decelerate the rotational velocity of the earth and to enlarge the distance between the earth and the moon. This mechanism is called the tidal friction. Regarding the effect of tidal friction between the earth and the moon on changing the earth's obliquity, there have been many researches since 1960's (Kaula, 1964; MacDonald, 1964; Goldreich, 1966; Turcotte *et al.*, 1977; Mignard, 1982; Conway, 1982).

Tidal friction is a very slow process with the time scale of 10^9 years, and the way it acts is fairly complicated and variational, dependent on the earth-moon distance and the mode of tidal torque (ocean-continent configuration). Since the moon does not revolve in the equatorial plane of the earth, the earth's rotation carries the tidal bulge out of the moon's orbital plane. Net effect of lunar torques acting on the tidal bulge tends to decrease the component of angular momentum of the earth which is perpendicular to the lunar orbital plane. Obliquity of the earth thus tends to increase and the inclination of the lunar orbital plane tends to decrease.

Although actually it is very difficult to explain qualitatively whether tidal friction increased the obliquity of the earth or not, it is probably certain that, for these 4 billion years except the early era of lunar formation, tidal friction has been gradually increasing the obliquity. Almost all results of many researches support this fact. Figure 2 shows one of the newest results of the numerical calculation on the dynamical evolution of the earth-moon system (Abe *et al.*, 1992). We can easily see the gradual increasing trend of the obliquity.

2.2 Climate friction

Next we will consider about the climate friction which is frequently discussed recently. Since there are large ice sheets on the earth, it is sufficiently possible that the variation of the moment of inertia due to the development and consumption of ice sheets exerts on feedback effect on the rotational motion of the earth. Development and consumption of the ice sheets cause the deformation for the rocky part of the earth, which has inevitably finite time lag of deformation because of the viscous characteristics of the earth. Recently some papers report that this time lag may give some significant effect to the incoming/outgoing balance of rotational angular momentum of the earth and drive the secular change of the obliquity (Rubincam, 1990; Rubincam, 1993; Bills, 1994; Ito *et al.*, 1995). This mechanism is called the climate friction and it has been suddenly in the spotlight recently as a typical example which clearly shows the physical process of multisphere interaction on the earth. Figure 3 is one of the calculated results from some numerical models, showing secular change in the obliquity resulted from the climate friction. We can clearly see the trend of secular increase of obliquity with the oscillations of higher frequencies (these higher oscillations are produced by short periodic motion ($O(10^4) \sim O(10^5)$ years) of the precession of the orbital plane and the coupling between the spin axis and the orbital plane (Ward *et al.*, 1979; Ward & Rudy, 1991). Rate of the secular increase in obliquity by the climate friction is strongly dependent on the viscosity of the mantle, especially the lower part. According to Ito *et al.* (1995) and Rubincam (1992), averaged increasing rate of the earth's obliquity is symbolically expressed as

$$\overline{\left(\frac{d\varepsilon}{dt}\right)} \simeq \frac{3n^2 \frac{d\Omega}{dt} \sin I \cos\varepsilon}{4\omega\Xi} \Delta H_0 \sin\xi \tag{4}$$

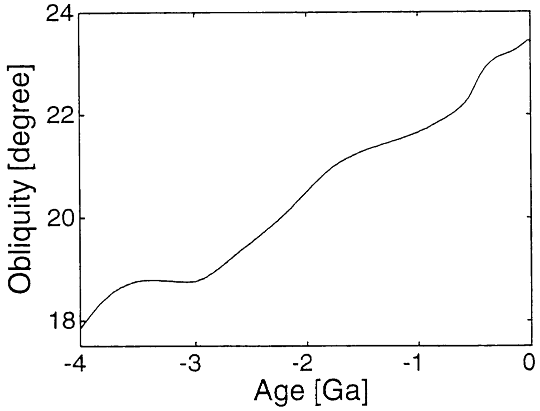

Fig. 2. One of the typical examples of the variation of the obliquity due to the tidal friction after Abe *et al.* (1992). It is calculated from −4Ga to the present, showing the gradual increase from about 18° to 23°.

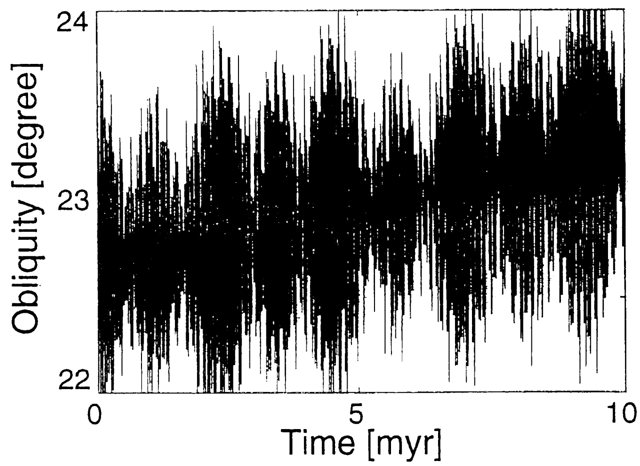

Fig. 3. One of the typical examples of secular change of the obliquity due to the climate friction after Ito *et al.* (1995). We can see that the monotonic trend is superposed on the high frequency oscillation of the obliquity. Periods of shorter periodic oscillation hardly change. Time integration is forward during ten million years with the time step of one thousand years, using the present observational data as the initial values. This case shows the result when we assume the viscous relaxation time lag of the solid earth (ξ in equation (4)) as 3000 years and the maximum amplitude of the dynamical ellipticity (ΔH_0 in equation (4)) as 5.0×10^{-5}, which seems plausible on the actual earth.

where

$$\Xi \equiv -\left(\alpha_0 \cos\bar{\varepsilon} + \frac{d\Omega}{dt}\cos I\right) \tag{5}$$

and I is orbital inclination and Ω is longitude of ascending node, ω is the rotational angular velocity of the earth, α_0 is the average value of the precession constant, ΔH_0 is the maximum amplitude of the dynamical ellipticity of the earth H, and the ξ is the phase lag angle of the deformation of the solid earth against the ice-loading event. If we assume $\Delta H_0 \sim O(10^{-5})$ and $10° \leq \xi \leq 30°$ during ten million years, we get

$$d\varepsilon \simeq 0.1° \sim 0.3°.$$

In Ito *et al.* (1995) we assumed the linear relationship between the variation of solar insolation and the variation of ice sheet amount. Of course the rate of secular increase in obliquity caused by the climate friction strongly depends on the actual linearity of the surface climate system of the earth and the distribution pattern of ocean, continent and ice-sheets (Källen *et al.*, 1979; Pollard, 1983; Snieder, 1985; Abe-Ouchi, 1993). This problem is fairly difficult including the question that whether the oceanic sediments linearly record the insolation variation or not (Trendall, 1973; Trendall & Morris, 1983). To what degree the climate friction can be significant for the obliquity history of the earth is still in some ambiguity, waiting for the progress of future researches.

2.3 Giant impact

Giant impact of the outer stellar bodies to the earth has continued to provide a material of discussion especially in relation to the origin of the moon. It has long been suggested that the primordial earth acquired its initial obliquity by impact with a huge (Mars-sized) body. This view is strongly supported as the widely accepted single-giant-impact hypothesis of lunar origin (Cameron & Ward, 1975; Hartmann *et al.*, 1986; Taylor, 1975; Melosh, 1990). Wide appeal of this hypothesis arises from its apparent explanation of the angular momentum and orbital characteristics of the earth-moon system, and the distinctive geochemical composition of the moon. If a giant impact of such large scale occurred after the stage of lunar formation, obliquity must also be strongly affected and changed a lot. For example, obliquity of Uranus at present as large as 97.9°, which is considered to be caused by the large impact of planetary bodies at the late stage of accretion (Korycansky *et al.*, 1990). This state of Uranus is believed to have been kept throughout the whole history of that planet since the rotational environments such as no large satellite comparable to the earth's moon, and the long distance from the sun, have protected Uranus from the influence of tidal effects, which is very different from the case of the earth (Tremaine, 1991).

However there is little possibility of such large impacts after the era of heavy bombardment, because there would be no such large protoplanets (such as Mars-sized) in this solar system after that era. Of course many smaller meteorites must have crashed into the earth, like Shoemaker-Levy 9 into Jupiter in the summer of 1994 (Watanabe *et al.*, 1994), but the effects of these smaller impacts are completely negligible. Dachille (1963) calculated that an impactor 32km in diameter (about a few times the diameter of K/T-boundary bolide), with a density of 3.5 g/cm^3 and traveling at 72 km/s would alter the obliquity only by $2''$ under optimum conditions of impact. He also showed that an impactor of 10 times larger (320km diameter) with same density and same velocity would produce a maximum shift of obliquity only up to $32'$. Furthermore the dynamical effects of terrestrial impact over time would have been random and would not have led to cumulative change in obliquity.

2.4 Spin-orbital resonance and chaotic character of the solar system

Next we note an interesting process called the spin-orbital resonance of the rotational axis, and the chaotic behavior of the dynamics in the solar system. Equation of motion which describes the precessional motion of the spin axis expressed by the unit vector $\boldsymbol{s}$, given by Ward (1974) or Bills (1990b) in the averaged form over both diurnal and seasonal cycles, is as follows:

$$\frac{d\boldsymbol{s}}{dt} = \alpha(\boldsymbol{s} \cdot \boldsymbol{n})(\boldsymbol{s} \times \boldsymbol{n}) \tag{6}$$

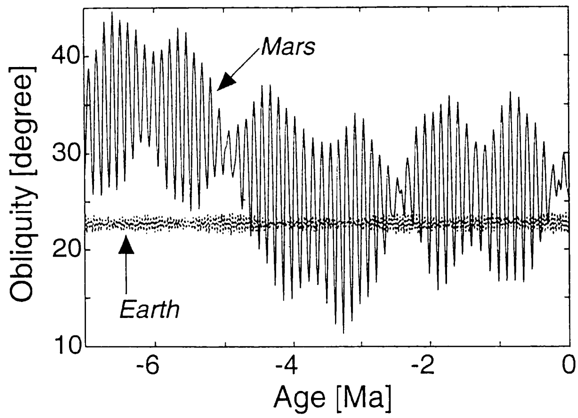

Fig. 4. Obliquity oscillation of Mars in contrast to the earth. Solid line expresses the obliquity of Mars, dashed line the earth. Notice the huge difference of the variational amplitude between Mars and the earth. It is because the variational range of the orbital elements and the dynamical ellipticity of Mars are quite large, and in addition, typical spin-orbital resonance occurs in the case of Mars. Here we used the result of Laskar (1988) as the quasi-periodic motion of the planets' orbital elements.

where the instantaneous orbital normal unit vector $\boldsymbol{n}$ is expressed by orbital inclination I and longitude of ascending node Ω as $\boldsymbol{n} = (\sin I \sin\Omega, -\sin I\cos\Omega, \cos I)$ where I and Ω are measured with respect to the invariable (space fixed) plane which is perpendicular to the total angular momentum vector of the solar system (Nobili *et al.*, 1989). And a rate constant α is called the precession constant representing the magnitude of the gravitational torque obtained by the equatorial bulge of the earth. Here the orbital elements such as I, Ω, eccentricity e and longitude of perihelion with respect to the fixed vernal equinox ϖ, are usually expressed as the following forms

$$e\sin\varpi = \sum_j M_j \sin(s_j t + \epsilon_j) \tag{7}$$

$$e\cos\varpi = \sum_j M_j \cos(s_j t + \epsilon_j) \tag{8}$$

$$\sin\frac{I}{2}\sin\Omega = \sum_j N_j \sin(s'_j t + \delta_j) \tag{9}$$

$$\sin\frac{I}{2}\cos\Omega = \sum_j N_j \cos(s'_j t + \delta_j). \tag{10}$$

Amplitudes M_j, N_j, frequencies s_j, s'_j and initial phases ϵ_j, δ_j are, for example, listed in the tables of Ward (1979) or Brouwer & van Woerkom (1950). In the first order (or linear) approximation we can easily integrate the equation of motion (6) and express the solution of the obliquity variation using (7), (8), (9), (10) as

$$\varepsilon = \Theta - \sum_j N'_j \left(\sin(s'_j t + \alpha t\cos\Theta + \delta_j - \Phi - \Omega_0) - \sin(\delta_j - \Phi - \Omega_0)\right) \tag{11}$$

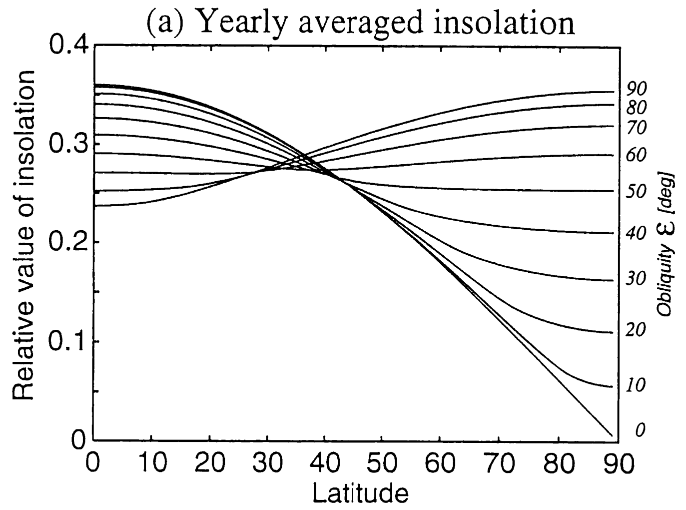

Fig. 5. One of our typical calculational results about the reverse zonation of the solar insolation distribution caused by the large obliquity. (*a*) shows the averaged annual insolation (normalized to the solar constant at 1AU) as a function of latitude for various values of the obliquity. When the obliquity ε is small, insolation amount is more at the low latitude area and less at the high latitude area, but when the obliquity becomes large and gets over the threshold value ~54°, distribution pattern of the insolation reverses, less at the low latitude region and more at the high latitude region.

where

$$N'_j \equiv \frac{s'_j N_j}{s'_j + \alpha \cos \Theta} \tag{12}$$

Θ is a constant and the solution of the 0-th order approximation (constant obliquity with uniform precession), and the constants Φ and Ω_0 are the present values of the precession angle and the longitude of ascending node. The effect of the precession of the spin axis is well illustrated by the above expression. If $\alpha \to 0$, the planet cannot get any gravitational torque from the other planetary bodies and the obliquity is completely dominated by the movement of the orbital plane. On the other hand, as $\alpha \to \infty$, the $\{N_j\}$ are suppressed, and the obliquity approaches the constant Θ in spite of the movement of the orbital plane. In a sense the spin axis has the ability to track the movement of the orbit normal. This latter type of motion is fairly common in the solar system, being exhibited by many of the natural satellites and probably Mercury as well (Colombo, 1966; Peale, 1969).

Equation (12) also admits a third possible behavior. Since $s'_j < 0$, if $|s'_j + \alpha \cos \Theta| < 1$, the amplitude of the j-th term in (12) would be enlarged instead of suppressed. This is the essential mechanism of the spin-orbital resonance, and it is indeed realized in the case of Mars (Fig. 4). Spin-orbital resonance can alter the obliquity largely in the time scale of 10^7 years. Though it cannot be called the secular variation, it is very much interesting in the viewpoint of classical celestial mechanics. For example in Fig. 5 to illustrate the reverse insolation zonation, we can physically explain the fairly large variational amplitude in the case of $\varepsilon_0 = 60°$ by this spin-orbital resonance.

By the way, precession constant α depends on the orbital mean motion n, the angular rotational velocity ω, polar and equatorial moment of inertia C and A (Brouwer & Clemence,

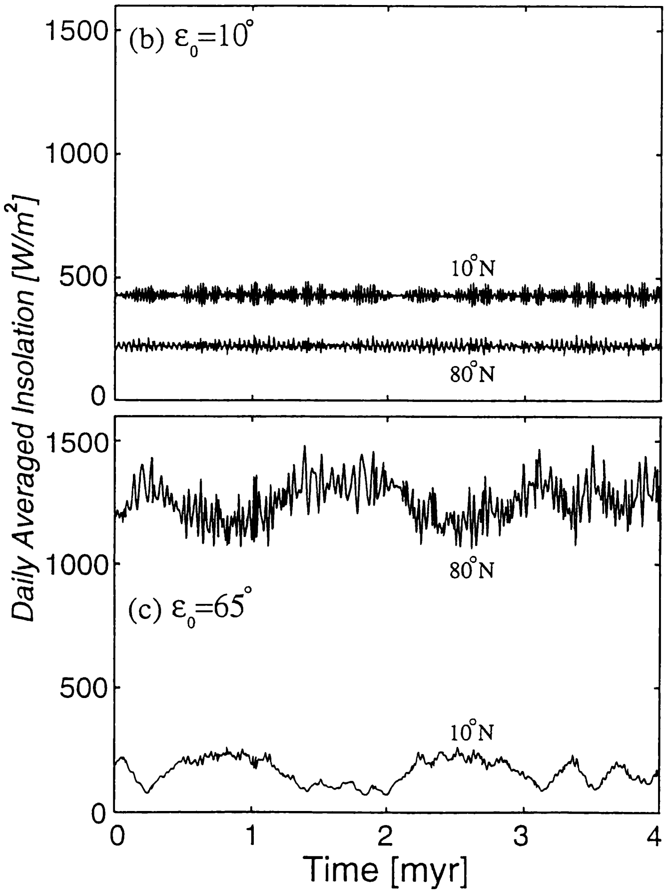

Fig. 5 (continued). (*b*) and (*c*) indicate the daily averaged insolation variation onto the latitude of 10°N and 80°N at the summer solstice. (*b*) is the case of small initial obliquity ($\varepsilon_0 = 10°$), and (*c*) is the case of large initial obliquity ($\varepsilon_0 = 65°$). The unit is W/m^2. We performed the forward time integration for the four million years using the present observational data as the initial values. In the case of large obliquity (*c*), variational amplitude becomes also large because of the occurrence of typical spin-orbital resonance (Ward *et al.*, 1979).

1961; Stacy, 1992). In the case of the earth, usually we only count the effect of the moon and the sun (valid assumption within the time scale of $10^6 \sim 10^7$ years), so α becomes

$$\alpha = \frac{3n^2}{2\omega} H \left(\left(1 - {e_s}^2\right)^{-\frac{3}{2}} + \frac{M_m}{M_s} \left(\frac{a_s}{a_m}\right)^3 \left(1 - {e_m}^2\right)^{-\frac{3}{2}} \left(1 - \frac{3}{2}\sin^2 i_m\right) \right). \tag{13}$$

Here M_s and M_m are the mass of the sun and the moon, a_s and a_m is the length of semimajor axis of the earth's orbital plane and the moon's orbit, e_s and e_m is eccentricity of the moon's orbit and the earth's orbit, and i_m is the inclination of the moon's orbit with respect to the earth's orbital plane, respectively. α consists of the sum of two contribution from the sun

$$\alpha_s = \frac{3G}{2\omega} H \frac{M_s}{{a_s}^3} \left(1 - {e_s}^2\right)^{-\frac{3}{2}} \tag{14}$$

and from the moon

$$\alpha_m = \frac{3G}{2\omega} H \frac{M_m}{{a_m}^3} \left(1 - {e_m}^2\right)^{-\frac{3}{2}} \tag{15}$$

with the effect of moon's inclination angle i_m (Smart, 1953; Sharaf and Budnikova, 1967). G denotes the gravitational constant. However in the very long time interval (maybe over 10^8 years), effect from the other planets can not be neglected, so the precession constant should be the form

$$\alpha = \alpha_s + \alpha_m + \alpha_{\rm Mercury} + \alpha_{\rm Venus} + \alpha_{\rm Mars} + \alpha_{\rm Jupiter} + \alpha_{\rm Saturn} + \cdots. \tag{16}$$

Recently, orbital motion of the planets in this solar system is believed chaotically evolved (Sussman & Wisdom, 1992; Laskar, 1990). In the case when we count the effect of these chaotic motion of the planetary motion, surprisingly, obliquity of most of the inner planets also behaves chaotically, almost independent of the initial value ε_0 (Laskar & Robutel, 1993). However since the spin axis of the earth is fortunately stabilized by the massive moon (Ward, 1982), earth is not considered to have experienced the chaotic (irregular and large-amplitude) variation of the obliquity which might have caused the serious effect on the surface climate system through the radical insolation variation (Fig. 6). If the moon did not exist, obliquity of the earth would have become chaotic and could have taken a large value (Laskar *et al.*, 1993a; Laskar *et al.*, 1993b). Physical mechanism of this chaotic behavior is strongly nonlinear and complicated, and many researchers are now beginning to devote themselves to this kind of problems (Wisdom, 1991; Berger *et al.*, 1992; Laskar *et al.*, 1992a; Laskar *et al.*, 1992b).

2.5 Core-mantle dissipative coupling

Any differential motion between the liquid core and the solid mantle produces friction and dissipates energy. The solar torque on the equatorial bulge causes the mantle to precess, but this gravitational torque on the liquid core is insufficient to cause the core to precess just at the same rate (Peale, 1974; Sasao *et al.*, 1980). Pressure force at the core-mantle boundary will precess the core with the mantle if the ellipticity of the core-mantle boundary exceeds the ratio of the rotation to the precession periods (Poincare, 1910). The relative motion between the core and the mantle dissipates energy which must come from the rotation. But the average external torque causing the precession lies in the orbital plane, thereby conserving the component of the spin angular momentum perpendicular to the orbital plane. The energy must then come from the component of the angular momentum in the orbital plane thereby reducing its magnitude. Hence, a core-mantle dissipation tends to drive the obliquity $\varepsilon \to 0°$ for $0° < \varepsilon < 90°$, and $\varepsilon \to 180°$ for $90° < \varepsilon < 180°$ (Goldreich & Peale, 1966; Goldreich & Peale, 1970).

The mechanical couples or torques at the core-mantle boundary most commonly discussed in regard to the transfer of angular momentum between core and mantle are only three plausible candidates: (1) *viscous* (or *turbulent*) *coupling*, a function of the viscous friction in either a laminar or turbulent boundary layer at the core-mantle boundary, and dependent of either the kinematic or effective viscosity of the outer core; (2) *electromagnetic coupling*, a function of magnetic field strengths and electrical conductivity in the core and mantle; and (3) *topographic* (or *inertial*) *coupling*, in which bumps or depressions at the core-mantle boundary modify the flow of fluid past the boundary and the pressure distribution on the mantle. Though a fourth possibility might seem to be a gravitational torque between the core and the mantle, this must be dismissed when it is remembered that the interior gravitational attraction due to any spheroidal

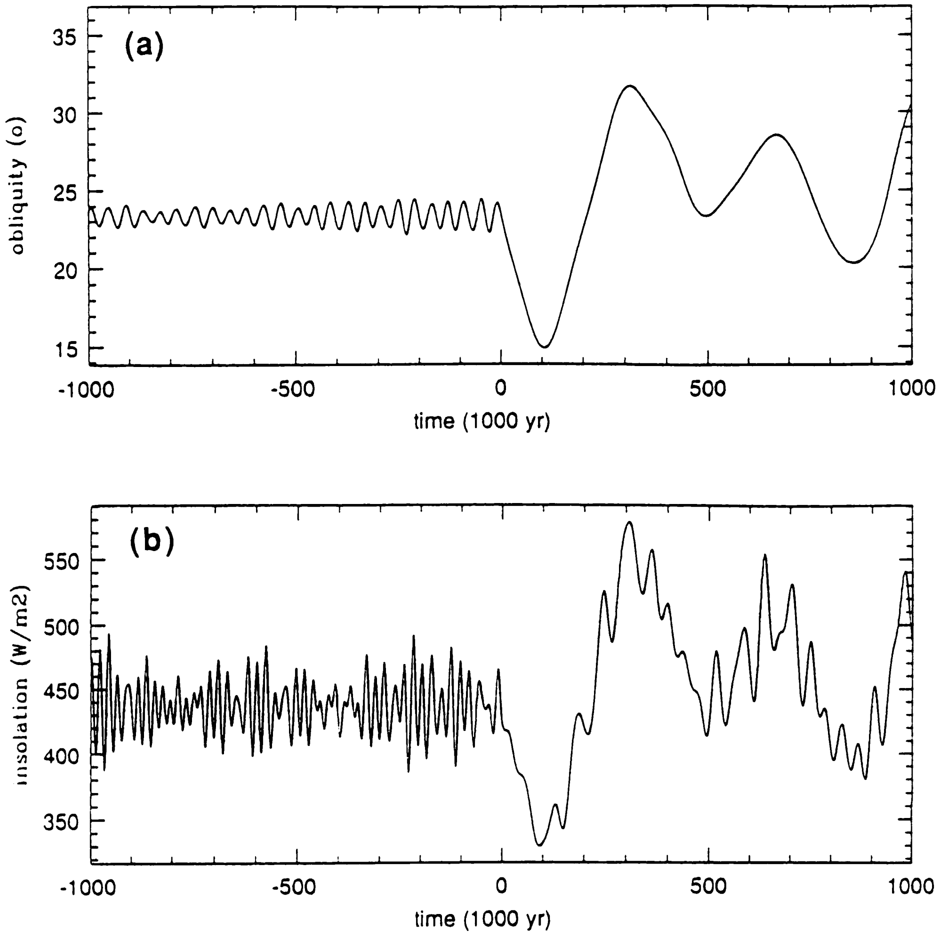

Fig. 6. Changes in the obliquity (a) and insolation (b) at 65°N, resulting from the suppression at $t = 0$ of the moon. Moon is present from -1Myr to 0, and absent from 0 to $+1$Myr. We can see the moon is an important stabilizer for the rotational motion of the earth. Adapted from Laskar *et al.* (1993a).

shell of constant density material would be strictly zero, and that the same ought to be true very nearly of the various layers of the mantle (Toomre, 1966; Peale, 1976).

One of the theoretical expression of the dynamics of angular momentum exchange by frictional coupling between the core and the mantle during precession is as follows: (Rochester, 1976)

$$\frac{d\varepsilon}{dt} = \left(\frac{e_0 - e_1}{e}\right)^2 \frac{{A_0}^2 {A_1}^2}{2\kappa A^3} \Omega^2 \sin 2\varepsilon. \tag{17}$$

Here A_0, e_0 are the equatorial moment of inertia and dynamical ellipticity of the mantle (which precesses with frequency $\Omega = 7.74 \times 10^{-12}$rad/s), A_1, e_1 are the corresponding quantities for the core, and A, e are those for the entire earth. κ expresses the strength of the core-mantle coupling and is a function of physical parameters listed above. However, our present knowledge about the physical properties of the earth's interior do not support the hypothesis that these dissipative coupling has been playing a significant role in the history of the earth's obliquity. For example, as far as the viscosity of the fluid core is concerned, even the the highest value (10 m^2/s) which is deduced from the phase lag between the 18.6-year nutation and its tidal excitation (Toomre, 1974), produces the secular rate of change in the obliquity $-0''.0004$/cy ($\sim -1°/10^8$ years). If we take the most reliable value of the viscosity (10^{-6} m^2/s) from the theoretical estimation (Gans, 1972), the secular rate of decrease becomes $-5 \times 10^{-7''}$/cy ($\sim -0.007°/10^8$ years) which is negligible compared to the other effects such as tidal friction. Although there is a large amount of ambiguity in the estimation of the electrical conductivity of lower mantle (Shankland *et al.*, 1993) or the topographic bumps at the core-mantle boundary (Yoshida & Hamano, 1993), the unified opinion now is "Unless the actual value of the core viscosity approaches the upper limit of 10 m^2/s set by Toomre, core-mantle coupling can be ignored in modeling the dynamical evolution of the earth-moon system" (Rochester, 1976).

3. Climatic enigma: huge obliquity in Precambrian era?

Although there remain plenty of unknowns about the obliquity history of the earth, almost all geophysicists and astronomers do not expect that the obliquity of the earth have experienced the drastic change during the whole history of the earth. However, an Australian geologist George E. Williams, who has found a lot of geological consequences of the LOD-variations at ancient ages (Williams *et al.*, 1978; Williams, 1989a; Williams, 1989b; Williams, 1990) recently presented in his recent paper (Williams, 1993) an amazing statement that "In the age of Precambrian, obliquity of the earth was much larger than the present, and possibly reached 60° to 70°". For the rest of this paper, we will give a brief consideration to the evidences and possibilities of this astounding hypothesis.

3.1 Geological evidence in Precambrian

We had better start from arranging the deducing process of Williams which finally led him to that extraordinary conclusion. At first, the found some sand wedges from the southern part of Australia which were typical examples of periglacial topography, and the age of these wedges were supposed to be −0.65Ga (at the end part of Proterozoic). Sand wedge is likely to be formed at the region where the annual difference of temperature is very large and the process of freezing/melting is highly active. On the modern earth they are only formed at, say, "cold regions" at the inland area of high latitude such as Siberia, or at the high mountain regions such as Andes or Himalaya. What we should be surprised at is the fact that at the places where these sand or ice wedges are found, inclination angles of the palaeogeomagnetic field are fairly small, which means that their palaeolatitudes were fairly low. Analogy with the common sense in Quaternary, these low-latitude regions would be warm enough to prevent sand wedges to develop. On the other hand, although in the China continent the inclination angle of palaeomagnetic field is relatively high, which means that it was located at the high latitude region and conditions of large annual difference of temperature and active freezing/melting process were filled, no evidences were found to show the existence of sand or ice wedges. What does this enigma mean?

One easily thinks of the global refrigeration. However as mentioned above, the north China block which seemed to occupy the high palaeolatitudes during the Late Proterozoic yet apparently was not glaciated, and detailed palaeogeographic reconstructions for late Proterozoic south Australia do not reveal any drastic lowering of sea level during the major glaciations of the late Proterozoic (Preiss, 1987). In addition, even a 30% reduction in solar luminosity at 4Ga years ago did not seem to produce the global freezing (Sellers, 1990), which makes it difficult to suppose the global freezing in the increasing solar luminosity afterwards. Also from other negative consequences such as the survival of long-evolved organisms and other shallow water biota (Runnegar, 1991), global glaciation cannot be supported by our present knowledge.

Someone proposed a strange hypothesis that there were the large ice rings around the earth like Saturn at those ages and they obstructed the insolation onto the earth. But there is no way to confirm the existence of such rings and no explanation why these rings have disappeared now. In addition, some numerical calculation reports that the distribution of the solar insolation will be kept normal even if there existed such rings (Brinkman & McGregor, 1979), so the hypothesis of large ice rings should be abandoned.

Another hypothesis is that the dipole moment of the geomagnetic field at those ages was very small, and the magnetic pole was largely off the rotational axis. However, since the periglacial consequences of Proterozoic are found at several places with very long time interval, it is not likely that the geomagnetic dipole axis had continued to be off the rotational axis during such long era. Validity of the geocentric axial dipole model for the earth's magnetic field in the geological past can be tested by comparing palaeomagnetic data with independent indicators of past latitude. Phanerozoic palaeoclimatic indicators and the frequency distribution of palaeomagnetic inclination angles strongly support the geocentric axial dipole model for the Phanerozoic (Evans, 1976), and hence the geomagnetic field of Precambrian time is deduced to be like that of Phanerozoic (Embleton & Williams, 1986). The observation that the quadrupole moments of the magnetic fields of Uranus and Neptune are comparable to their dipole component actually indicates that non-axis dipole-quadrupole planetary magnetic fields are indeed possible for the earth. However, Rädler and Ness (1990) concluded that simple reoriention of the magnetic field of the terrestrial or Jovian planets cannot produce the Uranian magnetic field. Magnetic fields of Uranus and Neptune may result from the unique interior composition and state of those planets (Connerney *et al.*, 1987; Connerney *et al.*, 1991; Ness *et al.*, 1989).

However in spite of these speculations, interpretation of Williams is different: if the obliquity of the earth is near 23° as the present, distribution of the solar insolation does not betray our intuition, *i.e.* maximum at the equator and minimum at the poles. But if the obliquity increases more and more and finally crosses over a threshold, the state of this distribution will turn over, *i.e.* maximum at the poles and minimum at the equator (Fig. 5). Most possible cause of the formation of Proterozoic sand wedges in the low palaeolatitude area would be, he insists, due to this "reverse climatic zonation" brought by such a very large obliquity. And moreover, consequences of periglacial topography are also discovered from the site of older rocks of early Proterozoic (~2.5 Ga) where inclination angles of palaeogeomagnetism are also fairly small. Turning over of the distribution of solar insolation which might have caused the reverse climatic zonation occurs when the obliquity becomes larger than about 54° (it is an easy geometrical calculation. *cf.* Ward (1974)). Thinking of every sort of problems collectively, Williams suggested the new version of the obliquity history of the earth (Fig. 7).

3.2 Is it really possible?

Many papers are published about the initial value of obliquity after the early era of planetary accretion. Now it is considered that the very large obliquity of 70° or 80° is sufficiently possible, dependent on the collisional velocity of planetesimals, or the mass ratio between the planetesimals and the parent bodies (Hartmann *et al.*, 1986). Williams assumed this high value as the initial obliquity. Then, if the initial value of obliquity had been so large, how would it have been reduced to the present value? Especially, did the drastic change of obliquity at the end period of Proterozoic really occur? He ascribes all this drastic decreasing of obliquity to the energy dissipation process at the core-mantle boundary. Actually, no other process can be a candidate for the decrease of the obliquity. As seen above, both two frictions (tidal and climate)

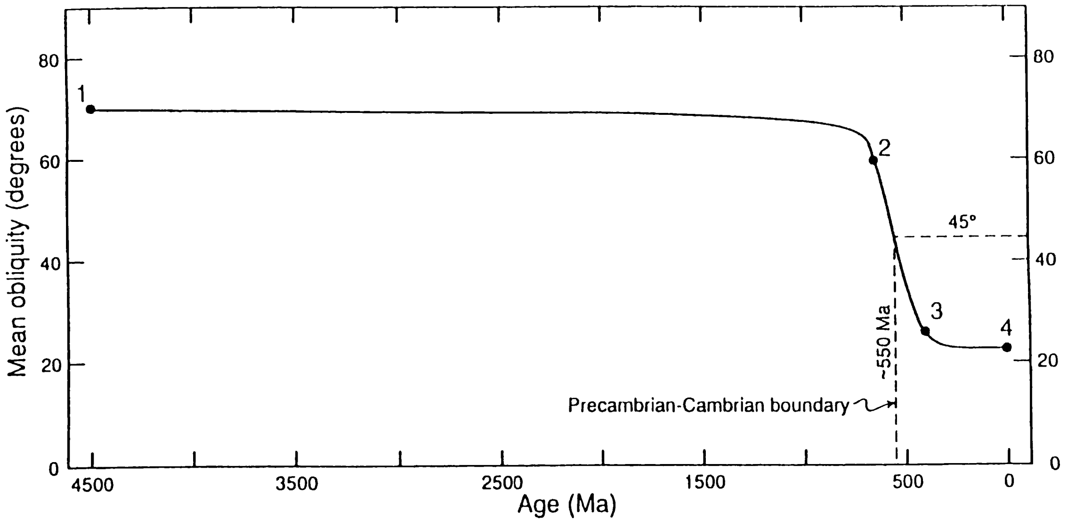

Fig. 7. Proposed curve of mean obliquity $\bar{\varepsilon}$ against time, consistent with the single giant impact hypothesis for the moon's origin and interpretation of the geological record, adapted from Williams (1993). The four control points are: (1) $\bar{\varepsilon} \simeq 70°$ at 4500Ma; (2) $\bar{\varepsilon} \simeq 60°$ at 650Ma; (3) $\bar{\varepsilon} \simeq 26°$ at 430Ma; and (4) $\bar{\varepsilon} \simeq 23°$ at present. These data indicate that there occurred the rapid decreasing event of the obliquity around 550Ma, which is taken as the Precambrian–Cambrian boundary.

only act to increase the obliquity. Giant impact of the outer stellar bodies, though it may have actually happened, can be eliminated from the candidate because from the standpoint of impact flux of the meteorites there is little possibility for such huge impacts to occur particularly at the last part of Proterozoic that we are concerned here. Spin-orbital resonance is very interesting from the viewpoint of classical celestial mechanics, but its time scale is rather short (10^7 years) and it is better called the oscillation of the obliquity with irregular frequencies and large amplitudes, rather than the secular motion (Fig. 5(c)).

However as mentioned above, since the core-mantle dissipative coupling is expected to be very weak and seems negligible in the obliquity history or the dynamical evolution of the earth-moon system, we cannot solve this enigma at all. Basis on which Williams stands is the theory of core-mantle coupling advocated by Aoki and Kakuta of Tokyo Astronomical Observatory in Japan in early 1970's (Aoki, 1969; Aoki & Kakuta, 1971). They claimed the possible large effect of core-mantle dissipative coupling on the secular decrease of obliquity. However, the observational data are now negative to them. If the obliquity has kept its large value during the whole Precambrian, it means that there was scarcely any dissipation of rotational energy throughout that era. So what could cause the abrupt energy dissipation at the end of Precambrian? If the friction works, it should work at much earlier era and have reduced the obliquity. If the friction does not work, it should not work at all and keep the large obliquity. Obliquity of Uranus takes the astonishingly large value of 97.9°, and this state is considered to have been kept from the initial epoch of planetary accumulation which means almost no existence of energy dissipation. Why could the earth's obliquity have decreased alone?

Of all the ambiguous factors about the secular decrease of obliquity, most ambiguous one is the value of palaeolatitude obtained from the palaeomagnetism of the ancient rocks. For example, according to the standard results of the reconstruction of the continental distribution (mainly done by the palaeomagnetism together with some biological or climatic evidences) such as Scotese (1984) or Smith *et al.* (1973), Australian continent, in which the typical Proterozoic periglacial sand wedges were found, is located at a low latitude region or near the equator from the late Proterozoic to early Phanerozoic, consistent with the claim of Williams. But it is impossible to avoid the essential arbitrariness attached to the reconstruction of continental distribution of the age more than 0.2 Ga for which all of the ocean bottoms which record the direct evidence of the palaeomagnetism have been all subducted out. For example, mutual locations of the continents are fairly different between Scotese (1984) and Smith *et al.* (1973). And moreover, some other results showed that the Australian continent was not at the low latitude region (*cf.* Morel & Irving (1978)). Especially in Uchimura (1994), Australia seems to wander around the antarctic region in those ages! We have to overcome these inconsistencies about the palaeolatitude of Precambrian to solve the enigma of ancient periglacial consequences. But anyway, we can say that the contradiction of very cold climate in low palaeolatitude during the late Precambrian which Williams pointed out brings a new problem in palaeomagnetism and celestial mechanics.

4. Concluding remarks

We reviewed the possibilities of the variation in the obliquity in the history of the earth, especially the long time scale ones (secular variation). But there are too many unknowns now, especially about the core-mantle coupling, and it is still impossible to draw any specific conclusion at the present stage. However, since it is very important to know the whole history of the earth's obliquity to investigate the evolution of the surface climate system, we have to continue the effort to solve out the unknowns by geological, geophysical and astronomical methods, including the task of figuring out the enigmatic problem of periglacial consequences in low palaeolatitude in Precambrian.

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