スペクトラルニュートリノ輻射輸送法を用いた大質量星の重力崩壊計算

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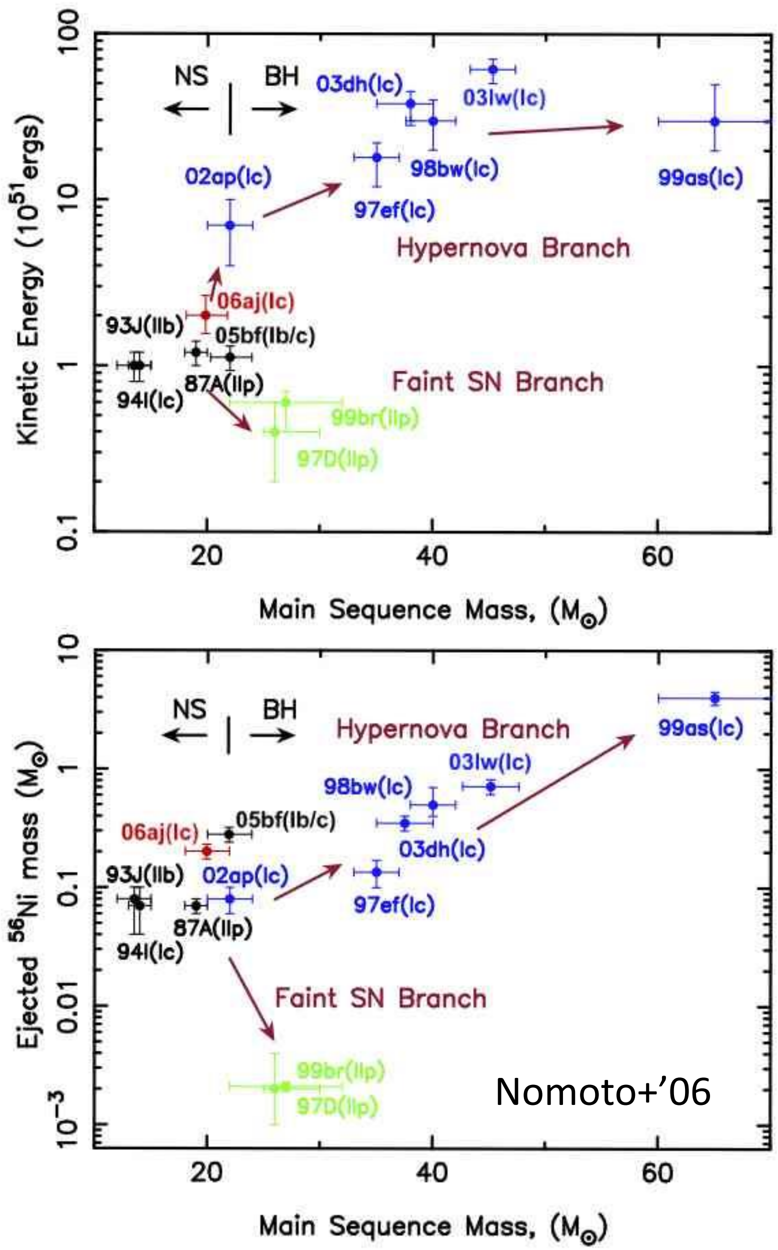
Gaps Between Observations and Theory



Our final goal

First principal simulation

Current status

Successful Explosion,
but low explosion energy

We still have to explain

- Explosion energy
- Two branches
- Ejected elements ……..

Toward Full 3DGR Multi-Energy ν-Transport Models

より“現実”に近づけた数値コードでの計算が必要

今後超新星計算に取り入れるべき内容

- 一般相対論
- 3次元
- ニュートリノ輸送

ただし、Full GR Boltzmann eq. (6+1D)での計算[1]は
現在の計算機資源（多分次世代でも）では現実的に難しい。

[1]Sumiyoshi&Yamada,’12, Nagakura+,’14

Toward Full 3DGR Multi-Energy ν-Transport Models

そこで、ニュートリノ輻射に関しては角度依存をつぶしたモーメンタム法を採用した[1]。 [1]Shibata+,'11

$$\left(J_{(\varepsilon)}, H^{\mu}_{(\varepsilon)}, L^{\mu\nu}_{(\varepsilon)}\right) \equiv \varepsilon^3 \int d\Omega f(\varepsilon,\Omega)(1, l^{\mu}, l^{\mu} l^{\nu})$$

$$\begin{aligned} T^{\mu\nu}_{(\varepsilon)} &= E_{(\varepsilon)} n^{\mu} n^{\nu} + F^{\mu}_{(\varepsilon)} n^{\nu} + F^{\nu}_{(\varepsilon)} n^{\mu} + P^{\mu\nu}_{(\varepsilon)} \\ &= J_{(\varepsilon)} u^{\mu} u^{\nu} + H^{\mu}_{(\varepsilon)} u^{\nu} + H^{\nu}_{(\varepsilon)} u^{\mu} + L^{\mu\nu}_{(\varepsilon)}. \end{aligned}$$

（1次の角運動量までを輸送するM1法）

$$\left\{\begin{aligned} &\nabla_{\mu} G^{\mu\nu} = \nabla_{\mu} T^{\mu\nu} = 0 \\ &T^{\alpha\beta}_{(\mathrm{total})} = T^{\alpha\beta}_{(\mathrm{fluid})} + \int d\varepsilon \sum_{\nu \in \nu_e, \bar{\nu}_e, \nu_x} T^{\alpha\beta}_{(\nu,\varepsilon)} \end{aligned}\right.$$

BSSN equations (17 variables)

$$
\begin{aligned}
(\partial_t - \mathcal{L}_\beta)\tilde{\gamma}_{ij} &= -2\alpha\tilde{A}_{ij} \\
(\partial_t - \mathcal{L}_\beta)\phi &= -\alpha K/6 \\
(\partial_t - \mathcal{L}_\beta)\tilde{A}_{ij} &= e^{-4\phi}\left[\alpha(R_{ij} - 8\pi(S_{ij} + P_{ij})) - D_i D_j \alpha\right]^{\mathrm{trf}} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{\gamma}^{kl}\tilde{A}_{jl}) \\
(\partial_t - \mathcal{L}_\beta)K &= -\Delta\alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + K^2/3) + 4\pi\alpha(S_0 e^{-6\phi} + E + \gamma^{ij}(S_{ij} + P_{ij})) \\
(\partial_t - \beta^k\partial_k)\Gamma^i &= -16\pi\tilde{\gamma}^{ij}(S_j e^{-6\phi} + F_j) \\
&\quad -2\alpha(\frac{2}{3}\tilde{\gamma}^{ij}K_{,j} - 6\tilde{A}^{ij}\phi_{,j} - \tilde{\Gamma}^i_{jk}\tilde{A}^{jk}) \\
&\quad +\tilde{\gamma}^{jk}\beta^i_{,jk} + \frac{1}{3}\tilde{\gamma}^{ij}\beta^k_{,kj} - \tilde{\Gamma}^j\beta^i_{,j} + \frac{2}{3}\tilde{\Gamma}^i\beta^j_{,j} + \beta^j\tilde{\Gamma}^i_{,j} - 2\tilde{A}^{ij}\alpha_{,j}
\end{aligned}
$$

Hydrodynamic equations (6 variables)

$$\partial_t\rho_* + \partial_i(\rho_* v^i) = 0,$$

$$
\begin{aligned}
&\partial_t\sqrt{\gamma}S_i + \partial_j\sqrt{\gamma}(S_i v^j + \alpha P\delta^j_i) = \\
&\quad -\sqrt{\gamma}\left[S_0\partial_i\alpha - S_k\partial_i\beta^k - 2\alpha S^k_k\partial_i\phi + \alpha e^{-4\phi}(S_{jk} - P\gamma_{jk})\partial_i\tilde{\gamma}^{jk}/2 + \alpha\int d\varepsilon S^\mu_{(\varepsilon)}\gamma_{i\mu}\right]
\end{aligned}
$$

$$
\begin{aligned}
&\partial_t\sqrt{\gamma}\tau + \partial_i\sqrt{\gamma}(\tau v^i + P(v^i + \beta^i)) = \\
&\quad \sqrt{\gamma}\left[\alpha K S^k_k/3 + \alpha e^{-4\phi}(S_{ij} - P\gamma_{ij})\tilde{A}^{ij} - S_i D^i\alpha + \alpha\int d\varepsilon S^\mu_{(\varepsilon)}n_\mu\right]
\end{aligned}
$$

$$\partial_t(\rho_* Y_e) + \partial_i(\rho_* Y_e v^i) = \sqrt{\gamma}\alpha m_u\int\frac{d\varepsilon}{\varepsilon}(S^\mu_{(\nu_e,\varepsilon)} - S^\mu_{(\bar{\nu}_e,\varepsilon)})u_\mu,$$

Neutrino Radiation equations (12x20 variables)

$$\partial_t\sqrt{\gamma}E_{(\varepsilon)} + \partial_i\sqrt{\gamma}(\alpha F^i_{(\varepsilon)} - \beta^i E_{(\varepsilon)}) + \sqrt{\gamma}\alpha\partial_\varepsilon\left(\varepsilon\tilde{M}^\mu_{(\varepsilon)}n_\mu\right) = \sqrt{\gamma}(\alpha P^{ij}_{(\varepsilon)}K_{ij} - F^i_{(\varepsilon)}\partial_i\alpha - \alpha S^\mu_{(\varepsilon)}n_\mu$$

$$\partial_t\sqrt{\gamma}F_{(\varepsilon)i} + \partial_j\sqrt{\gamma}(\alpha P_{(\varepsilon)}{}^j_i - \beta^j F_{(\varepsilon)i}) - \sqrt{\gamma}\alpha\partial_\varepsilon\left(\varepsilon\tilde{M}^\mu_{(\varepsilon)}\gamma_{i\mu}\right) = \sqrt{\gamma}[-E_{(\varepsilon)}\partial_i\alpha + F_{(\varepsilon)j}\partial_i\beta^j + (\alpha/2)P^{jk}_{(\varepsilon)}\partial_i\gamma_{jk} + \alpha S^\mu_{(\varepsilon)}\gamma_{i\mu}]$$

Numerical Method

Explicit updating part　　　Implicit updating part

$$\partial_t \mathbf{Q} + \boxed{\mathbf{S}_{\mathrm{adv,s}}} + \boxed{\mathbf{S}_{\mathrm{adv,e}}} + \boxed{\mathbf{S}_{\mathrm{grv}}} + \boxed{\mathbf{S}_{\nu\mathrm{m}}} = 0,$$

$$\mathbf{Q} = \begin{bmatrix} \rho_* \\ \sqrt{\gamma} S_i \\ \sqrt{\gamma}\tau \\ \rho_* Y_e \\ \sqrt{\gamma} E_{(\nu,\varepsilon)} \\ \sqrt{\gamma} F_{(\nu,\varepsilon)_i} \end{bmatrix}$$

Explicit updating part

$$\frac{\mathbf{Q}^* - \mathbf{Q}^n}{\Delta t} + {\mathbf{S}_{\mathrm{adv,s}}}^n + {\mathbf{S}_{\mathrm{grv}}}^n = 0$$

Implicit updating part

$$\frac{\mathbf{Q}^{n+1} - \mathbf{Q}^*}{\Delta t} + {\mathbf{S}_{\mathrm{adv,e}}}^{n+1} + {\mathbf{S}_{\nu\mathrm{m}}}^{n+1} = 0$$

Numerical Method

G ==> geometrical variables
Q ==> conservative variables
P ==> primitive variables

$$\mathbf{G}^n, \mathbf{Q}^n \;\&\; \mathbf{P}^n(\mathbf{G}^n, \mathbf{Q}^n)$$

↓ Geometrical updating part

$$\mathbf{G}^{n+1}, \mathbf{Q}^n \;\&\; \mathbf{P}^n(\mathbf{G}^{n+1}, \mathbf{Q}^n)$$

↓ Explicit updating part, Eq.(32)

$$\mathbf{G}^{n+1}, \mathbf{Q}^* \;\&\; \mathbf{P}^*(\mathbf{G}^{n+1}, \mathbf{Q}^*)$$

$$\mathbf{f}(\mathbf{P}^{n+1}) \equiv \frac{\mathbf{Q}(\mathbf{P}^{n+1}) - \mathbf{Q}^*}{\Delta t} + \mathbf{S}_{\mathrm{adv,e}}(\mathbf{P}^{n+1}) + \mathbf{S}_{\nu\mathrm{m}}(\mathbf{P}^{n+1}) = 0$$

$$\mathbf{P} = \begin{bmatrix} \rho \\ u_i \\ s \\ Y_e \\ E_{(\nu,\varepsilon)} \\ F_{(\nu,\varepsilon)_i} \end{bmatrix}$$

→ (6+12x20) variables

↓

$$\mathbf{G}^{n+1}, \mathbf{Q}^{n+1} \;\&\; \mathbf{P}^{n+1}(\mathbf{G}^{n+1}, \mathbf{Q}^{n+1})$$

Numerical Method

Newton法を用いて以下の式を解く

$$\mathbf{f}(\mathbf{P}^{n+1}) \equiv \frac{\mathbf{Q}(\mathbf{P}^{n+1}) - \mathbf{Q}^{*}}{\Delta t} + \mathbf{S}_{\mathrm{adv,e}}(\mathbf{P}^{n+1}) + \mathbf{S}_{\nu\mathrm{m}}(\mathbf{P}^{n+1}) = 0$$

実際の我々の解き方　$\mathbf{f}(\mathbf{P}^{n+1}) = 0$

do for i=0,1,2,3,,,,,

$$\boxed{\frac{\partial \mathbf{f}(\mathbf{P}^{I})}{\partial \mathbf{P}} \delta \mathbf{P}^{I} = -\mathbf{f}(\mathbf{P}^{I})}$$

$$\mathbf{P}^{I+1} = \mathbf{P}^{I} + \delta \mathbf{P}^{I},$$

until $\dfrac{|\delta \mathbf{P}^{I}|}{|\mathbf{P}^{I}|} < tol$

Numerical Method

$$\frac{\partial \mathbf{f}(\mathbf{P}^I)}{\partial \mathbf{P}}\delta\mathbf{P}^I = -\mathbf{f}(\mathbf{P}^I)$$

$$\mathbf{P} = \begin{bmatrix} \rho \\ u_i \\ s \\ Y_e \\ E_{(\nu,\varepsilon)} \\ F_{(\nu,\varepsilon)_i} \end{bmatrix}$$

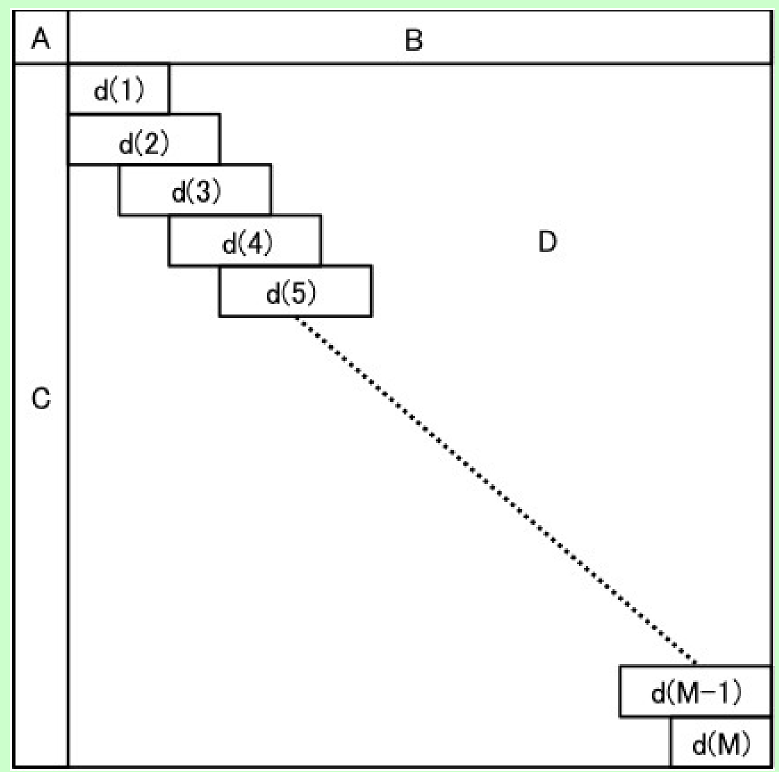
ρ → バリオン数は変化しない

matrix → A(5,5), B(5,240)

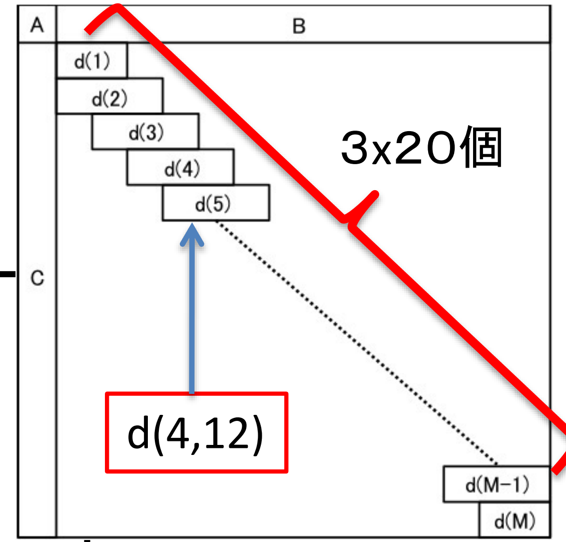
matrix → C(240,5), d(4,12)

(5+12x20) variables

Jacobian $\frac{\partial \mathbf{f}(\mathbf{P}^I)}{\partial \mathbf{P}}$ の形



$$\frac{\partial \mathbf{f}(\mathbf{P}^I)}{\partial \mathbf{P}}\delta \mathbf{P}^I = -\mathbf{f}(\mathbf{P}^I)$$



元の方程式

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix}$$

の列と行を入れ替えた方程式

$$\begin{pmatrix} D & C \\ B & A \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} g \\ f \end{pmatrix}$$

に対してGaussの消去法を適用するようにして解く方法があります。

1. $C' = D^{-1}C, g' = D^{-1}g$ → 反復法
2. $f' = f - Bg', S = A - BC'$
3. $x = S^{-1}f', y = g' - C'x$

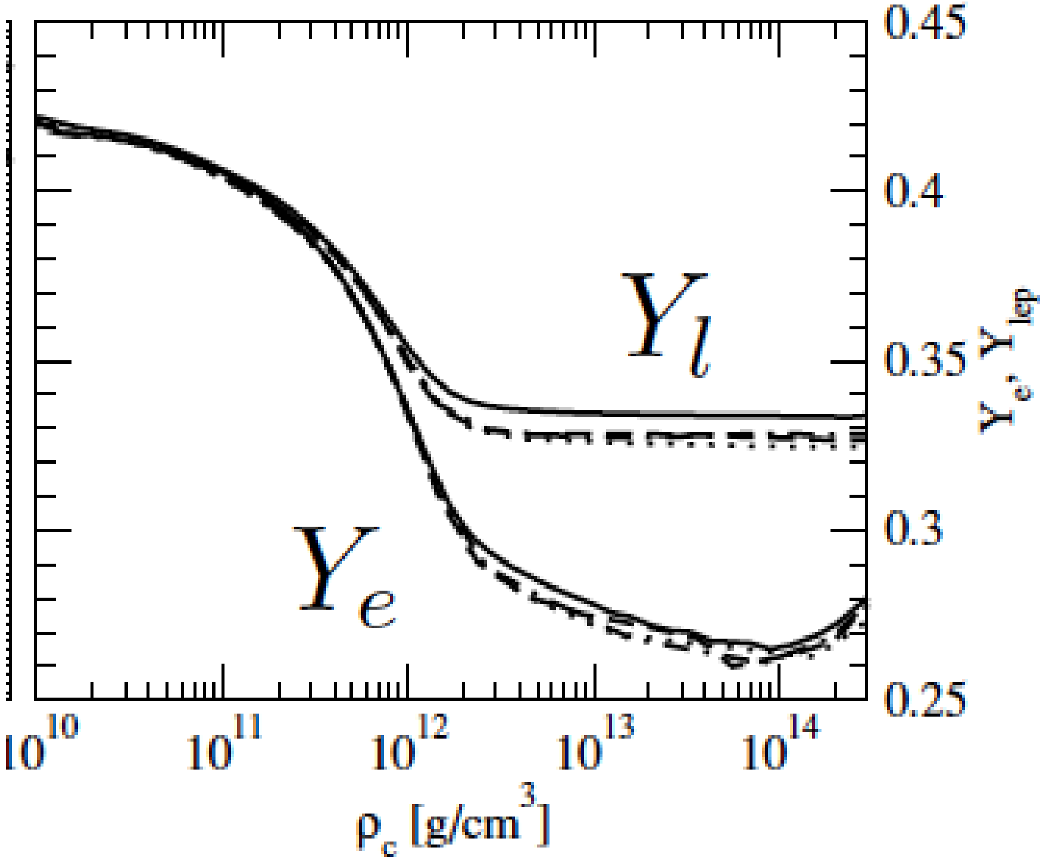
2.–3. 直接法

通常Step 1のDの方程式は反復法で解き、
一方Step 3の方程式は小規模なので直接法で解きます

筑波大学　今倉氏提供

About Lepton Number Conservation

超新星計算においてレプトン数保存は非常に重要



~10^{12}g/cc 以降でニュートリノは流体にトラップされる(Sato, '75)

Y_lはコア質量を決める重要な因子

About Lepton Number Conservation

$$\partial_t\left(\frac{\rho_* Y_e}{m_{\rm u}}\right) + \partial_i\left(\frac{\rho_* Y_e v^i}{m_{\rm u}}\right) = \sqrt{\gamma}\alpha \int \frac{d\varepsilon}{\varepsilon}\left(S^{\mu}_{(\nu_e,\varepsilon)} - S^{\mu}_{(\bar{\nu}_e,\varepsilon)}\right)u_{\mu}$$

$$\partial_t(q^0_{(\varepsilon)}\sqrt{\gamma}\alpha) + \partial_i(q^i_{(\varepsilon)}\sqrt{\gamma}\alpha) - \sqrt{\gamma}\alpha\partial_\varepsilon\left(\varepsilon q^{\alpha\beta}_{(\varepsilon)}\right)\nabla_\beta u_\alpha = -\frac{\sqrt{\gamma}\alpha}{\varepsilon} S^{\mu}_{(\varepsilon)} u_{\mu},$$

$$q^{\alpha}_{(\varepsilon)} \equiv (\mathcal{J}_{(\varepsilon)} u^{\alpha} + \mathcal{H}^{\alpha}_{(\varepsilon)})$$

$$q^{\alpha\beta}_{(\varepsilon)} \equiv (\mathcal{H}^{\alpha}_{(\varepsilon)} u^{\beta} + \mathcal{L}^{\alpha\beta}_{(\varepsilon)})$$

where $(\mathcal{J}, \mathcal{H}^{\alpha}, \mathcal{L}^{\alpha\beta}) \equiv \varepsilon^{-1}(J, H^{\alpha}, L^{\alpha\beta})$

$$\partial_t(\rho_* Y_l) + \partial_i\left(\rho_* Y_e v^i + m_{\rm u}\sqrt{\gamma}\alpha \int d\varepsilon\left[q^i_{(\nu_e,\varepsilon)} - q^i_{(\bar{\nu}_e,\varepsilon)}\right]\right) = 0.$$

$$\begin{aligned} Y_l &= Y_e + Y_{\nu_e} - Y_{\bar{\nu}_e} \\ &= Y_e + \frac{m_{\rm u}}{\rho u^t}\int d\varepsilon(q^0_{(\nu_e,\varepsilon)} - q^0_{(\bar{\nu}_e,\varepsilon)}) \end{aligned}$$

About Lepton Number Conservation

実際の我々の基礎方程式が粒子数保存を満たしているか？
特に拡散領域で。

$$\partial_t\left(\frac{\rho_* Y_e}{m_u}\right)+\partial_i\left(\frac{\rho_* Y_e v^i}{m_u}\right) = \sqrt{\gamma}\alpha\int\frac{d\varepsilon}{\varepsilon}(S^\mu_{(\nu_e,\varepsilon)}-S^\mu_{(\bar{\nu}_e,\varepsilon)})u_\mu$$

$$\boxed{\partial_t\sqrt{\gamma}E_{(\varepsilon)}+\partial_i\sqrt{\gamma}(\alpha F^i_{(\varepsilon)}-\beta^i E_{(\varepsilon)})+\sqrt{\gamma}\alpha\partial_\varepsilon(\varepsilon\tilde{M}^\mu_{(\varepsilon)}n_\mu) = -\sqrt{\gamma}\alpha S^\mu_{(\varepsilon)}n_\mu}$$

In Newtonian & diffusion $\boxed{F^i\sim\frac{4}{3}Ev^i}$ approximation

この関係とエネルギーシフトが重要

$$\begin{aligned}\partial_t E+\partial_i F^i+\partial_\varepsilon(-\varepsilon P^{ij}\partial_i v_j) &\sim \partial_t E+\partial_i\left(\frac{4}{3}Ev^i\right)-\frac{E}{3}\delta^{ij}\partial_i v_j\\ &\sim \partial_t E+\partial_i\left(\frac{4}{3}Ev^i\right)-\partial_i\left(\frac{1}{3}Ev^i\right)\\ &\sim \partial_t E+\partial_i E v^i.\end{aligned}$$

$q^0\sim\mathcal{J}\sim\mathcal{E}-2\mathcal{F}^i v_i\sim\mathcal{E}\sim E/\varepsilon$

各エネルギービンのニュートリノ数は流体の速度で運ばれる

先行研究との比較

progenitor : “s15s7b2” in Woosley & Weaver 1995

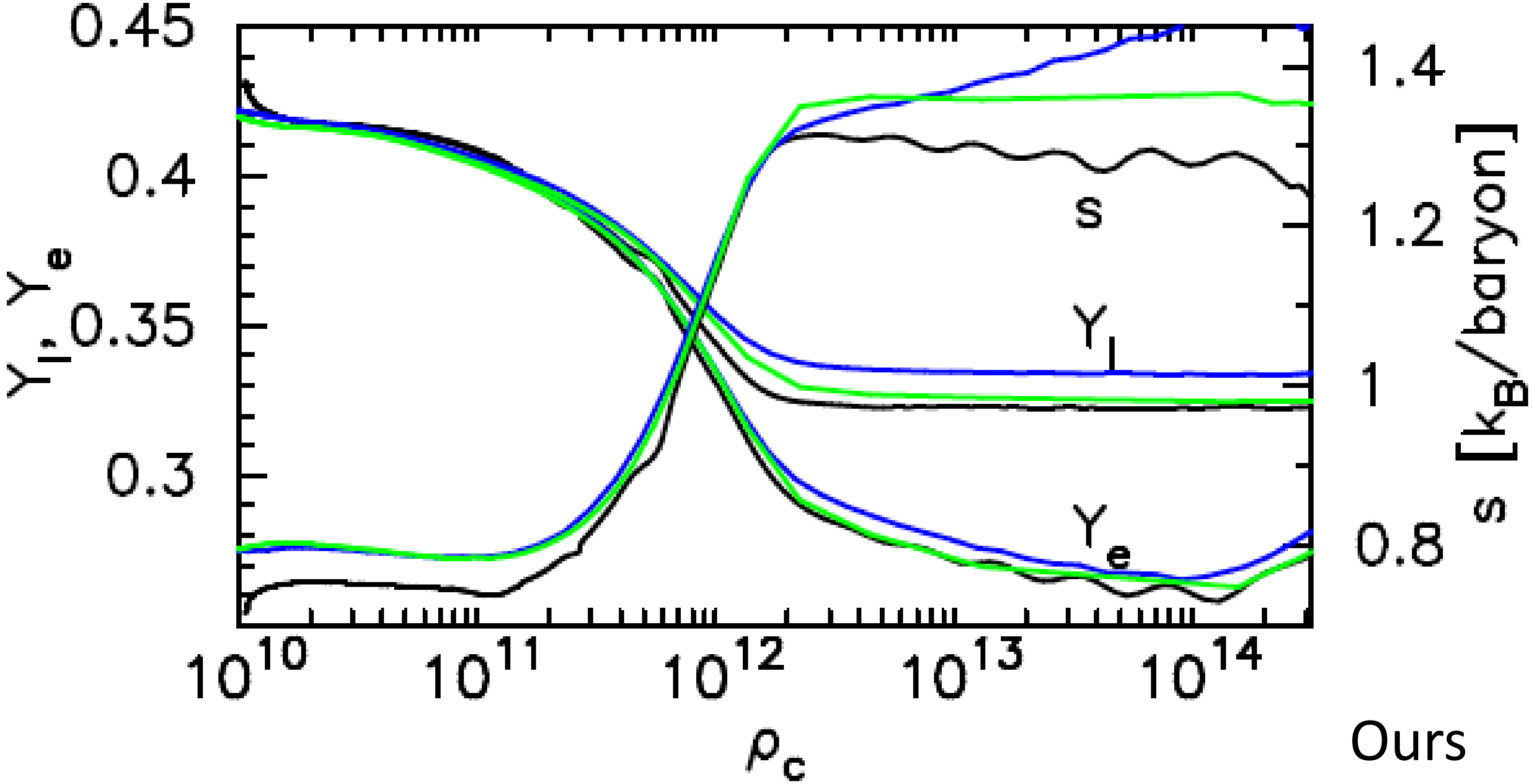
EOS : Lattimer&Swesty,’91 (K=220MeV)

３次元直交座標系を用いた

Process	reference
$n\nu_e \leftrightarrow e^- p$	Bruenn (1985); Rampp & Janka (2002b)
$p\bar{\nu}_e \leftrightarrow e^+ n$	Bruenn (1985); Rampp & Janka (2002b)
$\nu_e A \leftrightarrow e^- A'$	Bruenn (1985); Rampp & Janka (2002b)
$\nu p \leftrightarrow \nu p$	Bruenn (1985); Rampp & Janka (2002b)
$\nu n \leftrightarrow \nu n$	Bruenn (1985); Rampp & Janka (2002b)
$\nu A \leftrightarrow \nu A$	Bruenn (1985); Rampp & Janka (2002b)
$\nu e^{\pm} \leftrightarrow \nu e^{\pm}$	Bruenn (1985)
$e^- e^+ \leftrightarrow \nu\bar{\nu}$	Bruenn (1985)
$NN \leftrightarrow \nu\bar{\nu}NN$	Hannestad & Raffelt (1998)

先行研究との比較

(1) Central evolution during collapse phase

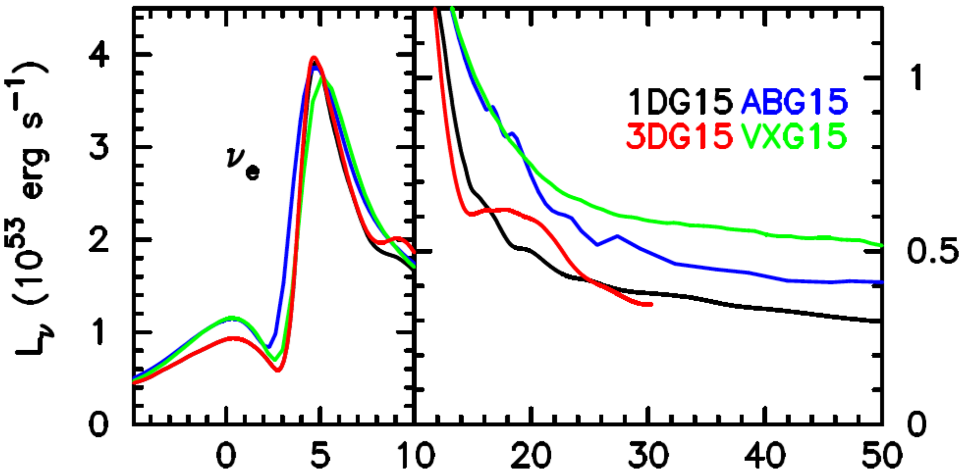


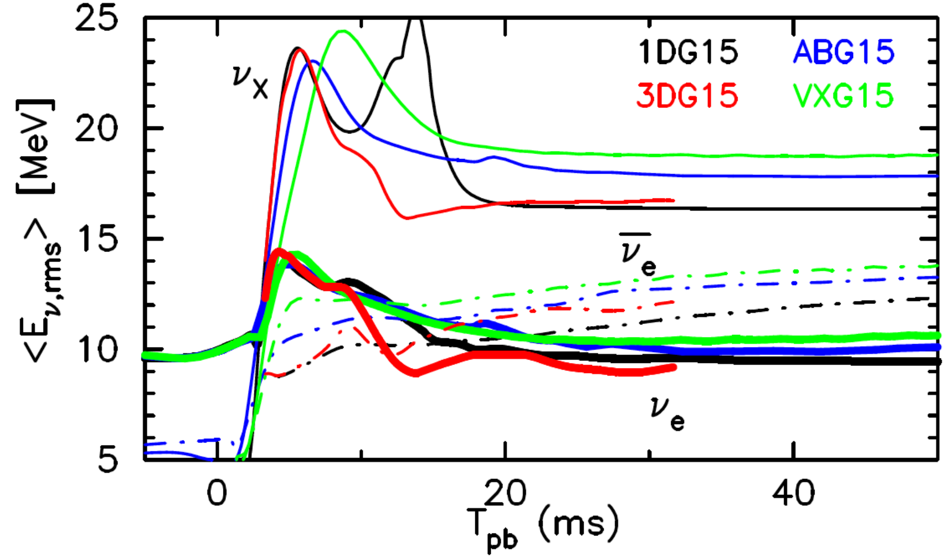
Ours
Agile-Boltztran
Vertex

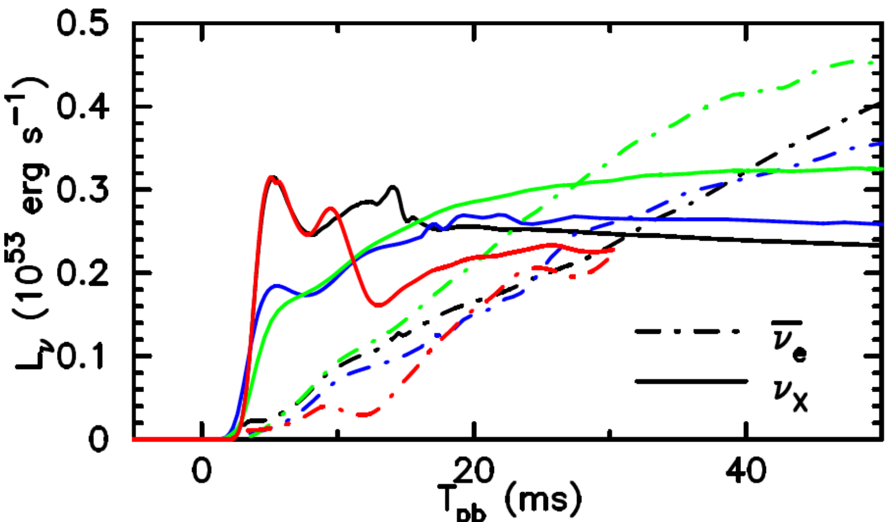
Our momentum scheme correctly captures central evolution

先行研究との比較

(2) Neutrino profiles







Neutrino luminosities and energies show consistent results with AGILE-Boltztran (KT+, in prep.)

Toward Full 3DGR Multi-Energy ν-Transport Models

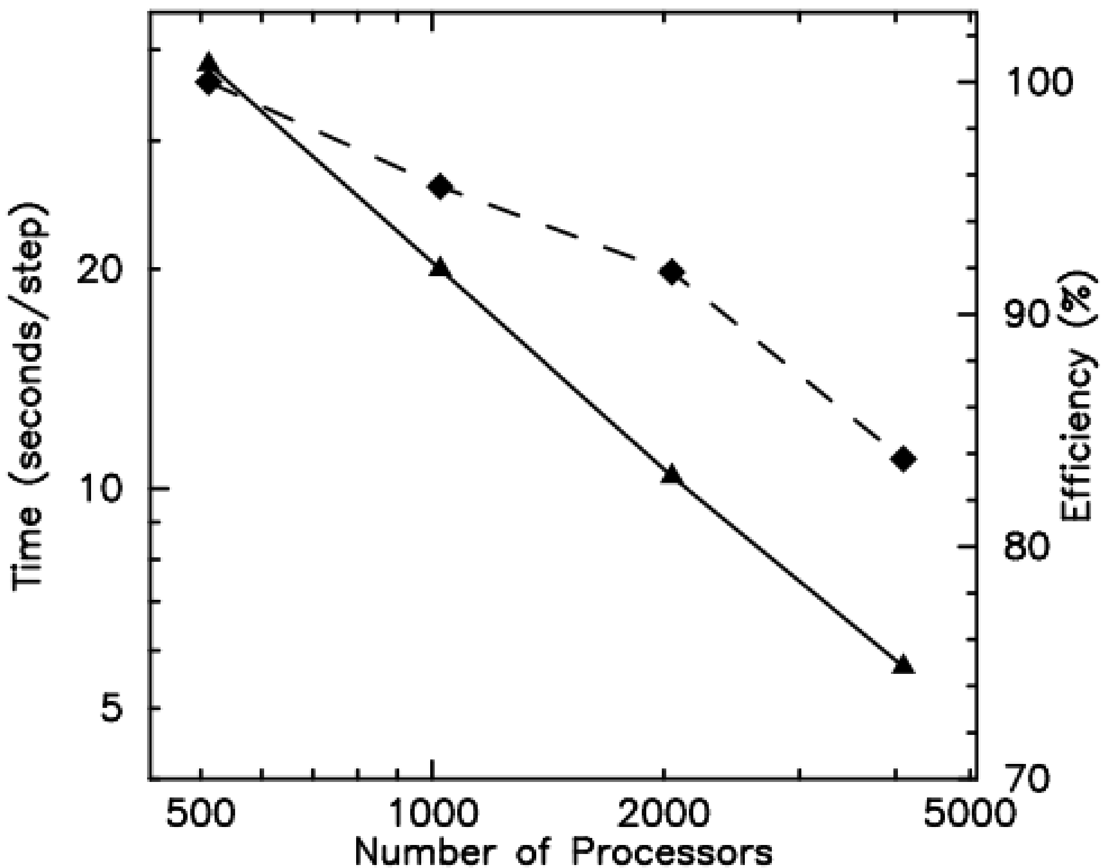
Garching group との比較

	Ours	Mueller+,'14
Closure relation	M1 analytical	Variable Eddington
Dimension	3D (cartesian)	2D (spherical)
GR (metric solver)	BSSN	CFC approximation

まとめ

- 3D Full GR, Multi-energy neutrino hydro codeを開発
  - 各保存量(エネルギー、モーメンタム、レプトン数)を満たす形式
  - 陽解法、陰解法の混在
  - 陰解法に置ける行列反転も高速化の工夫
- 先行研究(1D GR Boltzmann)と無矛盾な結果を得た

計算効率について



•Implicit part は各数値メッシュで閉じているので、並列化の効率は比較的良い
•チューニング後のxcでの実行効率は約32%, 260Gflops