重力相互作用計算の工夫 II

周期境界問題

矢作　日出樹

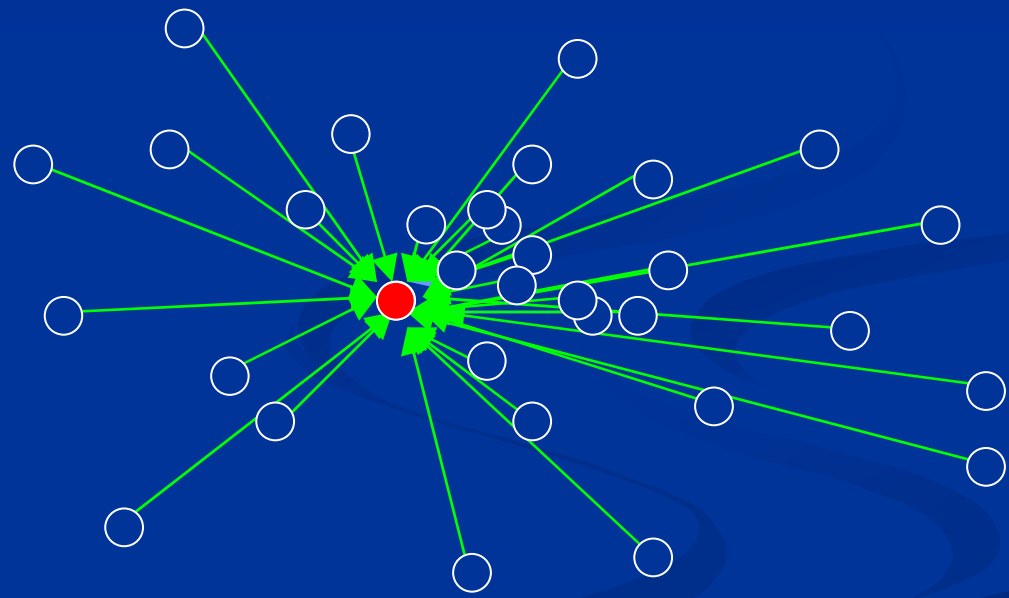
国立天文台天文学データ解析計算センター

N体法

- 開放境界(天体の単体形成)
  - 直接総和法
  - ツリー法
- 周期境界(統計的性質)
  - PM
  - P^3M
  - AMR

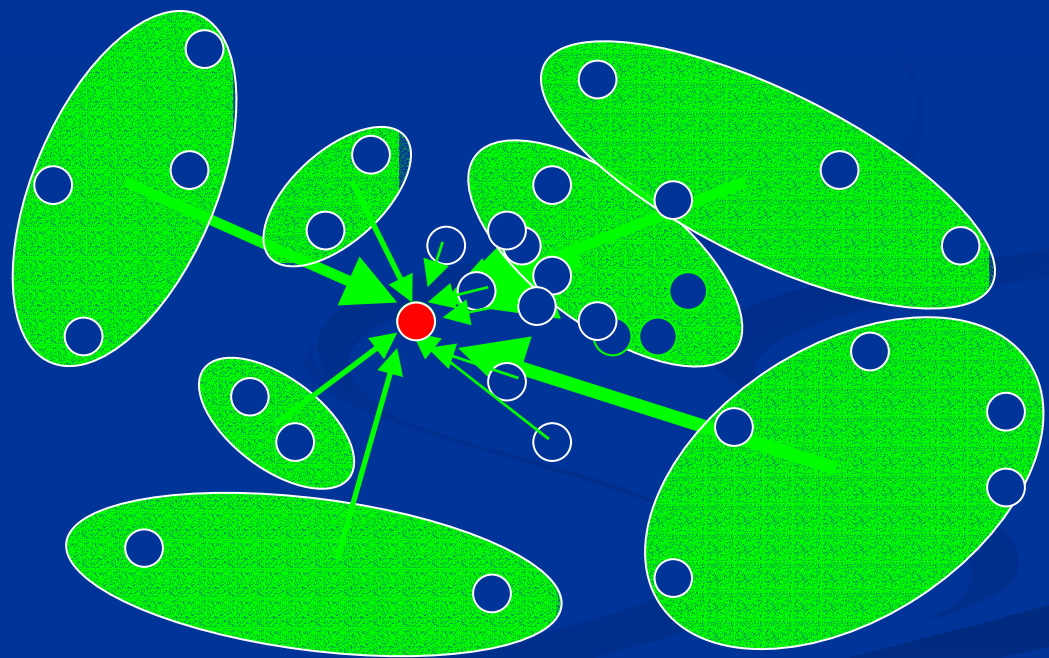
N-body Methods

- Direct sum method
  - Calculate force from each particle
  - $O(N^2)$



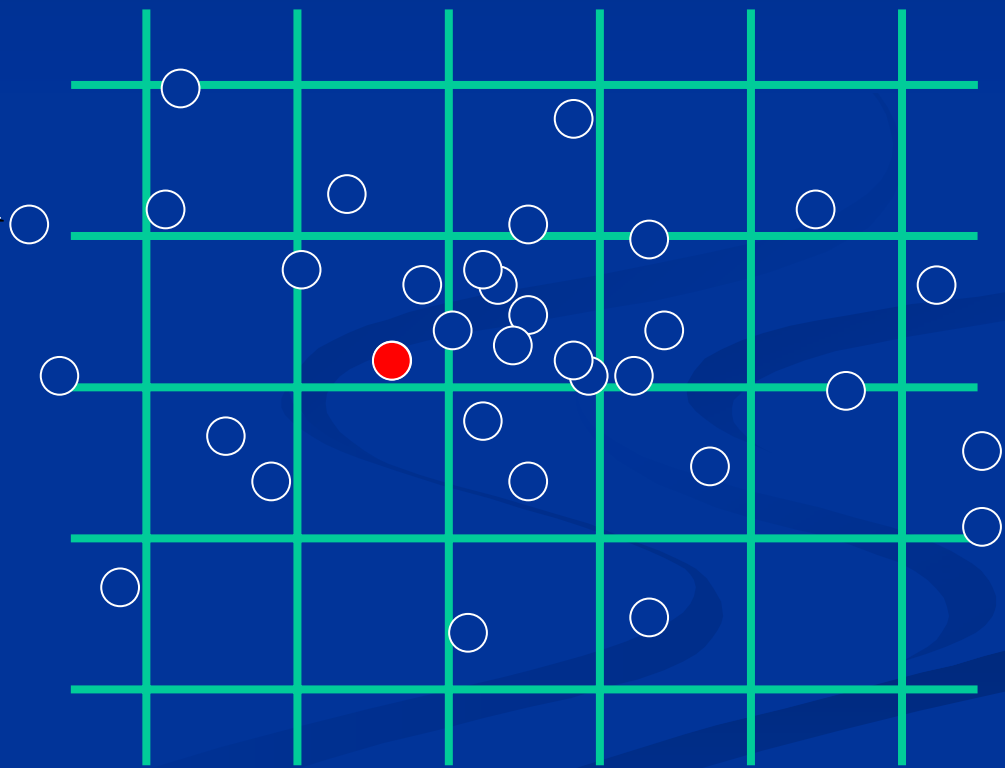
N-body Methods

- Tree method
  - Distant particles are bundled up
  - $O(N \log N)$



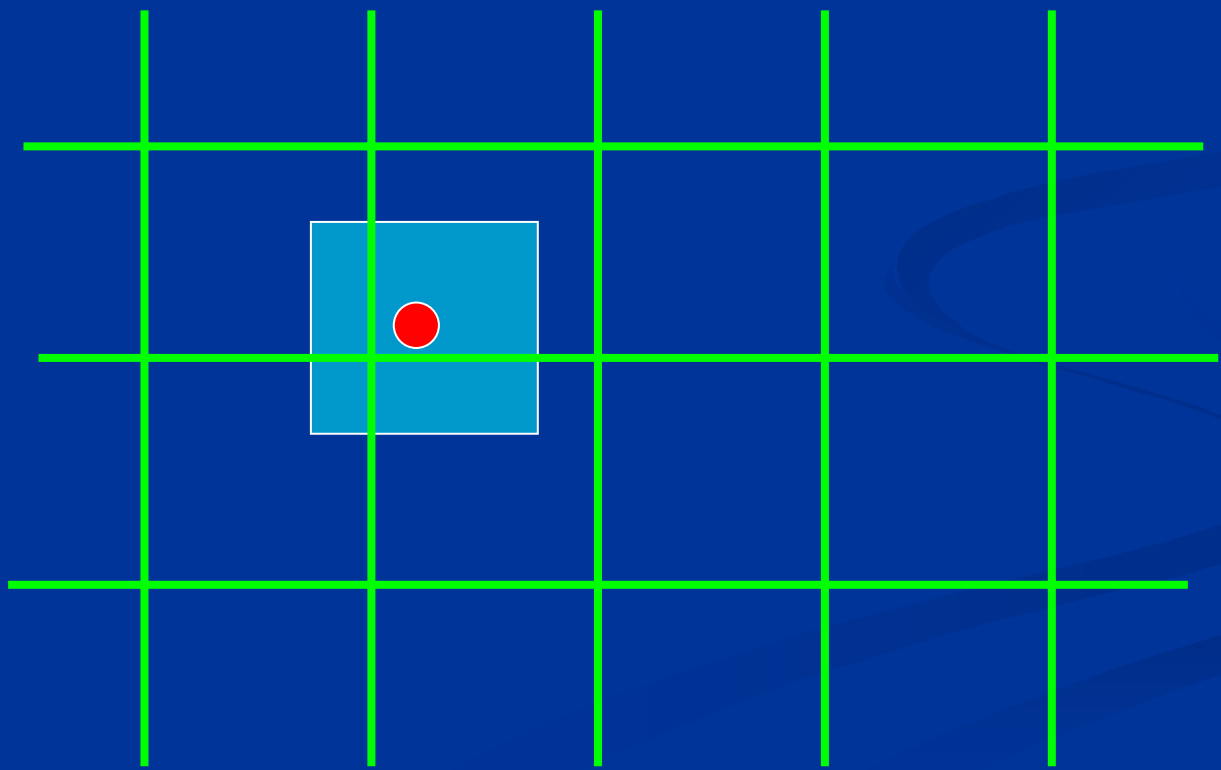
N-body Methods

- Particle-mesh (PM) method
  - Mass assignment
  - Poisson solver
  - Force interpolation
  - $O(N \log N)$



PM法

- Mass assignment (Cloud-in-Cell Scheme)



PM法

- Mass assignment (Cloud-in-Cell Scheme)

```
for (i=0; i<np; i++){
    ix=(int) pos[ip][0]; dx=pos[ip][0] – (double)ix;
    iy=(int) pos[ip][1]; dy=pos[ip][1] – (double)iy;
    iz=(int) pos[ip][2]; dz=pos[ip][2] – (double)iz;
    rho[ix  ][iy  ][iz  ] += mass[ip] * (1-dx) * (1-dy) * (1-dz);
    rho[ix  ][iy  ][iz+1] += mass[ip] * (1-dx) * (1-dy) * (  dz);
    rho[ix  ][iy+1][iz  ] += mass[ip] * (1-dx) * (  dy) * (1-dz);
    rho[ix  ][iy+1][iz+1] += mass[ip] * (1-dx) * (  dy) * (  dz);
    rho[ix+1][iy  ][iz  ] += mass[ip] * (  dx) * (1-dy) * (1-dz);
    rho[ix+1][iy  ][iz+1] += mass[ip] * (  dx) * (1-dy) * (  dz);
    rho[ix+1][iy+1][iz  ] += mass[ip] * (  dx) * (  dy) * (1-dz);
    rho[ix+1][iy+1][iz+1] += mass[ip] * (  dx) * (  dy) * (  dz);
}
```

PM法

- Poisson Solver (Convolution method)
  - $\Delta\phi = 4\pi G\rho$
  - $k^2\phi_k = 4\pi G\rho_k$
  - ρ ー(FFT)→ ρ k ー(conv.)→ ϕ k ー(FFT)→ ϕ

```
fft3d_real_forward (n, rho);
for (kx=0; kx<n; kx++){
    for (ky=0; ky<n; ky++){
       for (kz=0; kz<n; kz++){
                    phi[kx][ky][kz]=4.0 * PI * G * rho[kx][ky][kz]/ (kx*kx+ky*ky+kz*kz);
       }
    }
}
fft3d_real_backward(n, phi);
```

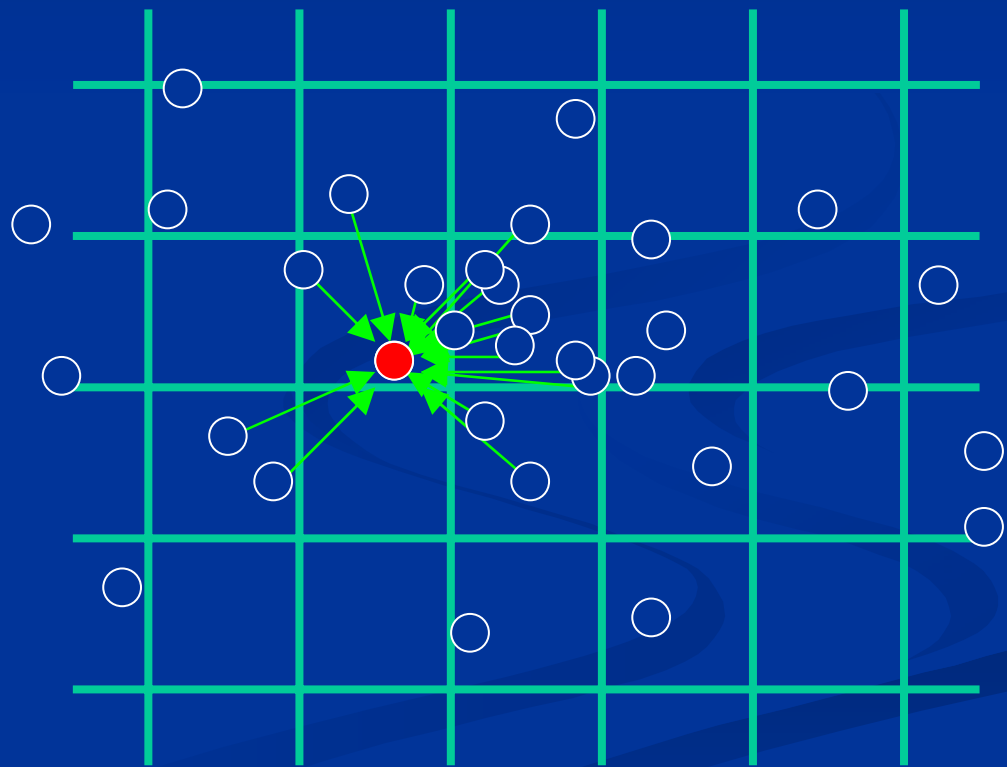
PM法

- Force interpolation (Cloud-in-Cell Scheme)

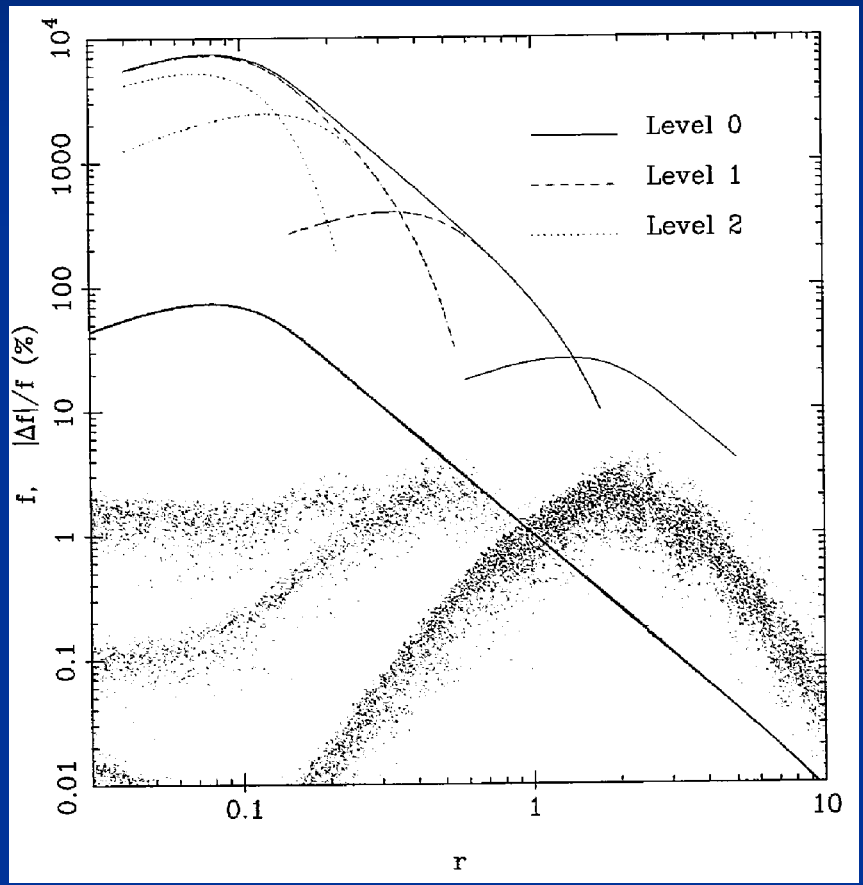
```
for (i=0; i<np; i++){
    ix=(int) pos[ip][0]; dx=pos[ip][0] – (double)ix;
    iy=(int) pos[ip][1]; dy=pos[ip][1] – (double)iy;
    iz=(int) pos[ip][2]; dz=pos[ip][2] – (double)iz;
    acc[ip] =
      grv[ix  ][iy  ][iz  ] * (1-dx) * (1-dy) * (1-dz)+
      grv[ix  ][iy  ][iz+1] * (1-dx) * (1-dy) * (  dz)+
      grv[ix  ][iy+1][iz  ] * (1-dx) * (  dy) * (1-dz)+
      grv[ix  ][iy+1][iz+1] * (1-dx) * (  dy) * (  dz)+
      grv[ix+1][iy  ][iz  ] * (  dx) * (1-dy) * (1-dz)+
      grv[ix+1][iy  ][iz+1] * (  dx) * (1-dy) * (  dz)+
      grv[ix+1][iy+1][iz  ] * (  dx) * (  dy) * (1-dz)+
      grv[ix+1][iy+1][iz+1] * (  dx) * (  dy) * (  dz);
}
```

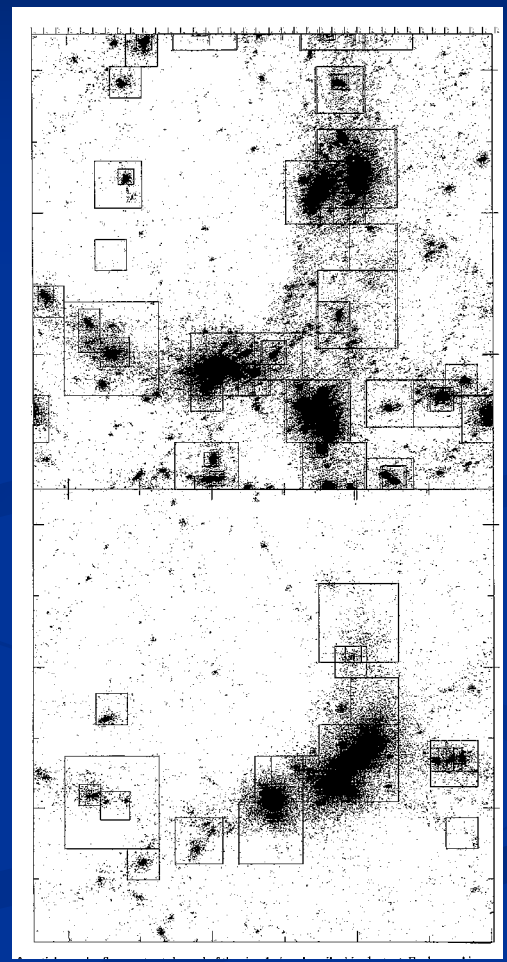
N-body Methods

- Particle-particle particle-mesh (P^3M) method
  - Distant particles
    - PM
  - Nearby particles
    - Dierct sum



AP^{3}M法

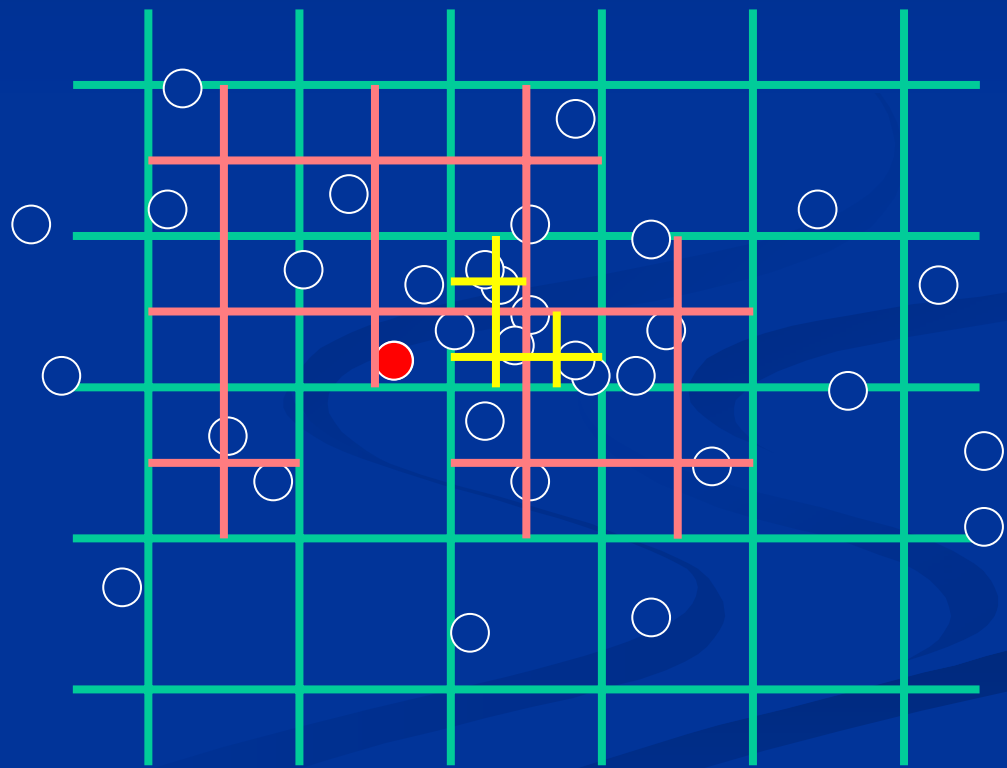




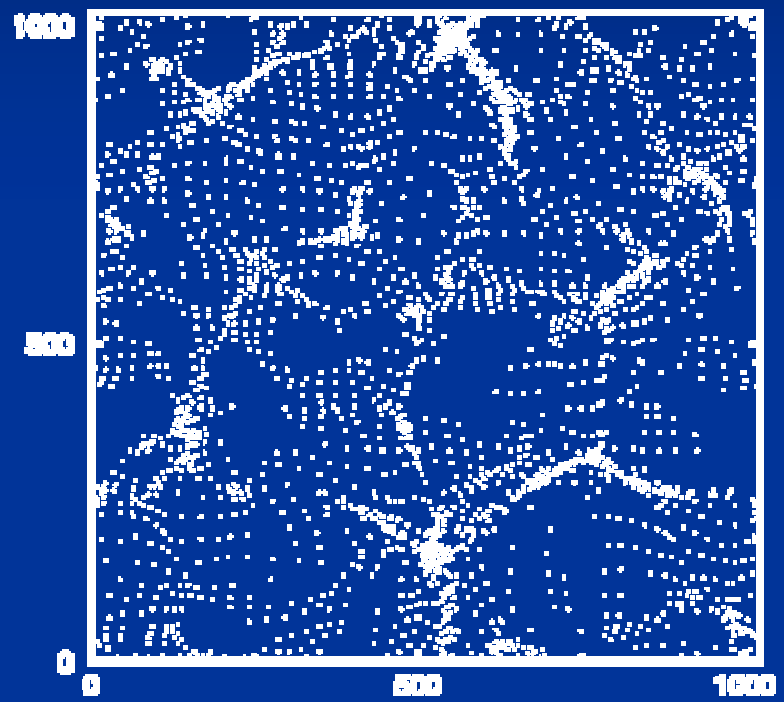
Couchman (1991)

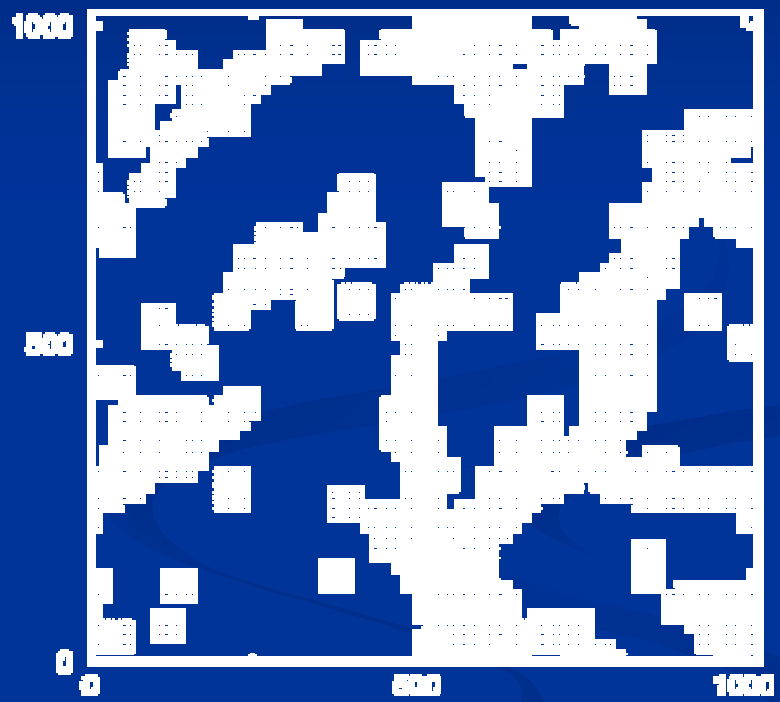
N-body Methods

- Adaptive Mesh Refinement
  - Divide cells only where higher resolution required



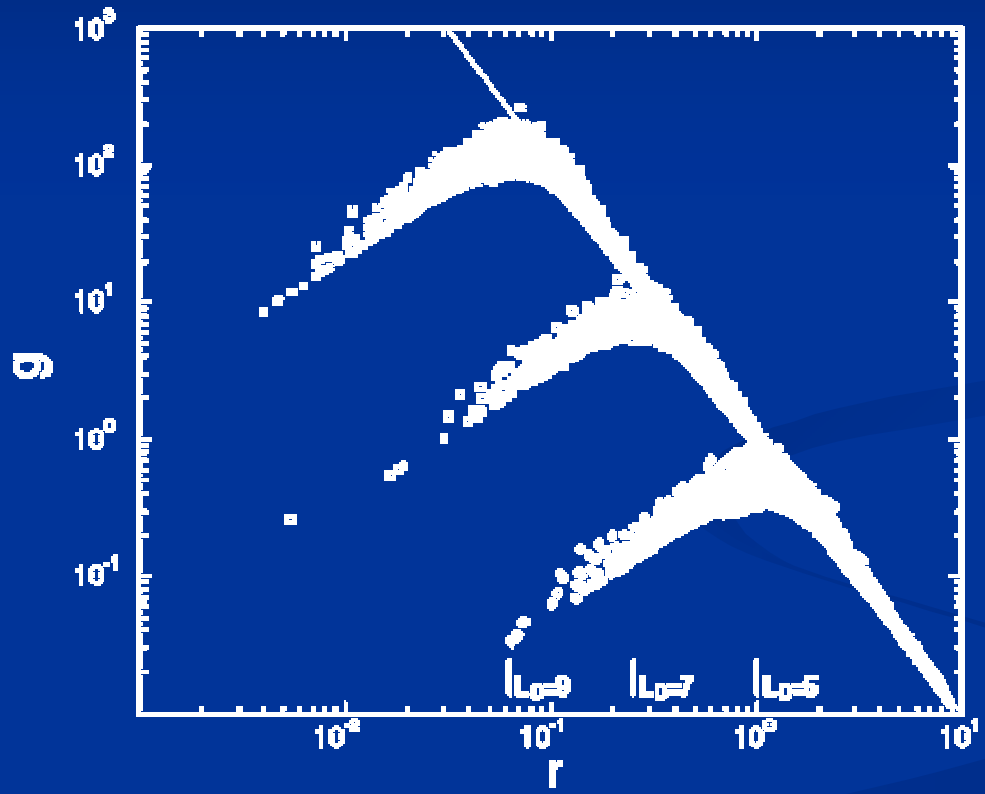
AMR法





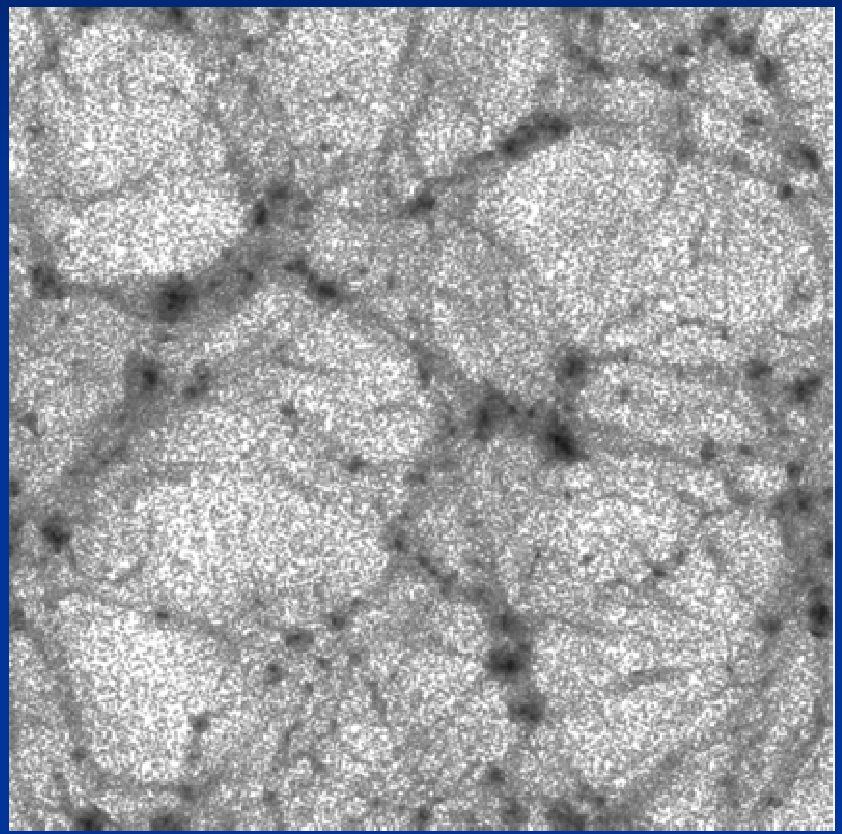
Yahagi & Yoshii (2001)

AMR法

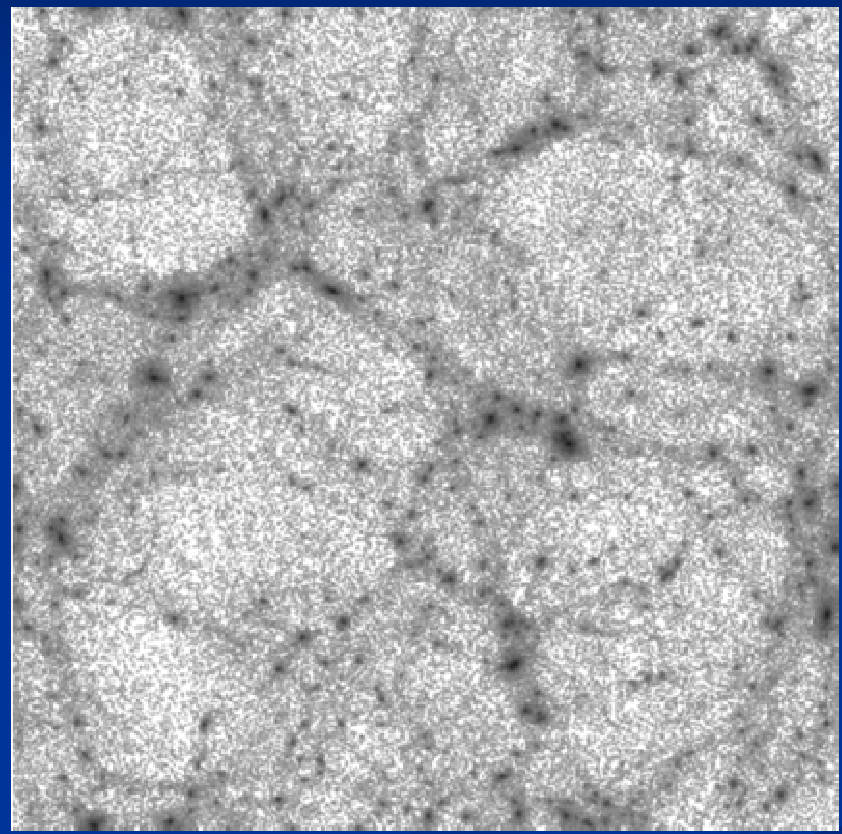


Yahagi & Yoshii (2001)

AMR法



PM



AMR

Yahagi & Yoshii (2001)

習うより慣れましょう

- ツリー法
  - Treecode by Barnes (http://www.ifa.hawaii.edu/~barnes/software.html)
- P^3M
  - p3m by Bertschinger (ftp://arcturus.mit.edu/Software/)
- AP^3M
  - hydra by Couchman et al. (http://hydra.mcmaster.ca/hydra/)
- AMR
  - MLAPM by Knebe (http://astronomy.swin.edu.au/staff/aknebe/MLAPM/)

終