Preprint from *Geophysical Research Letters*

Evolutionary history of the Milankovitch cycles over 4000000000 years

Takashi Ito, Mineo Kumazawa, Yozo Hamano, and Takafumi Matsui

October 20, 1999 Rev. 1.60 (submitted)

Evolutionary history of Milankovitch cycles over 4000000000 years

TAKASHI ITO, YOZO HAMANO, AND TAKAFUMI MATSUI

Department of Earth and Planetary Physics, Faculty of Science, University of Tokyo

MINEO KUMAZAWA

Department of Earth and Planetary Science, Faculty of Science, Nagoya University

Abstract

Solar insolation variation due to the gravitational perturbation among the planetary bodies in the solar system, so called Milankovitch cycle, is widely believed as a major cause of the climatic change such as the glacial-interglacial cycles in Quaternary, and its typical frequencies are supposed to be constant during Quaternary. However, the periods of the Milankovitch cycles must have been largely changed in the longer time scale of billion years following to the dynamical evolution of the earth-moon system. Deccelerated rotational velocity of the earth has been making the dynamical ellipticity of the earth smaller and lengthening the major periods of both Milankovitch cycles and tidal cycles. We have studied the relation between the frequencies of the Milankovitch cycles and the rotation rate of the earth on the basis of the theoretical and computational analysis on the earth-moon system with several assumptions involved. Our conclusion is that this cyclicity which can be recorded in the sediments are mutually related well as a function of the dynamical ellipticity and the absolute age. This fact implies that we can establish the standard time scale for measuring the relative age, in other words, the lap time clock or the chronometer for decoding the whole history of the earth, by comparing the stripes in BIF and other sediments of Archean or Proterozoic with a set of theoretical Milankovitch cycle and tidal cycle frequencies. Here we present the preliminary reference model of the evolution of the Milankovitch cycles and tidal cycles and attempt to establish the lap time clock which will be a potential device for our project to clarify evolutionary history of our earth's environment back to -4Ga.

Introduction

Solar insolation onto the surface of the planet has been said to vary periodically and cause long period climate change. Time scale of this cycle is $10^4 \sim 10^6$ years, and it has been considered as a pacemaker of the glacial-interglacial cycles in Quaternary on the earth (Hays *et al.*, 1976). This is called the Milankovitch cycle and its mechanism has been theorized quantitatively from so early times because it consists of classical celestial mechanics about point masses and rigid bodies (Milankovitch, 1941). However in view of the longer time scale ($O(10^9)$years), secular change of the earth-moon distance and rotational velocity of the earth must have caused the great effect on the Milankovitch cycles. This effect was qualitatively pointed out by Walker and Zahnle (1986) up -2.5Ga, and quantitatively computed by Berger *et al.* (1992) only up to -0.5Ga. As insisted by Berger, it needs careful handling to trace the Milankovitch cycles back to the older era, because the relevant dynamics is not linear that the variation of the Milankovitch cycles have been supposed to be chaotic (Sussman & Wisdom 1992, Laskar 1990). Nevertheless we have a strong demand to identify and decode the cyclicity observed in the sediments such as banded iron formation of Archean or Proterozoic. The cyclicity recognized in the sediments was probably caused by the climate change due to Milankovitch cycles as estimated from the sedimentation rate. Therefore we have examined a possibility to clarify the evolutionary history of the Milankovitch cycles.

The Milankovitch cycle is defined as the long period variation of solar insolation on to the top of the earth's atmosphere. This oscillation of the solar insolation is considered to trigger the glacial and interglacial cycles in Quaternary. The power spectrum of the insolation variation of these four million years by the standard Fourier transformation is shown in Figure 1 (upper). Apparently we can see four sharp peaks. The peaks of 19Kyr and 23Kyr are due to the oscillation of precession angle, and the peaks of 41Kyr and 54Kyr is due to the oscillation of obliquity (though the peak of 54Kyr is rather weak). Milankovitch cycles mainly consist of these four components (we call them Mp1, Mp2, Mo1 and Mo2 respectively). Here we pay our attention to these frequencies in the Archean or Proterozoic. The length of these periods (19, 23, 41, 54Kyr) are thought to be nearly constant during the short time scale of Quaternary.

In view of the longer time scale (10^9years), these typical periods of Milankovitch cycles must have been largely changed following the dynamical evolution of the earth-moon system. Although the chaotic planetary perturbation is sure to exist, we noticed that the effect of the planetary perturbation is non-systematic and minor in the variation of Mp1, Mp2, Mo1 and Mo2 (Ito *et al.* (1993), submitting) so the evolution of the Milankovitch cycles can be traced back to the ancient times when the earth was spinning much faster and the moon was much nearer. The purpose of the present paper is (1) to discuss about some special assumptions needed for the calculation, and (2) to present some results of the possible evolution paths of the Milankovitch cycles on the basis of the standard calculational result of the dynamical evolution of the earth-moon system.

Equation of motion

The annually averaged equation of motion of the rotational axis of the planet (in this case, the earth) is derived from the Euler's equation of rigid body rotation viewing from the inertial coordinate system (Ward, 1974; Bills, 1990)

$$\frac{d\boldsymbol{s}}{dt} = \alpha(\boldsymbol{s}\cdot\boldsymbol{n})(\boldsymbol{s}\times\boldsymbol{n}) \tag{1}$$

where $\boldsymbol{s} = (s_x, s_y, s_z)$ is the spin axis unit vector of the earth. $\boldsymbol{n}$ is the orbital normal unit vector of the earth and expressed by the orbital inclination I and longitude of the ascending node Ω as $\boldsymbol{n} = (\sin I \sin\Omega, -\sin I\cos\Omega, \cos I)$. Following Ward (1974) we obtain the time variation of the obliquity θ and the precession angle ϕ as $\theta = \cos^{-1}(\boldsymbol{s}\cdot\boldsymbol{n}), \phi = \sin^{-1}\left(\frac{s_y}{\sin\theta}\right)$ and calculate the solar insolation variation. As for the time series of the orbital elements I, Ω, and eccentricity e_s, longitude of perihelion with respect to the fixed vernal equinox ϖ, we use the solution of Laskar (1988)'s secular perturbation theory.

In equation (1) α is called the precessional constant representing the magnitude of the gravitational torque obtained by the equatorial bulge of the earth (present value is 54.96 arcsec/year) and expressed as

$$\alpha = \frac{3n^2}{2\omega}\frac{C-A}{C}\left(\left(1-{e_s}^2\right)^{-\frac{3}{2}} + \frac{M_m}{M_s}\left(\frac{a_s}{a_m}\right)^3\left(1-{e_m}^2\right)^{-\frac{3}{2}}\left(1-\frac{3}{2}\sin^2 i_m\right)\right) \tag{2}$$

where A and C are the polar and the equatorial moment of inertia of the earth, n is the earth's mean motion to the sun, e_m is the eccentricity of the moon's orbit, M_m and M_s is the mass of the moon and the sun, a_m and a_s is the length of the semimajor axis of the moon's orbit and the earth's orbit, and i_m is the inclination of the moon's orbit relative to the orbit of the earth. The frequencies of the Milankovitch cycles are determined by the time variation of $\boldsymbol{n}$, α, and the mean obliquity $\overline{\theta} \simeq \cos^{-1}\langle \boldsymbol{s}\cdot\boldsymbol{n} \rangle$ ($\langle\rangle$ means the time average). $\boldsymbol{n}$ represents the orientation of the earth's orbital inclination which is affected by the gravitational perturbation of other planets. We use the quasi-periodic variation of e_s shown by Laskar (1988) as mentioned before.

We utilize two kinds of computational results as the evolution of e_m and i_m: Abe *et al.* (1992) and Turcotte *et al.* (1977). e_m and i_m are just equal to zero in the simple model of Turcotte *et al.* (1977), and Abe *et al.* (1992) precisely calculated the change of these parameters (Figure 2). However the absolute values of e_m and i_m are originally so small that they have only slight effect on the change of the precessional constant α (ie., $1 - {e_m}^2$ and $1 - \sin^2 i_m$ is almost unity). Similarly $1 - {e_s}^2 \sim 1$ in a good approximation so actually we can consider that the precessional constant α is determined almost all by the relationship between three variables: dynamical ellipticity of the earth $\frac{C-A}{C}$, rotational angular velocity of the earth ω, and the earth-moon distance a_m. The changes of the earth-sun distance a_s, the masses M_s, M_m are not taken into account in this research.

About the evolution of the mean obliquity of the earth there are many hypothesis: climate friction (Rubincam, 1990), stochastic accumulation process (Dones and Tremaine, 1993), tidal evolution (Kaula, 1964). In this discussion we only take into account the tidal evolution effect calculated by Abe *et al.* (1992) because other factors are rather vague. But actually the mean obliquity at -4Ga of Abe *et al.* (1992) is about $18°$, and the difference of $\cos\overline{\theta}$ at -4Ga and the present is $\cos 18° - \cos 23.5° \approx 0.034$, which is fairly small. Below we explicitly put four assumptions for determining the evolution of α and consider about the validity of these assumptions in the next section.

Assumptions and their extent of validity

1. Effect of the possible chaos in the planetary motion is small: Recent researches have shown that the planetary motion in the solar system is chaotic (Sussman and Wisdom, 1992). Generally the term "chaotic motion of the solar system" indicates that the long term motion of the earth's orbital elements is not predictable by the numerical integration using the present values as the initial conditions. However, possible variation range of the fundamental frequencies g_i and s_i may not be so large (Laskar, 1990) that could significantly affect the frequencies of the Milankovitch cycles. Our intention here is to build a preliminary reference model of the evolution of the Milankovitch cycles, so we confirm the validity of this assumption.

2. Conservation of the angular momentum of the earth-moon system: The earth-moon system has been losing its angular momentum along the evolution due to the tidal torque from the sun. This effect is calculated by Abe *et al.* (1992) and the results are shown in Figure 3 (c). As we can see, the angular momentum of the earth-moon system has changed no more than 1% in these 4Ga, so this assumption is quite good as the first approximation.

3. Density structure of the earth's interior has not been changed: Though the timing of core formation of the earth is still not clear, it is sure that the earth underwent the core formation stage quite earlier in its accumulation history (Newsom, 1990). Of course some events like fractionation of the crust from the mantle or mode change of mantle convection from the one layer mode to the two layer mode may have changed the density structure of the earth, but we can say that the time variation of the density structure of the earth's interior has been enough small within the practically variable range for our purpose.

4. Dynamical ellipticity is proportional to the square of the rotational angular velocity (ω^2) : When the angular velocity ω is large, dynamical ellipticity of the earth $\frac{C-A}{C}$ are large, and the planet is more oblate. On the contrary when ω is small, $\frac{C-A}{C}$ are small. The accurate determination of the dynamical ellipticity as a function of rotational angular velocity ω in the case of the actual earth is complicated (Denis, 1986). But here we assume that the dynamical ellipticity $\frac{C-A}{C}$ is proportional to the square of rotational angular velocity (ω^2). This assumption is just correct in the case of constant density rotating body and justified well if the density structure of the earth is in hydrostatic equilibrium (Turcotte and Schubert, 1982). The effects of nonhydrostatic state and the variability of the density structure within the earth are supposed to be small. But of course they are one of the important subjects of our future work.

Results and Discussions

Below we show the calculational results of the evolution of the earth's rotational angular velocity ω and the earth-moon distance a_m by Abe *et al.* (1992) and Turcotte *et al.* (1977) in Figure 3. In Figure 3 (a) ω is replaced by LOD (Length of Day). The difference between two results in Figure 3 (a)(b)(c)(d)(e) is caused by the difference of the method of modeling of the tidal mechanism. Turcotte's method is a simple and analytical one, and Abe's method is completely numerical including the non-axis-symmetrical distribution of continents and oceans, and effect of the solar tide.

Then we can calculate the motion of the rotational axis of the earth by solving the equation of motion (1) and obtain ancient Milankovitch cycles. Figure 1 (lower) is one of the results, showing the power spectrum of daily average insolation time series of summer solstice at 65°N at -3Ga. The sampling duration is four million years and integration time step is one thousand years. Of course we can directly integrate the equation of motion (1) and digitize the peak frequencies manually, but typical frequencies of the Milankovitch cycles are known to consists of linear combinations of the fundamental frequencies of planetary perturbation g_i and s_i and precessional constant α (Berger and Loutre, 1987). In this paper we concentrated on the four peak frequencies (Mp1, Mp2, Mo1, Mo2) described in Table 1 and traced their evolution back to -4Ga. The results are summarized in Figure 3 (e). We can see that the major periods of Milankovitch cycles Mp1, Mp2, Mo1 and Mo2 were much shorter in ancient ages. We assume the constancy of the fundamental frequencies g_i and s_i as mentioned before. Since we are concentrating on the discussion of the frequency domain (frequencies of the Milankovitch cycles) we should take care only about the frequencies of the results, not about the phases (on the condition that the initial values are "moderate" or physically "reasonable" ones).

There is another point here to notice. In the diagrams of Figure 1, power of the obliquity term (Mo1 and Mo2) of -3Ga is apparently much lower than that of the present age. Magnitude of gravitational torque is much larger at -3Ga than the present so the amplitude of obliquity oscillation is suppressed (cf. Ward(1974)). This is the reason why the power of Mo1 and Mo2 becomes low in Figure 1 (lower). Strict mathematical formulations are given in equation (10) to (32) of Ward (1974).

The diagram with the horizontal axis of the absolute ages (Figure 3) is strongly dependent on the model of tidal dissipation mechanism between the earth and the moon. We can avoid this problem by replacing the absolute age with Length of Day (LOD) as the horizontal axis, because LOD is essentially a function only of the density structure of the earth, and the dynamical ellipticity or the rotational speed

peak	argument	resume $\rightarrow \frac{1}{\text{period}}$
Mp1	$g_4 + k$	$\frac{17.85+50.44}{3600\times 360} \rightarrow \frac{1}{18977}$
Mp2	$g_5 + k$	$\frac{4.249+50.44}{3600\times 360} \rightarrow \frac{1}{23697}$
Mo1	$s_3 + k$	$\frac{-18.88+50.44}{3600\times 360} \rightarrow \frac{1}{41064}$
Mo2	$s_6 + k$	$\frac{-26.33+50.44}{3600\times 360} \rightarrow \frac{1}{53753}$

Table 1. Origin of the present Milankovitch frequencies Mp1, Mp2, Mo1 and Mo2 after Berger & Loutre (1987). Here $k = 50.44$ (arcsec/year), where $\overline{\theta}$ denotes the time averaged obliquity. The unit of period is year. The factor 3600×360 denotes the conversion of unit from radian to arcsec.

of the earth. Therefore LOD-periods plot depends only on the rotational angular velocity ω. Figure 4 is the plot using LOD by the horizontal axis. We can obtain a similar result if we take the earth-moon distance as the horizontal axis instead of LOD.

Thus we can establish the standard measurement of relative age, ie., the lap time clock for decoding the history of the earth comparing the stripes on BIF with theoretical Milankovitch cycle frequencies. Since the Milankovitch cycles have been playing an significant role in the climate change on the earth, what we did in this discussion shows another possibility of the co-evolution of the Milankovitch cycles and the earth-moon system. In further researches we will make a detailed studies on the deviation due to these assumptions, and build the more precise model of the evolution of the Milankovitch cycles.

Acknowledgments. The authors have greatly benefitted from several stimulating advice and encouragement from Dr. B.F.Chao of Goddard Space Flight Center, NASA. Detailed and constructive reviews by Kooiti Masuda of Tokyo Metropolitan University have considerably improved the presentation of this paper. This paper was presented at the Multisphere Interaction, Evolution and Rhythm Symposium on December 4 at Earthquake Research Institute, University of Tokyo, 1992.

References

Abe, M., Mizutani, H., Tamura, Y., and Ooe, M. (1992) Tidal evolution of the lunar orbit and the obliquity of the earth, *Proc. ISAS Lunar Planet. Symp.*, **25**, 226–231.

Berger, A.L. and Loutre, M.F. (1987) Origine des fréquences des éléments astronomiques intervenant dans le calcul de l'insolation, in *Sci. Rep.*, 1987/13, Inst Astron Géophys G. Lemaître, Univ. Catholique de Louvain, Louvain-la-Neuve, 45–106.

Berger, A.L., Loutre, M.F., and Laskar, J. (1992) Stability of the astronomical frequency over the earth's history for Paleoclimatic studies, *Science*, **255**, 560–566.

Bills, B.G. (1990) The rigid body obliquity history of Mars, *J. Geophys. Res.*, **95**, 14137–14153.

Denis, C. (1986) On the change of kinetical parameters of the earth during geological times, *Geophys. J. R. Astron. Soc.*, **87**, 559–568.

Dones, L. and Tremaine, S. (1993) Why does the earth spin forward?, *Science*, **259**, 350–354.

Hays, J., Imbrie, J., and Shackleton, N. (1976) Variations in the earth's orbit: pacemaker of the ice ages, *Science*, **194**, 1121–1132.

Kaula, W.M. (1964) Tidal dissipation by solid friction and the resulting orbital evolution, *Rev. Geophys.*, **2**, 661–685.

Laskar, J. (1988) Secular evolution of the solar system over 10 million years, *Astron. Astrophys.*, **198**, 341–362.

Laskar, J. (1990) The chaotic motion of the solar system: A numerical estimate of the size of the chaotic zones, *Icarus*, **88**, 266–291.

Milankovitch, M. (1941) *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitproblem*, Vol. 133 of *Königlich Serbische Academie Publication*, Königlich Serbische Academie.

Newsom, H.E. (1990) Accretion and core formation in the earth: Evidence from siderophile elements, in *Origin of the Earth*, Oxford University Press, New York, 273–288.

Rubincam, D.P. (1990) Mars: Change in axial tilt due to climate?, *Science*, **248**, 720–721.

Sussman, G.J. and Wisdom, J. (1992) Chaotic evolution of the solar system, *Science*, **257**, 56–62.

Turcotte, D. and Schubert, G. (1982) *Geodynamics — Application of Continuum Physics to Geological Problems*, John Wiley & Sons, Santa Barbara.

Turcotte, D., Cisne, J., and Nordmann, J. (1977) On the evolution of the lunar orbit, *Icarus*, **30**, 254–266.

Walker, J.C.G. and Zahnle, K.J. (1986) Lunar nodal tide and distance to the Moon during the Precambrian, *Nature*, **320**, 600–602.

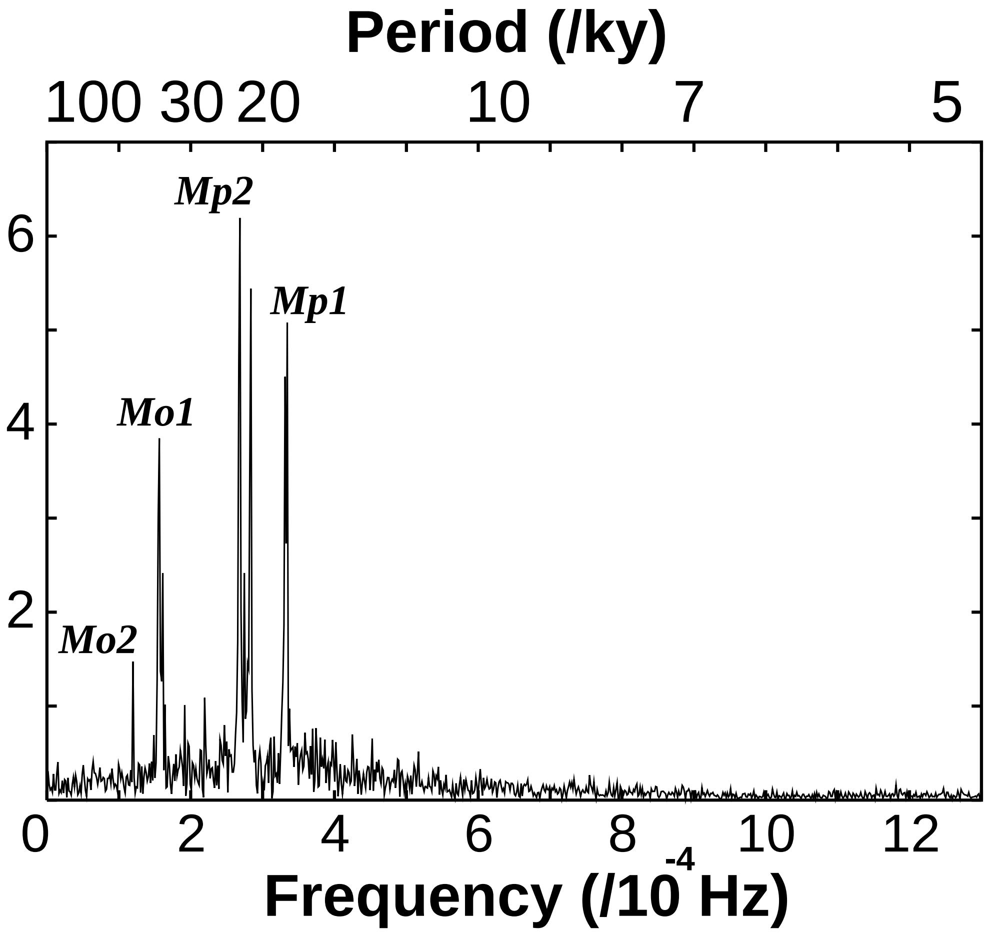
Ward, W.R. (1974) Climatic variations on Mars, 1, Astronomical theory of insolation, *J. Geophys. Res.*, **79**, 3375–3386.

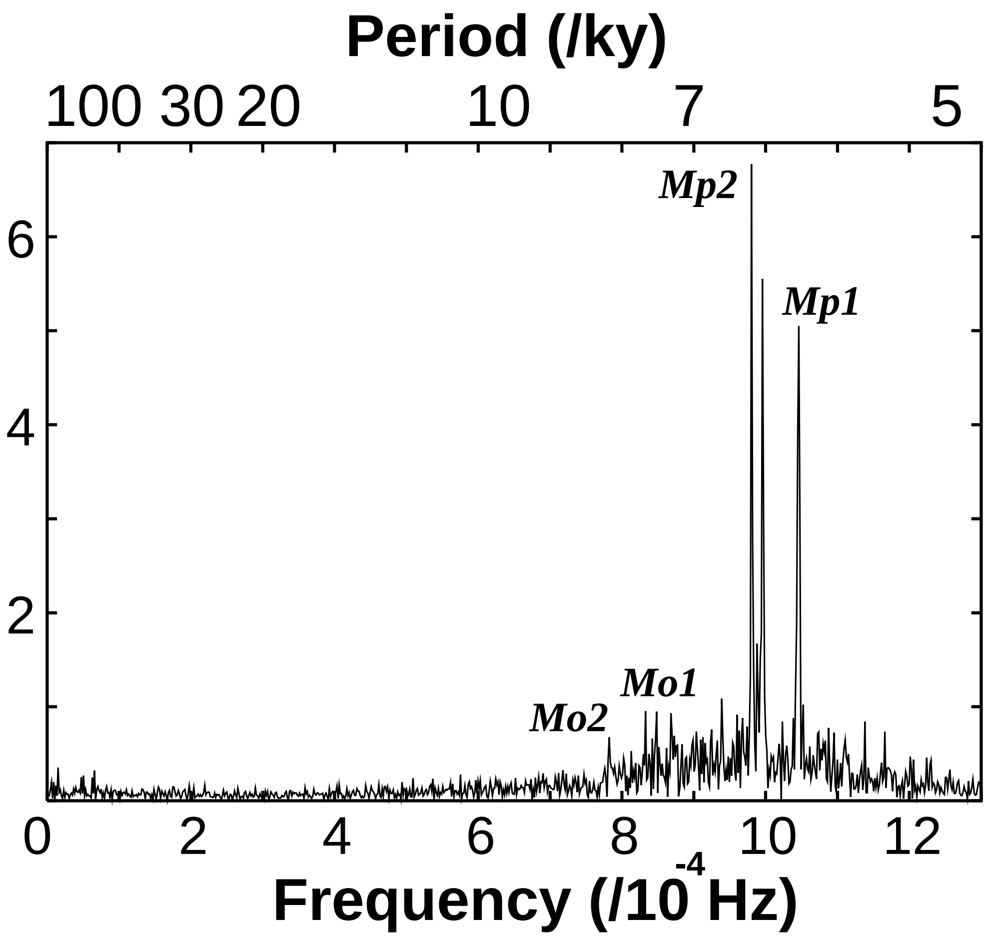
Figure 1. Power spectrum of the daily averaged solar insolation variation on 65°N at summer solstice. (Upper) present one. Notice on the peak of about 19Kyr (Mp1), 23Kyr (Mp2), 41Kyr (Mo1) and 54Kyr (Mo2). $Mp2$ is a doublet of two sharp spectral peaks owing to the modulation of several eigen frequencies of orbital parameters. But in practice these peaks are observed as a single and broad peak in the power spectrum of the time series of geological data such as δ^{18}O anomaly, because this doublet will be totally smoothed out by various processes of surface climate system on the earth (Hays *et al.*, 1976). (Lower) $-3Ga$ one. Notice that the major frequencies are all shifted toward the short periods, and the power of the contribution of obliquity (Mo1 and Mo2) are much lower than the present one.

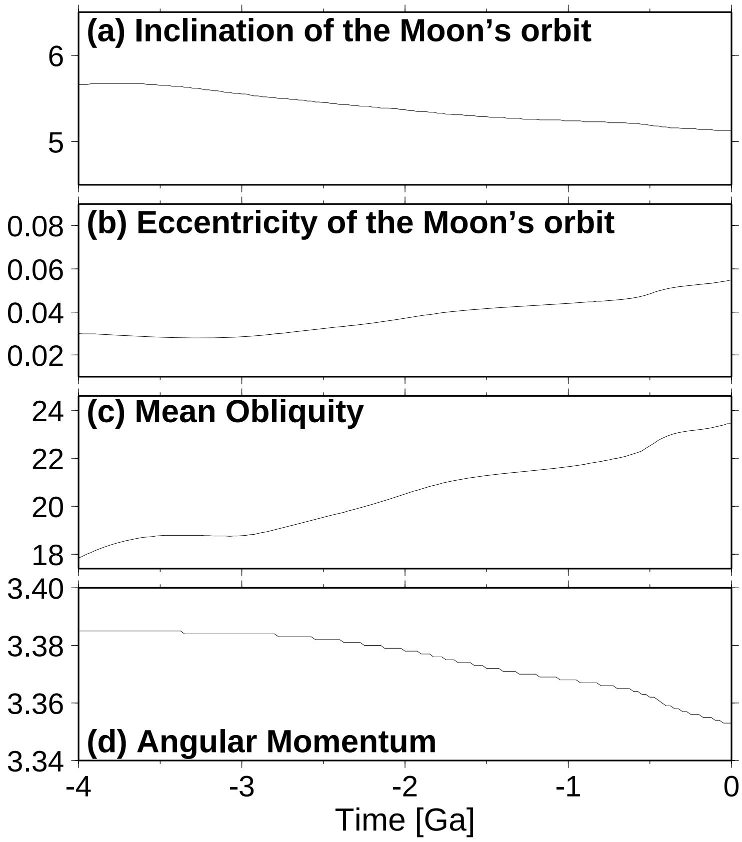
Figure 2. Orbital inclination (with respect to the earth's ecliptic) and eccentricity of the moon, mean obliquity of the earth and the total angular momentum of the earth-moon system after Abe *et al.* (1992). (a) Inclination of the moon i_m (degree) (b) eccentricity of the moon e_m (non-dimensional), (c) Mean obliquity (degree), (d) the angular momentum of the earth-moon system ($10^{34}\mathrm{Kg}\cdot\mathrm{m}^2/\mathrm{s}^2$). In the model of Turcotte *et al.* (1977) $e_m = i_m = 0$, and the mean obliquity is not taken into account (they only considered the secular change of the rotational angular velocity of the earth, no consideration about the precession or obliquity), and the total angular momentum is kept constant.

Figure 3. Transition diagram of the Milankovitch cycles including the dynamical evolution of the earth-moon system. (a) Length of Day (LOD, present value equals to the unity), (b) Distance between the earth and the moon (radius of the earth R_E), (c) Dynamical ellipticity of the earth (non-dimensional), (d) Precessional constant α (arcsec/year), (e) Evolution paths of the major periods of the Milankovitch cycles $Mp1$ (solid line), $Mp2$ (dashed line), $Mo1$ (dot-dash line), the unit is year. The solid lines in the column of (a)(b)(c)(d) are the result using the tidal model of Abe *et al.* (1992) and the dotted lines are using Turcotte *et al.* (1977). In (e) the three thick lines are from Abe *et al.* (1992) and the three thin lines are from Turcotte *et al.* (1977).

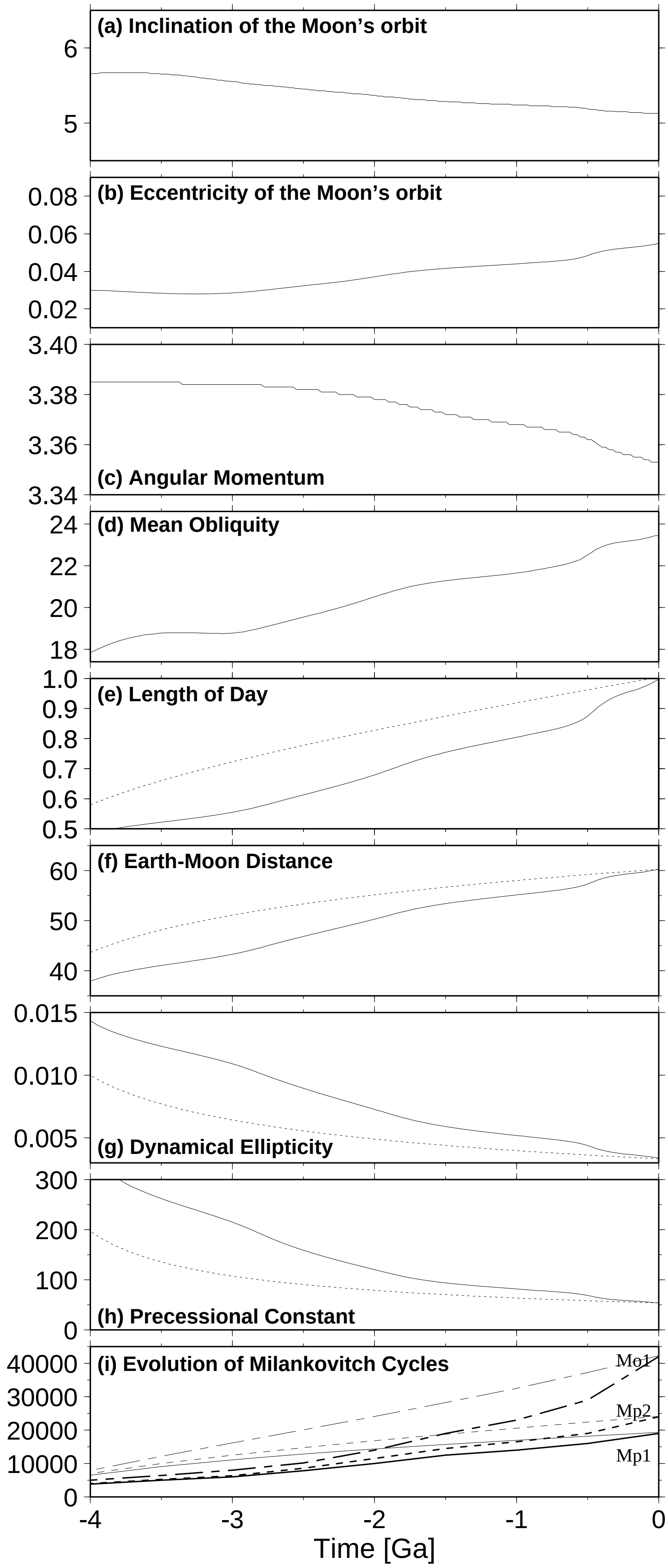
Figure 4. Same as the Figure 3 (e), but the horizontal axis is Length of Day (LOD) instead of absolute age. In this case there is no difference between each models.


Period (/ky)
100 30 20 10 7 5
Mp2
Mp1
Mo1
Mo2
6
4
2
0 2 4 6 8 10 12
Frequency (/10^-4 Hz)

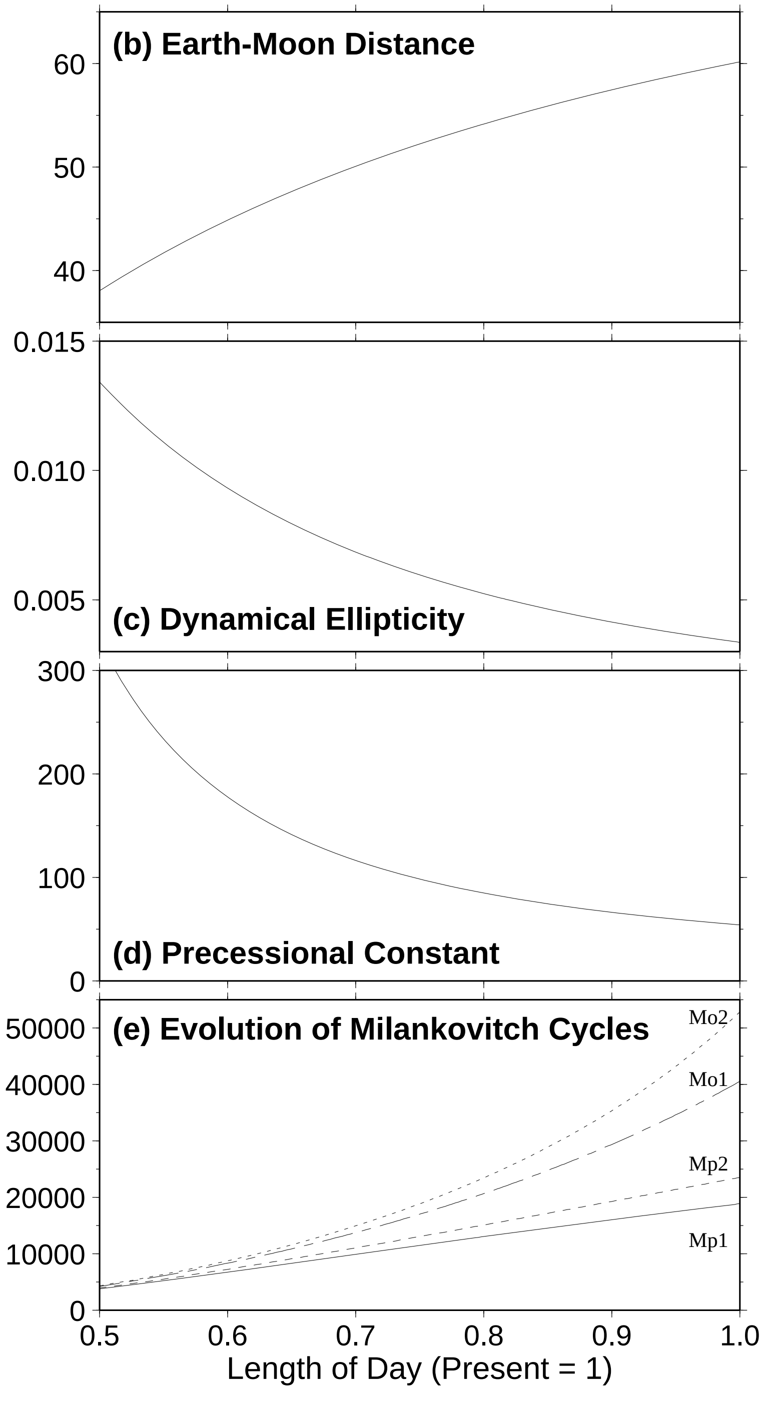

Period (/ky)
100 30 20 10 7 5
Mp2
Mp1
Mo1
Mo2
6
4
2
0 2 4 6 8 10 12
Frequency (/10^-4 Hz)



(a) Inclination of the Moon's orbit
6
5
(b) Eccentricity of the Moon's orbit
0.08
0.06
0.04
0.02
(c) Mean Obliquity
24
22
20
18
3.40
3.38
3.36
3.34
(d) Angular Momentum
-4
-3
-2
-1
0
Time [Ga]



(a) Inclination of the Moon's orbit
(b) Eccentricity of the Moon's orbit
(c) Angular Momentum
(d) Mean Obliquity
(e) Length of Day
(f) Earth-Moon Distance
(g) Dynamical Ellipticity
(h) Precessional Constant
(i) Evolution of Milankovitch Cycles
Mo1
Mp2
Mp1
Time [Ga]



(b) Earth-Moon Distance
(c) Dynamical Ellipticity
(d) Precessional Constant
(e) Evolution of Milankovitch Cycles
Mo2
Mo1
Mp2
Mp1
Length of Day (Present = 1)