Rayleigh-Taylor不安定性が相对論的ジェットで成長する条件

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What is a relativistic jet?

collimated bipolar outflow from gravitationally bounded object

\[
\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}
\]

\begin{itemize}
  \item active galactic nuclei (AGN) jet: \( \gamma \sim 10 \)
  \item microquasar jet: \( v \sim 0.9c \)
  \item Gamma-ray burst: \( \gamma > 100 \)
\end{itemize}
Morphological Dichotomy of the Jet

3C 31

FR I

Cygnus A.

FR II

- Morphology is one of the most fundamental property of the relativistic jet.
- A morphological dichotomy between FR I and FR II
  - A complex combination of several intrinsic and external factors
- Instabilities play an important role in the morphology and stability of the jet through the interaction between the jet and external medium.
Modeling for the growth of RTI

initially hydrostatic equilibrium

\[-\frac{\partial P}{\partial y} = \gamma^2 \rho h g = \gamma^2 \left( \rho + \frac{\Gamma}{\Gamma - 1} \frac{P}{c^2} \right) g\]

if \( y = 0 \), then \( P = P_0 \)

\[P = P_0 e^{-y/H} + \frac{\rho c^2(\Gamma - 1)}{\Gamma}(e^{-y/H} - 1)\]

\[\frac{1}{H} = \frac{\Gamma - 1}{\Gamma} \frac{\gamma^2 g}{c^2}\]

initial perturbation

\[\nu_y = \frac{\delta v}{4} (1 + \cos(2\pi x))(1 + \cos(2\pi x/10))\]

\(\delta v = 10^{-4}\)

restoring force of the oscillation

effective gravity

jet boundary

jet cross-section
Derivation of linear growth of RTI

equation of motion:  
\[
\gamma^2 \rho h \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] + \nabla P + \frac{\mathbf{v} \cdot \mathbf{P}}{c^2} \frac{\partial P}{\partial t} - \gamma^2 \rho h \mathbf{g} = 0
\]

assumption:  \( v_x, v_y \ll c_s \)
  temporal variation of the pressure is negligible

linearized equations:

equation of motion: \( x \)  
\[
\gamma^2 \rho h \frac{\partial v_x}{\partial t} = - \frac{\partial \delta P}{\partial x}
\]
equation of motion: \( y \)  
\[
\gamma^2 \rho h \frac{\partial v_y}{\partial t} = - \frac{\partial \delta P}{\partial y} - \gamma^2 \left( \delta \rho + \frac{\Gamma}{\Gamma - 1} \frac{\delta P}{c^2} \right) g
\]
continuity  
\[
\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0
\]
incompressible condition  
\[
\frac{\partial (\gamma \delta \rho)}{\partial t} = -v_y \frac{\partial (\gamma \rho)}{\partial y}
\]
conservation of entropy  
\[
\frac{\partial}{\partial t} \left( \frac{\delta P}{P} - \Gamma \frac{\delta \rho}{\rho} \right) + v_y \left( \frac{1}{P} \frac{\partial P}{\partial y} - \Gamma \frac{1}{\rho} \frac{\partial \rho}{\partial y} \right) = 0
\]
Derivation of linear growth of RTI

Equation of motion:

$$\gamma^2 \rho h \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] + \nabla P + \frac{\mathbf{v} \cdot \nabla P}{c^2} \frac{\partial P}{\partial t} - \gamma^2 \rho h g = 0$$

Assumption: $$v_x, v_y \ll c_s$$

Temporal variation of the pressure is negligible:

$$\delta \rho, \delta P, v_x, v_y \propto e^{i(kx - \omega t)}$$

Equation of motion: x

$$i \omega \gamma^2 \rho h v_x = ik \delta P$$

Equation of motion: y

$$i \omega \gamma^2 \rho h v_y = \frac{\partial \delta P}{\partial y} + \gamma^2 \left( \delta \rho + \frac{\Gamma}{\Gamma - 1} \frac{\delta P}{c^2} \right) g$$

Continuity

$$ik v_x = -\frac{\partial v_y}{\partial y}$$

Incompressible condition

$$i \omega \gamma \delta \rho = v_y \frac{\partial (\gamma \rho)}{\partial y}$$

Conservation of entropy

$$\frac{\delta P}{P} = \Gamma \frac{\delta \rho}{\rho} - \frac{v_y}{i \omega} \left( \frac{\gamma^2 \rho h g}{P} + \frac{\Gamma - 1}{\rho} \frac{\partial \rho}{\partial y} \right)$$
Derivation of linear growth of RTI

**Equation of motion:**

\[
\gamma^2 \rho h \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] + \nabla P + \frac{\mathbf{v} \cdot \frac{\partial P}{\partial t}}{c^2} - \gamma^2 \rho h g = 0
\]

**Assumption:** \( v_x, v_y \ll c_s \)

- Temporal variation of the pressure is negligible

**Dispersion relation**

\[
D_y (\omega^2 \gamma^2 \rho h D_y v_y) - k^2 \left( \omega^2 - \frac{g}{H} \right) \gamma^2 \rho h v_y = k^2 \gamma \left( D_y (\gamma \rho) + D_y \gamma \frac{\Gamma^2}{\Gamma - 1} \frac{P}{c^2} \right) g v_y
\]

\[
v_y = \begin{cases} 
  A e^{-ky}, & y > 0 \\
  A e^{ky}, & y < 0 
\end{cases}
\]

**Non-relativistic case**

\[
\omega = i \sqrt{g k \frac{\rho_j h_j' - \gamma_j \rho_c h_c'}{\gamma_j \rho_j h_j + \gamma_c \rho_c h_c}} \quad h' = 1 + \frac{\Gamma^2}{\Gamma - 1} \frac{P}{\rho c^2}
\]

\[
\omega = i \sqrt{g k \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}}
\]
Modeling for the growth of RTI

dispersion relation
$$\omega = i \sqrt{\frac{\gamma_j^2 \rho_j h_j' - \gamma_c^2 \rho h_c'}{\gamma_j^2 \rho_j h_j + \gamma_c^2 \rho h_c}}$$

equation of the inertia
$$\gamma_j^2 \rho_j h_j' \sim \gamma_j^2 P_j$$
$$\gamma_c^2 \rho h_c' \sim P_c$$
$$P_j \sim P_c$$
$$\gamma_j^2 \rho_j h_j' - \gamma_c^2 \rho h_c' > 0$$

RTI grows at relativistic jet boundary

growth of the RTI

jet cross-section

jet boundary

cocoon

simple setting

jet head
Numerical Setting: 3D Toy Model

- cylindrical coordinate
- relativistic jet ($z$-direction)
- ideal gas
- numerical scheme: HLLC (Mignone & Bodo 05)
- uniform grid: $\Delta r = \Delta z = 0.1$, $\Delta \theta = 2\pi/160$
two key parameters

- the effective inertia ratio of the jet to the ambient medium: 
  \[ \eta_{j,a} = \frac{\gamma_j \rho_j h_j}{\rho_a} \]

Neglecting the multi-dimensional effect, the propagation velocity of the jet head through a cold ambient medium can be evaluated by balancing the momentum flux of the jet and the ambient medium in the frame of the jet head (Marti+ 97, Mizuta+ 04):

\[ v_h = \frac{\sqrt{\eta_{j,a}}}{1 + \sqrt{\eta_{j,a}}} v_j \]

- dimension less specific enthalpy of the jet: \( h_j \)
In 3D case, we calculated the numerical flux to all directions.

### Basic Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial}{\partial t} (\gamma \rho) + \nabla \cdot (\gamma \rho \mathbf{v}) = 0 )</td>
<td>Mass conservation</td>
</tr>
<tr>
<td>( \frac{\partial}{\partial t} (\gamma^2 \rho h \mathbf{v}) + \nabla \cdot (\gamma^2 \rho h \mathbf{v} \mathbf{v} + P \mathbf{I}) = 0 )</td>
<td>Momentum conservation</td>
</tr>
<tr>
<td>( \frac{\partial}{\partial t} (\gamma^2 \rho h c^2 - P) + \nabla \cdot (\gamma^2 \rho h c^2 \mathbf{v}) = 0 )</td>
<td>Energy conservation</td>
</tr>
</tbody>
</table>

### Specific Enthalpy, Ratio of Specific Heats, Lorentz Factor

- Specific enthalpy: \( h = 1 + \frac{\Gamma}{\Gamma - 1} \frac{P}{\rho c^2} \)
- Ratio of specific heats: \( \Gamma = \frac{4}{3} \)
- Lorentz factor: \( \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \)
The amplitude of the corrugated jet interface grows due to the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities. Since the relativistic jet is continuously injected into the calculation domain, standing reconfinement shocks are formed.
As predicted analytically, the effective inertia of the jet becomes larger than the cocoon envelope for all the models.
The oscillation-induced Rayleigh-Taylor instability is responsible for the distortion of the cross section at $z = 30$.

Finger-like structures appeared in the cross-section at $z = 65$ and $90$ are outcome of both the Rayleigh-Taylor and Richtmyer-Meshkov instabilities.

$\partial_t (\gamma \rho f) + \nabla \cdot (\gamma \rho f \mathbf{v}) = 0$

passive tracer: $f$

$f = 1$: jet material

$f = 0$: ambient medium
Inherent Property of Relativistic Jet

Although the relativistic jet shows a rich variety of the propagation dynamics depending on its launching condition, the oscillation-induced Rayleigh-Taylor instability and secondary Richtmyer-Meshkov instabilities grow commonly at the jet interface and then induce a lot of finger-like structures.

- radial expansion (hot models)
- radial contraction (cold models)

After initial stage, the cold jet also follows the same evolution path as the hot jet and thus excites the Rayleigh-Taylor and Richtmyer-Meshkov instabilities.
Summary

dispersion relation
$$\omega = i \sqrt{gk \frac{\gamma_j^2 \rho_j h_j' - \gamma_c^2 \rho_c h_c'}{\gamma_j^2 \rho_j h_j + \gamma_c^2 \rho_c h_c}}$$

estimation of inertia
$$\gamma_j^2 \rho_j h_j' \sim \gamma_j^2 P_j$$
$$\gamma_c^2 \rho_c h_c' \sim P_c$$

$$P_j \sim P_c$$

$$\gamma_j^2 \rho_j h_j' - \gamma_c^2 \rho_c h_c' > 0$$

RTI grows at relativistic jet boundary